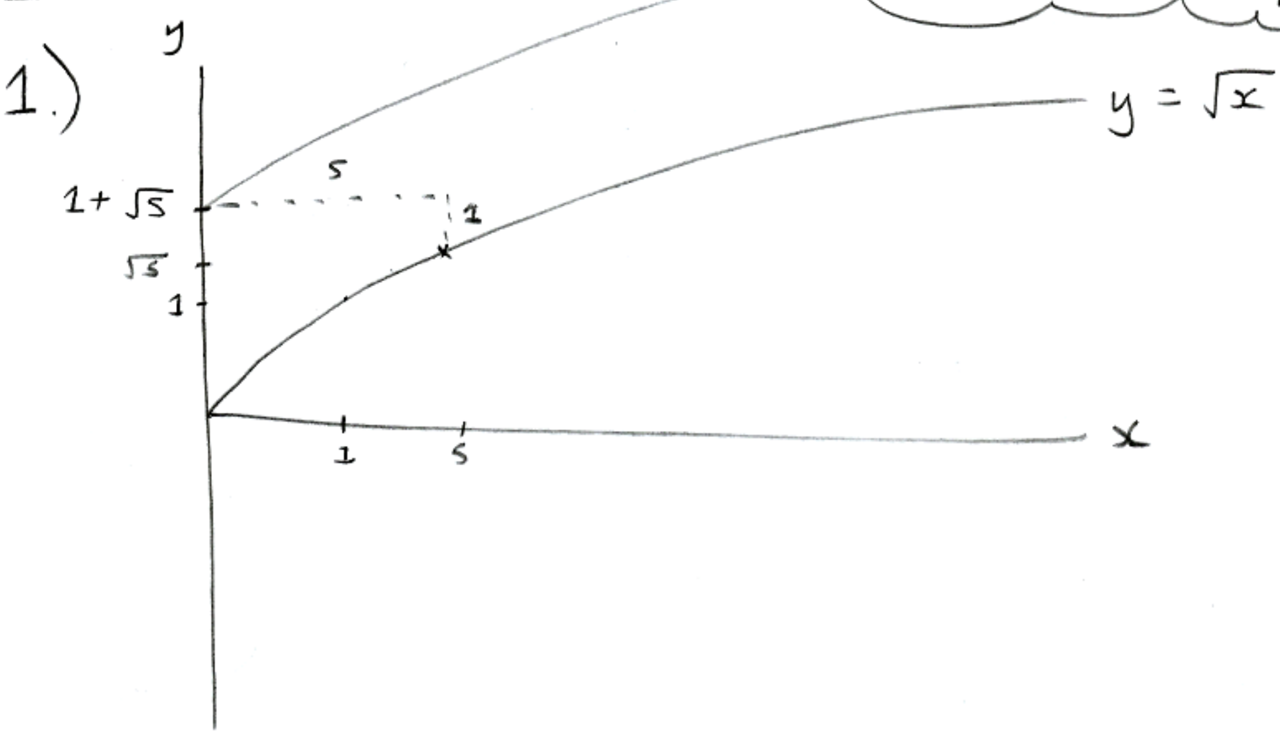
Practice AS-Level Mathematics Test

JDGM
18/XI/'17

1.)



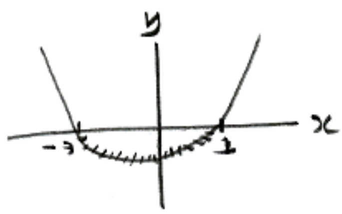
$y \rightarrow y-1$ is a shift by $+1$ in the y-direction.

$x \rightarrow x+5$ is a shift by -5 in the x-direction.

The vector is $\boxed{\begin{pmatrix} -5 \\ 1 \end{pmatrix}}$.

2.) $7 - x^2 - 2x > 4 \Rightarrow 0 > x^2 + 2x - 3$

$x^2 + 2x - 3 = (x+3)(x-1)$



$\therefore$ $\left.\begin{array}{l} x > -3 \\ x < 1 \end{array}\right\}$ $\boxed{-3 < x < 1}$

3.) i.) $\sqrt{3} = 3^{1/2} \quad \therefore \quad \boxed{p = 1/2}$

ii.) $1/9 = 1/3^2 = 3^{-2} \quad \therefore \quad \boxed{q = -2}$

iii.) $\sqrt{3} \times 3^x = 3^{x+1/2} = 1/9 = 3^{-2}$

$\therefore \quad x + 1/2 = -2$

$\boxed{x = -2\tfrac{1}{2}}$

4.) $$\frac{5\sqrt{2}+2}{3\sqrt{2}+4} = \frac{(5\sqrt{2}+2)(3\sqrt{2}-4)}{(3\sqrt{2}+4)(3\sqrt{2}-4)}$$

$$= \frac{15\cdot 2 - 20\sqrt{2} + 6\sqrt{2} - 8}{9\cdot 2 - 16} = \frac{(30-8)+(6-20)\sqrt{2}}{18-16}$$

$$= \frac{22 - 14\sqrt{2}}{2} = \boxed{11 - 7\sqrt{2}}$$

5.) $$(2+3x)^6 = \ldots + \underbrace{\binom{6}{3} 2^3 (3x)^3}_{} + \ldots$$

$\Rightarrow 20 \cdot 8 \cdot 27 \cdot x^3 = 4320x^3$

$\boxed{\text{Answer} = 4320}$

6) i) $y - -5 = 2(x - 4)$

$y + 5 = 2x - 8$

$\boxed{y = 2x - 13}$

ii) $3x - y = 4 \Rightarrow y = 3x - 4$

$m_1 m_2 = -1$

$3m_2 = -1$

$m_2 = {}^{-1}/_3$

$l_2: \quad y - 0 = {}^{-1}/_3 (x - 3)$

$\boxed{y = {}^{-1}/_3 x + 1}$

iii) Solve simultaneously.

$2x - 13 = {}^{-1}/_3 x + 1$

$6x - 39 = -x + 3$

$7x = 42$

$x = 6$

$y = {}^{-1}/_3 6 + 1 = -2 + 1 = -1$

l_1 intersects l_2 at $\boxed{(6, -1)}$

7.) i.) $f(x) = x^{3/2} - 8x^{-1/2}$

$$f(3) = 3^{3/2} - 8(3^{-1/2})$$

$$= 3\sqrt{3} - 8\left(\frac{1}{\sqrt{3}}\right)$$

$$= 3\sqrt{3} - 8\frac{\sqrt{3}}{3}$$

$$= \left(3 - \frac{8}{3}\right)\sqrt{3}$$

$$= \frac{9-8}{3}\sqrt{3} = \frac{1}{3}\sqrt{3} = \boxed{\frac{\sqrt{3}}{3}}$$

ii.) $f(x) = 0 = x^{3/2} - 8x^{-1/2}$

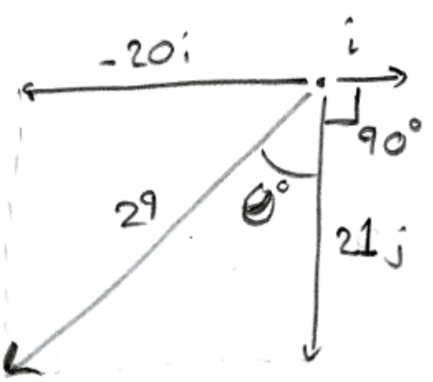
$$\Rightarrow x^{3/2} = 8x^{-1/2} \qquad (\times x^{1/2})$$

$$x^2 = 8$$

$$\boxed{x = \pm 2\sqrt{2}}$$

8.) i.) $\sqrt{20^2 + 21^2} = \sqrt{841} = \boxed{29}$

ii.)



$\theta° = \cos^{-1}\left(\frac{21}{29}\right) = 43.6°$

Angle between i and $-20i + 21j$ is $\boxed{133.6°}$

9.) i.) $t_2 = kt_1 - 7 = \boxed{3k - 7}$

$t_3 = kt_2 - 7 = k(3k-7) - 7$

$= \boxed{3k^2 - 7k - 7}$

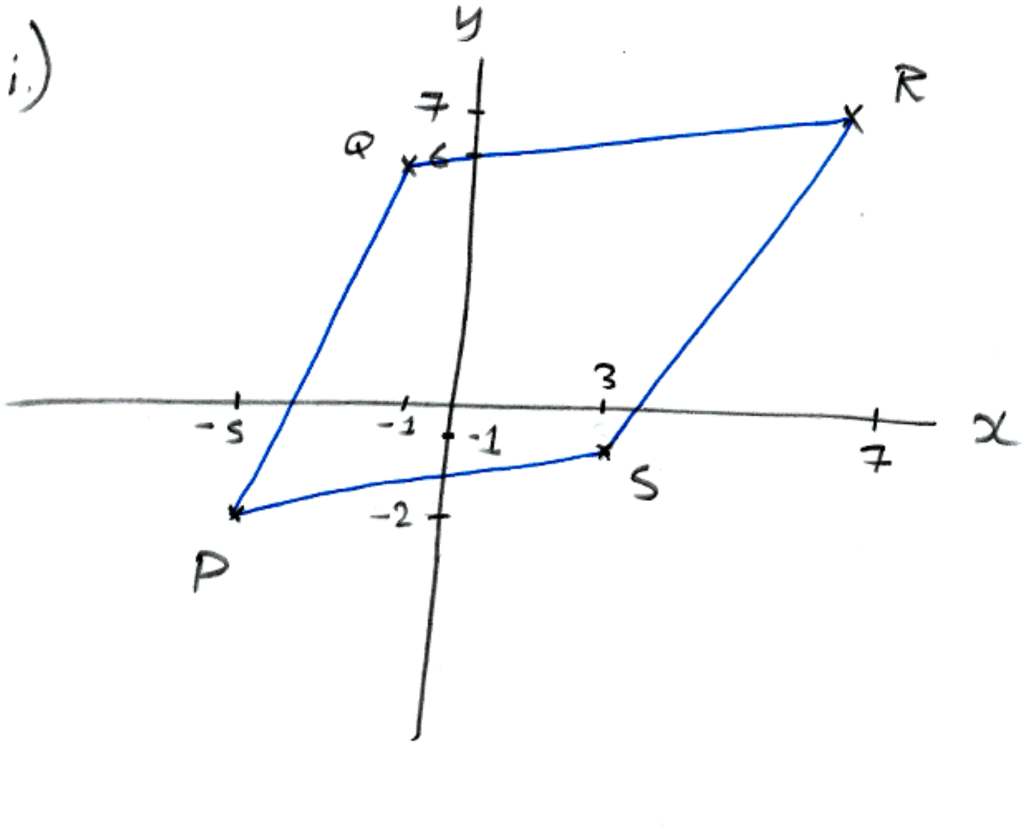
ii.) $t_3 = 3k^2 - 7k - 7 = 13$

$3k^2 - 7k - 20 = 0$

$(3k+5)(k-4) = 0$

$\boxed{k = 4 \text{ or } -5/3}$

10.) i.)



ii.) $PQ = \sqrt{(-5--1)^2 + (-2-6)^2} = \sqrt{(-4)^2 + (-8)^2}$

$= \sqrt{16 + 64} = \sqrt{80} = \sqrt{16}\sqrt{5}$

$= \boxed{4\sqrt{5}}$

iii.) $M = \left(\frac{1}{2}(-5 + -1), \frac{1}{2}(-2+6)\right)$

$= \left(\frac{1}{2}(-6), \frac{1}{2}(4)\right) = \boxed{(-3, 2)}$

iv.) $\overrightarrow{MS} = 6i - 3j$ $\quad (3--3)i + (-1-2)j$

$\overrightarrow{PQ} = 4i + 8j$

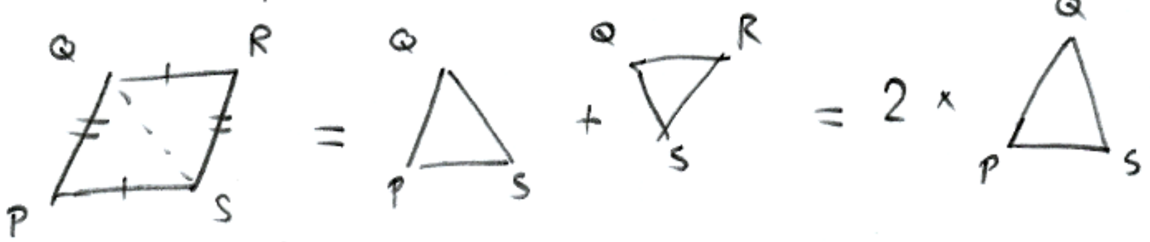
$m_1 = {}^{-3}/_{6} = {}^{-1}/_{2}$

$m_2 = {}^{8}/_{4} = 2$

$m_1 m_2 = -{}^{1}/_{2} \times 2 = 1$

$\therefore$ Perpendicular

iv.)



$|PQ| = 4\sqrt{5}$

$|MS| = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$\therefore$ Area PQS $= {}^{1}/_{2} \cdot 4\sqrt{5} \cdot 3\sqrt{5}$

$= 30$

$2\times$ Area PQS $= \boxed{60}$