

# Practice A-Level Mathematics Test

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**Time Allowed: 2 hours**

## Question 1

Differentiate  $y = 4x$  with respect to  $x$  from first principles.

(3 marks)

## Question 2

Prove that  $f(x) = \frac{2}{x}$  is a decreasing function. ( $x \in \mathbb{R}$ )

(3 marks)

## Question 3

Given that  $n$  is an integer, prove that 120 is a factor of  $n(n + 1)(n + 2)(n + 3)(n + 4)$ .

(4 marks)

## Question 4

Miles and Gwen are sometimes late for school.

$M$  = the event that Miles is late for school

$G$  = the event that Gwen is late for school

$P(M) = 0.25$ ,  $P(M \cap G) = 0.15$ , and  $P(M' \cap G') = 0.7$

On a randomly selected day, find the probability that

- i.) at least one of Miles or Gwen are late for school,
- ii.) Gwen is late for school

Given that Miles is late for school,

- iii.) find the probability that Gwen is late.

The teacher suspects that Miles being late for school and Gwen being late for school are linked in some way.

- iv.) determine whether or not  $M$  and  $G$  are statistically independent.
- v.) comment on the teacher's suspicion using your answer from iv.

(1+2+3+2+1 = 9 marks)

### **Question 5**

Sketch, on the same set of axes, the graphs of:

$$y = (x - 3)(x + 2) \text{ and } y = -x - 2$$

Find the area of the finite region between the graphs.

(3+6 = 9 marks)

### **Question 6**

A circle has equation  $(x - 2)^2 + (y + 2)^2 = 6$

Show that if the line  $y = mx$  does NOT intersect the curve then:

$$m^2 - 4m + 1 < 0$$

(6 marks)

### **Question 7**

An arithmetic series has eighteenth term 69 and thirty-fourth term 420.

Find the minimum number of terms for the sum of the series to exceed 2019.

(6 marks)

### **Question 8**

In this question  $g = 9.8\text{ms}^{-2}$ .

A dice is thrown vertically upwards from  $x$  metres above ground level.

Its initial speed is  $10\text{ms}^{-1}$ .

It hits the ground 2 seconds after being thrown.

Find  $x$ .

(4 marks)

### **Question 9**

A confectioner has a large number of sweets, all of which are either fudge or toffee.

The confectioner states that 75% of the sweets are fudge and 25% are toffee.

Jon selects 60 sweets at random and records the number of toffees.

i.) State, in context, two necessary assumptions for the binomial distribution to be applicable in this case.

ii.) Calculate the probability that 15 of Jon's sweets are toffee.

Kim selects 20 sweets at random.

iii.) Calculate the probability that at least 13 and at most 18 of the sweets are fudge.

(2+2+3 = 7 marks)

**Question 10**

You are given the equation  $4\cos^2x - 8\sin x = 7$ .

Show that this is equivalent to  $\sin x = -\frac{1}{2}$

(5 marks)

**Question 11**

For what values of  $x$  is the following curve concave?

$$y = 3x^5 - 130x^3 + 360x^2 + 10x + 7$$

(6 marks)

**Question 12**

A curve has equation  $y = \sqrt{x} + \frac{9}{x^2}$

for  $x > 0$

Find the stationary point(s) of the curve and state the nature (for each).

(7 marks)