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Application of Deep Learning for Solving Partial Differential Equations

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Motivation

- We explore the use of neural networks from deep learning for solving several PDEs, from 1D transient advection equation to 2D transient advection-diffusion equation.
- Neural Networks can provide certain advantages over traditional numerical methods[1].
- They may be less dependent on grid-size and may not subject to either numerical diffusion or oscillations. If generalizable, this simple yet powerful construction could lead to a new class of PDE solvers.

Neural Network (NN) Architecture

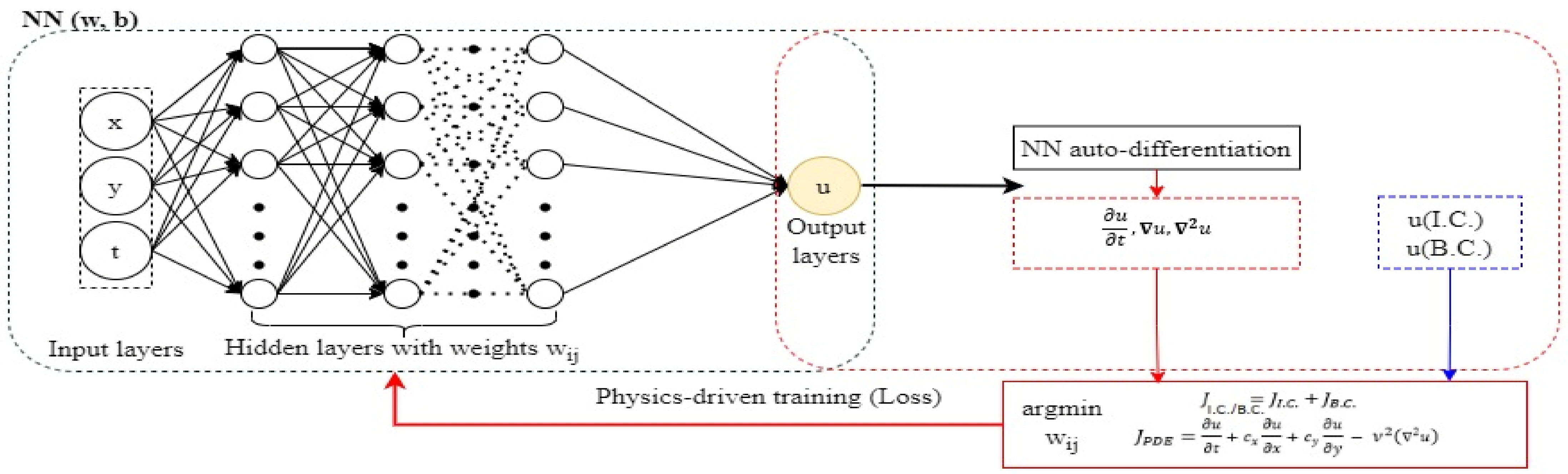

Fig. 1 Physics-Informed NN architecture with 8 hidden layers and 20 neurons per layer

Partial Differential Equations

- 1D Transient Advection

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

- 2D Steady State Diffusion

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

- 2D Transient Advection-Diffusion

$$\frac{\partial u}{\partial t} + c\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) = \nu^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Results and Discussion

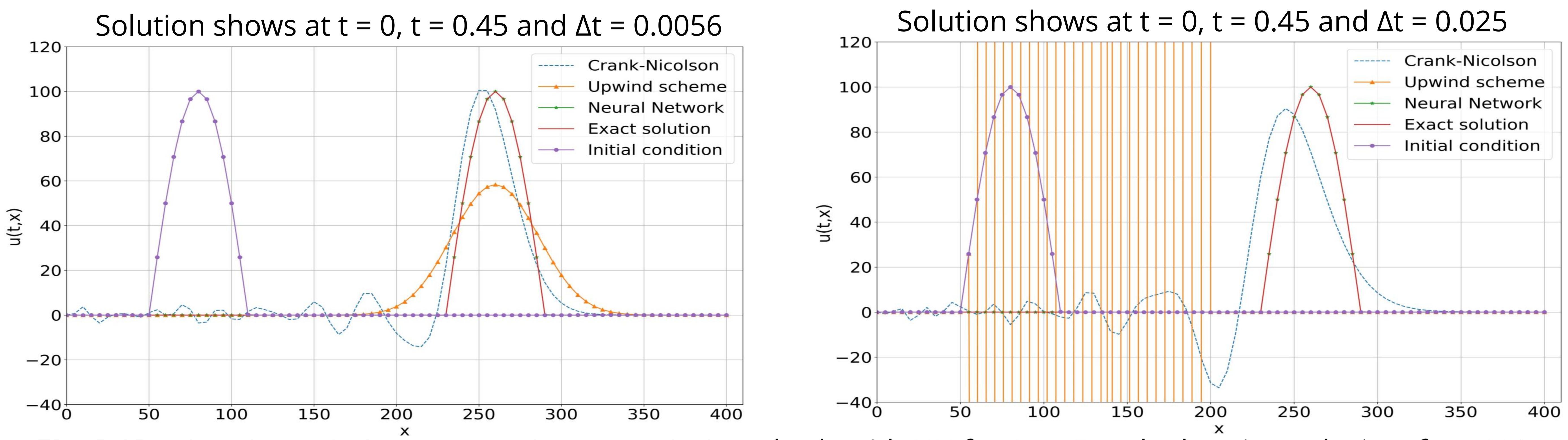

Fig. 2 1D advection solutions comparing numerical methods with NN for Δx = 5 and advection velocity of c = 400

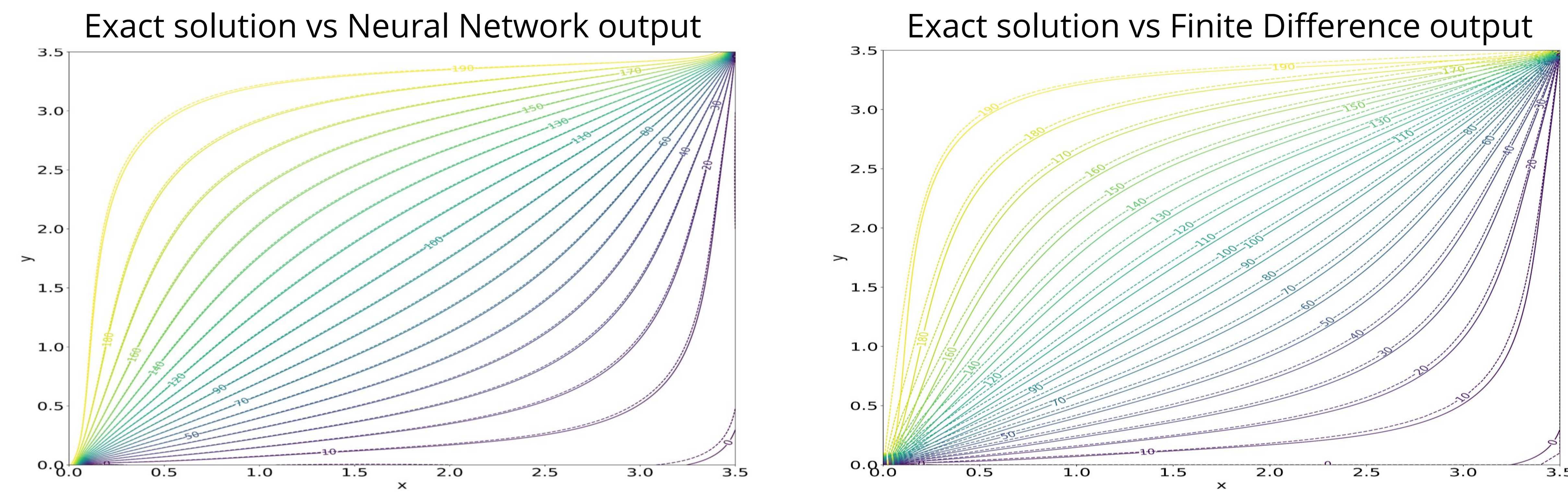

Fig. 3 2D steady state temperature contours in a square domain with Δx=Δy= 0.1

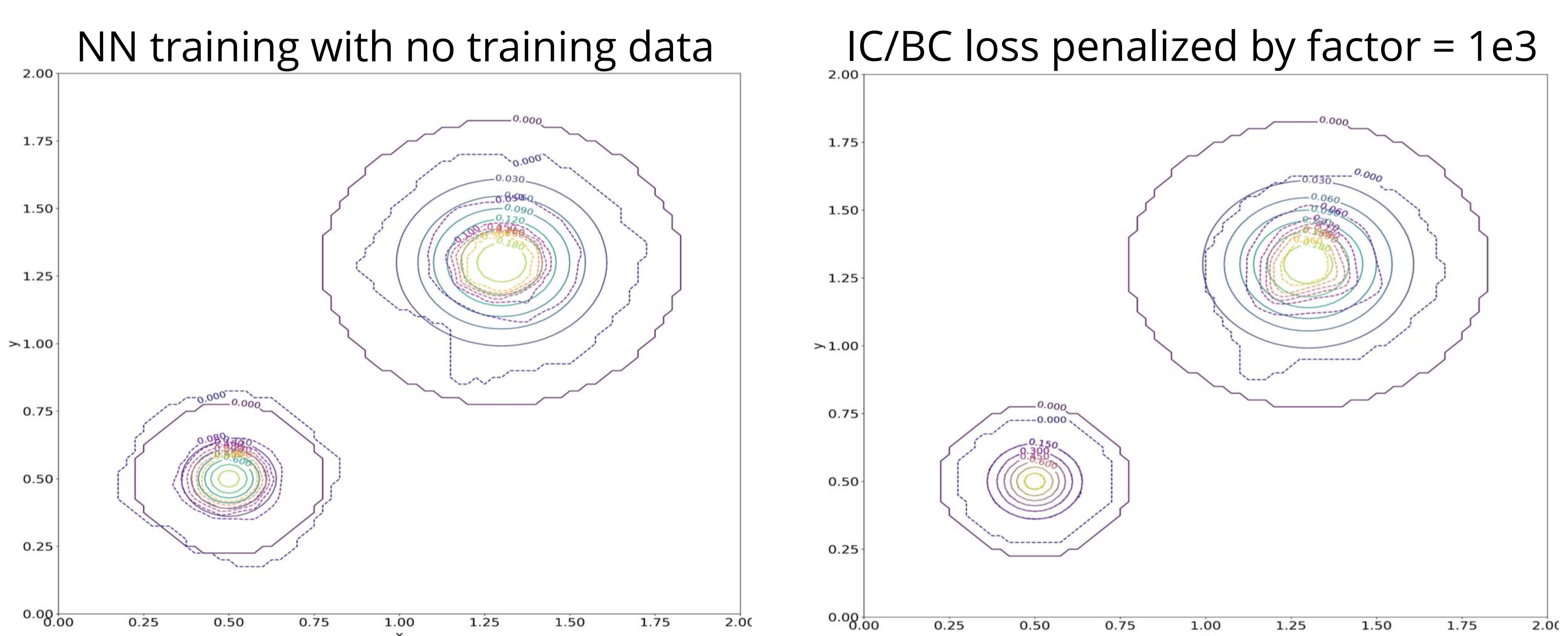

Fig. 4 2D advection diffusion contours at t = 0, t = 1s showing propagation of Gaussian pulse with advection velocity, c = 0.8. Δx = Δy = 0.025 and Δt = 0.01

- Current NN predicts a solution that satisfies IC/BC and PDE simultaneously:

$$u = u_{NN}(X, w, b)$$

- But the current architecture is struggling to find a solution in higher dimensions.
- We are working on constructing NN that only solves the PDE, while ICs/BCs are hard-coded in the network[2]:

$$u = u_{IC} + u_{BC} + \hat{u}_{NN}$$

where, $\hat{u}_{NN}$ is the solution of Neural Network without accounting for initial/boundary conditions.

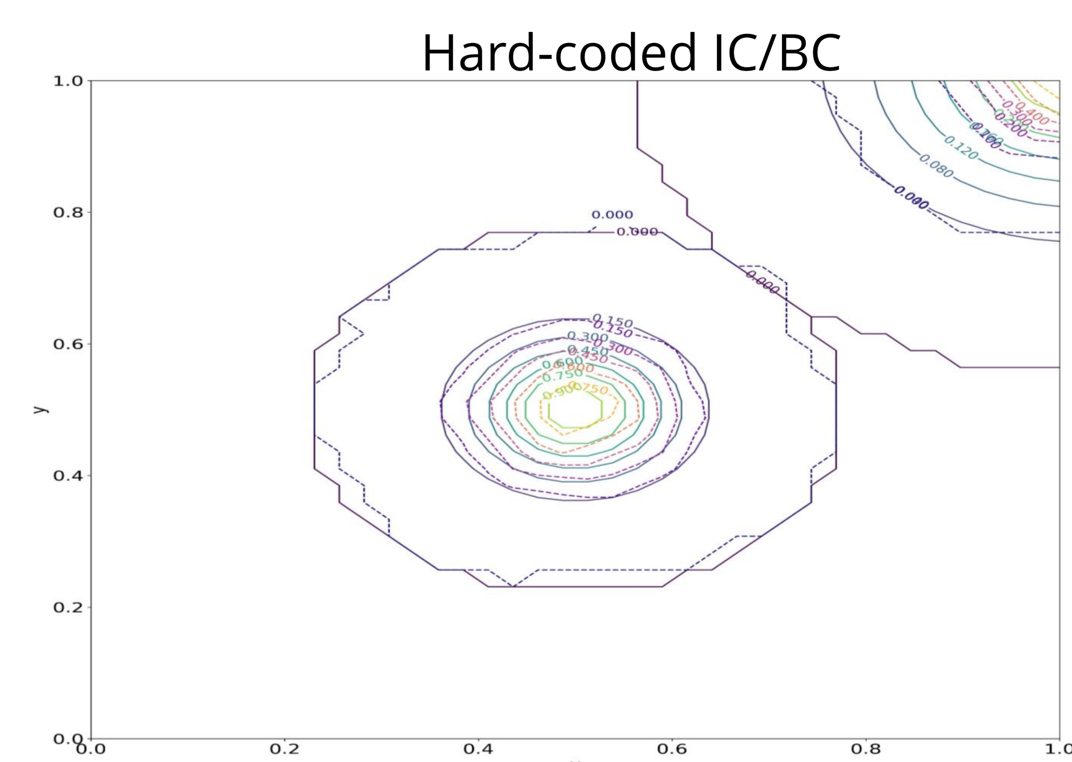

Fig. 5 2D advection-diffusion contours at t = 0, t = 1 in a 1x1 domain

Summary and Conclusions

- For lower dimensional PDEs, NN is able to provide fully-predictive solutions that are better than traditional numerical methods.
- For higher dimensional PDEs, we are still working on exploring potential advantages of NN-based PDE solvers.

Acknowledgements

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References: 1. Raissi, Maziar, et al. arXiv preprint arXiv:1711.10561 (2017) **2.** Lagaris, et al. IEEE transactions on neural networks 9.5 (1998)