CGP

Key Stage Three
Mathematics
Foundation Level

CGP

The Study Guide

It's another great book from CGP...

This book explains everything you'll need to get a solid grounding in KS3 Maths (ages 11-14) — with plenty of worked examples and practice questions to test your skills.

It's ideal if you're working at foundation level — it covers what would have been called Levels 3-6 in the old Curriculum.

CGP — still the best! ☺

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Then we work our socks off to get them out to you — at the cheapest possible prices.

Contents

Section 1 — Numbers

Section 2 — Algebra and Graphs

Section 3 — Ratio, Proportion & Rates of Change

Section 4 — Geometry and Measures

Section 5 — Probability and Statistics

Calculating Tips

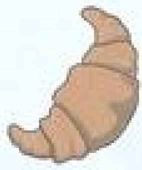
KS3 Maths — you're going to love it. There's plenty of learning to be done and plenty of fun to be had. But first, here are some tips that will help you on your way to maths glory.

Don't Be Scared of Wordy Questions

For the 'real-life' questions you've first got to work out what the question's asking you to do.

1) READ the question carefully.
   Think what bit of maths you might need to answer it.
2) Underline the BITS YOU NEED to answer the question — you might not have to use all the numbers they give you.
3) Write out the question IN MATHS and answer it, showing all your working clearly.

EXAMPLE: Ben's Bakery is branching out into mini pastries. All the mini pastries will weigh 50% as much as a normal pastry. If a croissant normally weighs 200 g, how much will a mini croissant weigh?



The "50%" tells you this is a percentages question (covered on page 21).
You need 200 g (the normal weight) and 50% (the percentage).
It doesn't matter what the shop is called or what the pastry is.
You want to work out 50% of 200 g, so:

50% of 200 g = 0.5 × 200 = 100 g

Don't forget the units in your final answer — this is a question about weight in grams, so the units will be g.

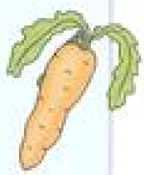
Working with Units

When you have a question involving units it's usually best to do the calculations without the units. Then you can add the units back in at the end — careful you don't forget about them.

Money questions always crop up — you'll have to keep an eye on the units and the decimal places.

EXAMPLE: Nicola buys a rare orange carrot for £30. After 2 months she sells the carrot for a 5% profit. How much profit does she earn on the carrot in a) pounds, and b) pence?

1) This percentages question is asking you to find 5% of £30. 0.05 × 30 = 1.5



2) The question uses pounds (£30), so 1.5 means 1.5 pounds.
   Answers in pounds should always be given to 2 decimal places.
   You might need to fill in the 2nd decimal place with a 0, like this: £1.50
3) Part b) asks for the answer in pence. £1 = 100p.
   So to change pounds into pence — times by 100: 1.50 × 100 = 150p

Wordy Questions? But this is maths, not English...

So pick out the bits you need from a question and keep an eye on those units. Try this one.

1) Caley's Clothing is having a sale — all clothing items are 50% of their original price.
   A shirt originally costs £80. What is the price of the shirt in the sale?

Calculating Tips

Here are a few tips to help you out with your calculations. They might seem a bit unusual and a little bit tricky, but learn them you must. They'll come in really handy later on.

BODMAS

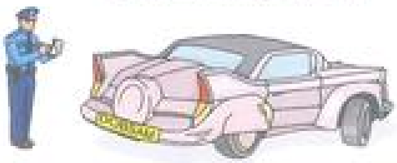
Brackets, Other, Division, Multiplication, Addition, Subtraction

BODMAS tells you the ORDER in which operations should be done:
Work out Brackets first, then Other things like squaring,
then Divide / Multiply groups of numbers before Adding or Subtracting them.

EXAMPLES:

1. Work out $4 + 6 \div 2$
   1) Follow BODMAS — do the division first... $4 + 6 \div 2 = 4 + 3$
   2) ...then the addition: $= 7$

If you don't follow the order of BODMAS, you get: $4 + 6 \div 2 = 10 \div 2 = 5$



2. Calculate $10 - 2^3$
   1) The cube is an 'other' so that's first: $10 - 2^3 = 10 - 8$
   2) Then do the subtraction: $= 2$

3. Find $(8 - 2) \times (3 + 4)$
   1) Start by working out the brackets: $(8 - 2) \times (3 + 4)$
   2) And now the multiplication: $= 6 \times 7$ $= 42$

Hidden Brackets in Fractions

This is a bit of a funny one — when you have a fraction with calculations on the top or bottom you have to imagine they're in brackets and do them first.

EXAMPLE: Work out $\frac{16}{6 + 6 \div 3}$.

1) Imagine the bottom of the fraction is in brackets. $\frac{16}{(6 + 6 \div 3)}$

2) Now follow BODMAS to do the calculation. $= \frac{16}{(6 + 2)} = \frac{16}{8} = 2$

EXAMPLE: Work out $\frac{20 \times 5}{4 + 2 \times 3}$.

1) Imagine the top and bottom are both in brackets. $\frac{(20 \times 5)}{(4 + 2 \times 3)}$

2) Now follow BODMAS to do the calculation. $= \frac{100}{(4 + 6)} = \frac{100}{10} = 10$

Before Operations Doctors Must Always Smile...

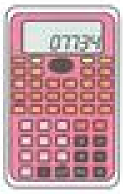
BODMAS is the important bit here — you'll have to learn what it stands for. Then try these:

1) Find the value of: a) $11 - 5 \times 2$ b) $3 \times 6 + 15 \div 5$ c) $(6 \times 3) \div 3^2$

2) Work out $\frac{10 \times 8}{4 \times 3 - 4}$ — remember hidden brackets and BODMAS.

Calculating Tips

Ever wondered what all those fancy buttons on your calculator do? Well you're about to find out. Careful though — reaching for your calculator isn't always the best option.



Calculators

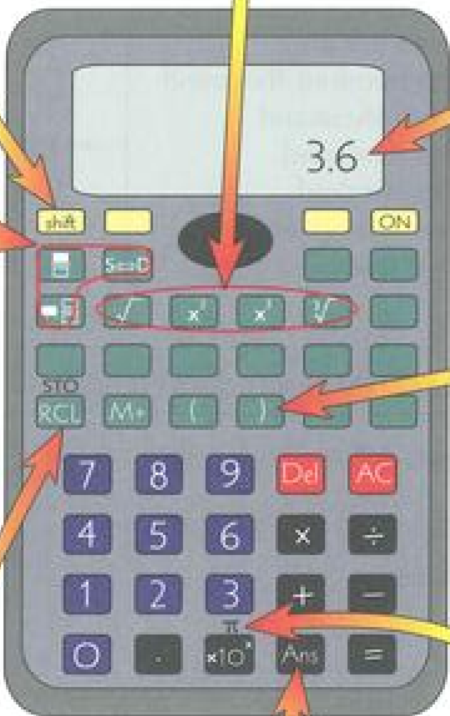
Make sure you know the important features on your calculator and how to use them.

SHIFT (OR 2ND FUNC)

Press this first if you want to use something written above a button (e.g. the pi (π) button).

SQUARE, CUBE AND ROOTS

E.g. 4 x^2 gives 4 squared = 16.
And $\sqrt[3]{\ }$ 27 gives the cube root of 27 = 3.



THE ANSWER

Before you jot down 3.6, think about what it means. E.g. in a money question, it might mean £3.60.

FRACTIONS

E.g. for $\frac{1}{4}$ press 1 ▭ 4.
(If you have a button that looks like $a\frac{b}{c}$ instead, use it in the same way.)
For $1\frac{3}{5}$ press 1 ▭ 3 ▭ 5 (you might have to press shift first).
To cancel down a fraction, enter it and press =.
Pressing the ▭ or S⇔D button also switches an answer between a fraction and a decimal.

BRACKETS

Calculators use BODMAS, so if there's part of a question you want the calculator to do first then put brackets in to tell it so.

MEMORY (STO, RCL & M+)

E.g. for $\dfrac{840}{12 \times 8}$:
Press 12 × 8 = and then STO M+ to store the bottom line in the memory.
Then press 840 ÷ RCL M+ =, and the answer is 8.75.

The 'Ans' button gives the number you got when you last pressed the '=' button.

PI (π) (See page 63)

The calculator stores the number for pi (= 3.141...). If it's above another button as shown here, press the shift button first.

With great power comes great responsibility — so here is some advice about using your calculator.

DON'T reach straight for your calculator. If you put a big calculation into your calculator all in one go you're quite likely to get the wrong answer.

DO show all your working — you're more likely to get the right answer by doing it in stages than doing it all in one go.

Got a favourite button? Mine's π for obvious reasons...

Your calculator might be a bit different to this one — so work out how to do the above on yours.

1) Using BODMAS work out $(11 - 9) \times (3 \times 2)^2$. Then try it on your calculator.

Ordering Numbers and Place Value

A nice easy page here — it's all about reading, writing and ordering whole numbers.

Split Big Numbers into Columns and Parts

1) First, you need to know the names of all the columns. E.g. for the number 3 232 594:

MILLIONS	HUNDRED-THOUSANDS	TEN-THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS
3	2	3	2	5	9	4

2) You can then split any number up into its parts, like this:

3 000 000	Three million
200 000	Two hundred thousand
30 000	Thirty thousand
2 000	Two thousand
500	Five hundred
90	Nine tens (ninety)
4	Four units

→ These add together to make 3 232 594.

Line up the columns so you can read the numbers clearly.

Look at Big Numbers in Groups of Three

To read or write a BIG number, follow these steps:

1) Start from the right-hand side of the number →
2) Moving left, ←, put a space every 3 digits to break it up into groups of 3.
3) Now going right, →, read each group of three as a separate number, as shown.

3 232 594

MILLIONS THOUSANDS The rest

So read as: 3 million, 232 thousand, 594
or write fully in words:
Three million, two hundred and thirty-two thousand, five hundred and ninety-four.

Ordering Numbers

The digits of a number are the individual values (0-9) that are written in each place value column. E.g. in 623, the digits are 6, 2 and 3.

EXAMPLE: Put these numbers in order from smallest to largest:

53 17 9 1729 754 3 548 88 2321

1) First put them into groups, the ones with fewest digits first:

1-digit	2-digit	3-digit	4-digit
9 3	53 17 88	754 548	1729 2321

2) Then just put each separate group in order of size:

3 9	17 53 88	548 754	1729 2321

Random related joke generator temporarily out of order...

Nothing too difficult on this page. Make sure you understand it all and have a go at these questions:

1) Write these numbers out fully in words: a) 9 905 285 b) 6 054 203
2) Put these numbers in order from smallest to largest: 54 442 2 98 304 45 59

Ordering Numbers and Place Value

You need to be able to put decimal numbers in order too. Again, it's all about the columns. After the decimal point the columns are called decimal places.

Split Decimals into Decimal Places

1) The decimal places have names too. E.g. for the number 173.753:

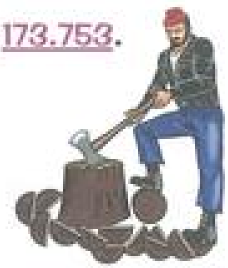
HUNDREDS	TENS	UNITS	DECIMAL POINT	TENTHS	HUNDREDTHS	THOUSANDTHS
1	7	3	.	7	5	3

2) You can split up decimals into parts too:

With decimals, line up the decimal points.

100.000	One hundred
70.000	Seven tens (seventy)
3.000	Three units
0.700	Seven tenths
0.050	Five hundredths
0.003	Three thousandths

→ These add together to make 173.753.



Putting Decimals in Order of Size

1) Do the whole number bit first, then the bit after the decimal point.
2) With numbers between 0 and 1, first group them by the number of 0s at the start. The group with the most 0s at the start comes first.

EXAMPLE: Write these numbers in order, from smallest to largest:

0.1 9.01 0.53 0.0011 0.027 0.023 0.0023 2.6

1) Do the whole-number bit first. Most of these decimals don't have a whole number bit.

0.1 0.53 0.0011 0.027 0.023 0.0023 2.6 9.01

2) The rest of the decimals are between 0 and 1, so group them by the number of 0s at the start.

2 initial 0s: 0.0011 0.0023 | 1 initial 0: 0.027 0.023 | no initial 0s: 0.1 0.53 | 2.6 9.01

3) Once they're in groups, just order them by comparing the first non-zero digits.
(If the first non-zero digits are the same, look at the next digit along instead.)

0.0011 0.0023 0.023 0.027 0.1 0.53 2.6 9.01

Order your decimals today and get free postage...

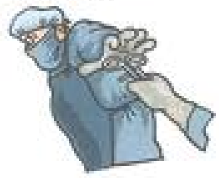
That was a lot of numbers — hopefully you've learnt how to order them. Try this:

1) Put these numbers in order from smallest to largest:

9.54 3.42 7.11 0.55 0.004 0.032 0.54 5.63 1.23 0.054

Add, Subtract, Multiply and Divide

These operations are the building blocks of maths — make sure you know how they work.



Using Opposite Operations

Adding and Subtracting

Start off with a number, add any number to it and then subtract the same number — you'll be back to the number you started with. This comes in handy when checking your answer.

EXAMPLE: Simon has 26p and steals 14p from his sister Emma. How much money does Simon have now?

Add the two amounts together to get the total. $26 + 14 = 40\text{p}$

Check your answer by using the opposite operation — you should get the amount you started with. $40 - 14 = 26\text{p}$ ✓

Multiplying and Dividing

Multiplying and dividing are opposite operations too. Start off with a number, multiply it by any number and then divide by the same number — you'll be back to the number you started with.

EXAMPLE: Michelle has 3 bags each containing 4 coconuts. She empties all of the coconuts into a box. How many coconuts are in the box?

Multiply the two numbers together to get the total. $3 \times 4 = 12$ coconuts

Check your answer by using the opposite operation — you should get the number of bags you started with. $12 \div 4 = 3$ bags ✓

Patterns in Calculations

Here are a couple of handy tricks — learn them and they'll make your life so much easier.

You can make addition questions easier by using opposite operations. If you add something to one number you have to subtract the same amount from the other number.

$$46 + 54 = 100 \quad (+4,\ -4)$$
$$50 + 50 = 100$$

$$14 \times 8 = 112 \quad (\times 2,\ \div 2)$$
$$28 \times 4 = 112 \quad (\times 2,\ \div 2)$$
$$56 \times 2 = 112$$

The same goes for multiplication questions. If you multiply one number by something you have to divide the other number by the same number.

EXAMPLE: How many 20 g bags of crisps are equal to three 120 g bags of crisps?

Here the total weight of the crisps doesn't matter. You can just use opposite operations until you have 20 g × ....

$$120\text{ g} \times 3 \quad (\div 2,\ \times 2)$$
$$60\text{ g} \times 6 \quad (\div 3,\ \times 3)$$
$$20\text{ g} \times 18$$

Always do the opposite of what you're asked to do...

Test out these opposite operations on the questions below. First work out the answer then check it.

1) a) $26 + 18$ b) $105 - 76$ c) 17×5 d) $124 \div 4$

2) Find the missing numbers: a) $96 + 67 = 100 + ...$ b) $8 \times 21 = 2 \times ...$

Addition and Subtraction

You ought to know how to do sums with just a pen and paper — so put away those calculators.

Adding

1) Line up the units columns of each number.

2) Add up the columns from right to left starting with the units: $2 + 1 + 6 = 9$.
Write a 9 at the bottom of the units column.

3) Add up the next column: $4 + 3 + 7 = 14$.
Write a '4' at the bottom of the tens column and carry over the '1' to the next column.

4) Add up the final column, including anything carried over: $3 + 2 + 0 + 1 = 6$.

EXAMPLE: Add together 342, 231 and 76.

$$\begin{array}{r} 342 \\ 231 \\ +\ \ 76 \\ \hline \end{array}$$

Line up the units columns

$$\begin{array}{r} 342 \\ 231 \\ +\ \ 76 \\ \hline 49 \\ {\scriptstyle 1}\ \ \end{array}$$

$2 + 1 + 6 = 9$

$4 + 3 + 7 = 14$ — write 4 and carry the 1

$$\begin{array}{r} 342 \\ 231 \\ +\ \ 76 \\ \hline 649 \\ {\scriptstyle 1}\ \ \ \ \end{array}$$

$3 + 2 + 0 +$ carried $1 = 6$

Subtracting

EXAMPLE: Zaara the zebra used to weigh 372 kg. Her current weight is 324 kg. How much weight has she lost (in kg)?

$$\begin{array}{r} 372 \\ -\ 324 \\ \hline \end{array}$$

Line up the units columns

2 is smaller than 4, so you can't do 2 – 4

$$\begin{array}{r} 3\overset{6}{\cancel{7}}\overset{12}{\cancel{2}} \\ -\ 324 \\ \hline \end{array}$$

Borrow 10 from the tens column

$$\begin{array}{r} 3\overset{6}{\cancel{7}}\overset{12}{\cancel{2}} \\ -\ 324 \\ \hline 8 \end{array}$$

$12 - 4 = 8$

$$\begin{array}{r} 3\overset{6}{\cancel{7}}\overset{12}{\cancel{2}} \\ -\ 324 \\ \hline 048 \end{array}$$

$6 - 2 = 4$

$3 - 3 = 0$

So Zaara has lost 48 kg.

1) Line up the units columns of each number.

2) Going right to left, take the bottom number away from the top number.
Here the top number (2) is smaller than the bottom number (4), so borrow 10 from the next column along. The '7' in the tens column becomes '6'.

3) You now have $2 + 10 = 12$ units, so you can do the subtraction: $12 - 4 = 8$.
Write the 8 at the bottom.

4) Subtracting the numbers in the next column along gives $6 - 2 = 4$, so write 4 at the bottom.

5) In the last column, $3 - 3 = 0$.

Carrying all of these numbers is tough work...

Test your new-found addition and subtraction skills on these teasers:

1) a) $113 + 645 + 39$ b) $1239 - 387$

2) Sam cycled 125 km, Jo cycled 87 km and Ian cycled 96 km. How far did they cycle altogether?

Adding and Subtracting Decimals

The methods for adding and subtracting decimals are just the same as the ones on the last page. But instead of lining up the units, always make sure you line up the decimal points.

Adding Decimals

1) Line up the decimal points and put one in the space for the answer too. Write in extra zeros to make the decimals the same length.
2) Add up the columns from right to left. The first column only has a 5 in it, so write 5 at the bottom of the column.
3) Add up the numbers in the next column along: 7 + 2 + 6 = 15. Write the '5' at the bottom and carry over the '1' to the next column.
4) Add up the next column, including anything carried over: 0 + 2 + 1 + 1 = 4.
5) The last column just has a 3, so write this in.

EXAMPLE: Work out 0.7 + 32.2 + 1.65.

$$\begin{array}{r} 0.70 \\ 32.20 \\ +\ 1.65 \\ \hline . \end{array}$$

Decimal points lined up

Extra zeros to make the decimals the same length

$$\begin{array}{r} 0.70 \\ 32.20 \\ +\ 1.65 \\ \hline .55 \\ {}^{1}\ \ \end{array}$$

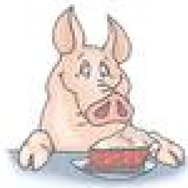
0 + 0 + 5 = 5

7 + 2 + 6 = 15 — write 5 and carry the 1

$$\begin{array}{r} 0.70 \\ 32.20 \\ +\ 1.65 \\ \hline 34.55 \\ {}^{1}\ \ \end{array}$$

0 + 2 + 1 + carried 1 = 4

There's only a 3 in the last column



Subtracting Decimals

1) At first, this question doesn't look like it has any decimals in it. But we need both numbers in pounds — so it becomes £5.00 – £0.91.
2) Set it out as usual, making sure you line up the decimal points and put one in for the answer.
3) Look at each column from right to left, taking away the bottom number from the top.
4) Borrow 10 from the next column along if you need to, as with whole numbers.

EXAMPLE: Ben has £5 and spends 91p on a pie. How much does he have left?

$$\begin{array}{r} £5.00 \\ -\ £0.91 \\ \hline . \end{array}$$

Decimal points lined up

0 is smaller than 1, so you can't do 0 – 1

$$\begin{array}{r} {}^{4}\ \ {}^{10}\ \\ £\cancel{5}.\cancel{0}0 \\ -\ £0.91 \end{array}$$

There's nothing to borrow here so you have to borrow from the next column

$$\begin{array}{r} {}^{4}\ \ {}^{9}\ {}^{10} \\ £\cancel{5}.\cancel{0}\cancel{0} \\ -\ £0.91 \\ \hline £4.09 \end{array}$$

Now borrow 10 from this column

10 – 1 = 9

9 – 9 = 0

4 – 0 = 4

Turn your pennies into pounds with decimal addition...

In real life, you'll come across decimals all over the place. Have a go at these questions:

1) Alia bought 3 items with prices £11.74, £7.12 and £0.76. What was the total cost?
2) Rob beat his 100 m sprint time of 13.22 seconds by 0.87 seconds. What is his new best time?

Multiplying by 10, 100, etc.

This stuff is easy peasy — I'm sure you'll have no problem flying through this page.

1) To Multiply Any Number by 10

Move the decimal point ONE place BIGGER and if it's needed, ADD A ZERO on the end.

E.g. 1.6 × 10 = 16
6213 × 10 = 62130
672.12 × 10 = 6721.2

2) To Multiply Any Number by 100

Move the decimal point TWO places BIGGER and ADD ZEROS if necessary.

E.g. 3.5 × 100 = 350
78 × 100 = 7800
3.7734 × 100 = 377.34

3) To Multiply by 1000 or 10 000, the same rule applies:

Move the decimal point so many places BIGGER and ADD ZEROS if necessary.

E.g. 99.67 × 1000 = 99670
1.729 × 10 000 = 17290

You always move the DECIMAL POINT this much:
1 place for 10, 2 places for 100,
3 places for 1000, 4 for 10 000 etc.

4) To Multiply by Numbers like 20, 300, 8000 etc.

MULTIPLY by 2 or 3 or 8 etc. FIRST,
then move the decimal point so many places BIGGER ( )
according to how many zeros there are.

EXAMPLE: Calculate 110 × 500.

1) First multiply by 5... 110 × 5 = 550
2) ...then move the decimal point 2 places. 550 × 100 = 55000

Learn these methods and you'll go from zero to hero...

Learning these multiplying methods isn't too strenuous. For a bit of a workout, try these:

1) Work out a) 4.9 × 100 b) 1729 × 10 c) 0.0222 × 1000
2) Work out a) 1.1 × 20 b) 3.30 × 2000 c) 700 × 500

Dividing by 10, 100, etc.

This is pretty easy stuff too. Just make sure you know it — that's all.

1) To Divide Any Number by 10

Move the decimal point ONE place SMALLER and if it's needed, REMOVE ZEROS after the decimal point.

E.g. $32.2 \div 10 = 3.22$

$6541 \div 10 = 654.1$

$4200 \div 10 = 420.\cancel{0} = 420$

2) To Divide Any Number by 100

Move the decimal point TWO places SMALLER and REMOVE ZEROS after the decimal point.

E.g. $333.8 \div 100 = 3.338$

$160 \div 100 = 1.6\cancel{0} = 1.6$

$1729 \div 100 = 17.29$

3) To Divide by 1000 or 10 000, the same rule applies:

Move the decimal point so many places SMALLER and REMOVE ZEROS after the decimal point.

E.g. $6587 \div 1000 = 6.587$

$978 \div 10\,000 = 0.0978$

You always move the DECIMAL POINT this much:
1 place for 10, 2 places for 100,
3 places for 1000, 4 for 10 000 etc.

4) To Divide by Numbers like 40, 300, 7000 etc.

DIVIDE by 4 or 3 or 7 etc. FIRST, then move the decimal point so many places SMALLER (i.e. to the left).

EXAMPLE: Calculate $180 \div 200$.

1) First divide by 2... $180 \div 2 = 90$

2) ...then move the decimal point 2 places smaller. $90 \div 100 = 0.9$

I bet you'll get tired before the decimal point does...

Knowing how to divide by multiples of 10 will be very handy. See if you can manage these questions:

1) Work out a) $3.33 \div 10$ b) $56112 \div 100$ c) $85.21 \div 1000$

2) Work out a) $1040 \div 20$ b) $3900 \div 300$ c) $56000 \div 800$

Multiplying Without a Calculator

Multiplying with a calculator is a piece of cake. The real challenge is multiplying without one.

Multiplying Whole Numbers

There are lots of methods you can use for this. Three popular ones are shown below. Just make sure you can do it using whichever method you prefer...

The Traditional Method

EXAMPLE:

a) Work out 32 × 18

Split it into separate multiplications, then add up the results in columns (right to left).

```
   3 2
 × 1 8
 2 5 6   This is 8 × 32
 3 2 0   This is 10 × 32
 5 7 6   This is 256 + 320
```

b) Work out 272 × 52

```
      2 7 2
 ×      5 2
      5 4 4   This is 272 × 2
  1 3 6 0 0   This is 272 × 50
  1 4 1 4 4   This is 544 + 13600
```

Other Methods

Here are a couple more methods you can use.

EXAMPLE: Work out 48 × 33

The Grid Method

1) Split up each number into its units and tens (and hundreds and thousands if it has them).
48 = 40 + 8 and 33 = 30 + 3

2) Draw a grid, with the 'bits' of the numbers round the outside.

	40	8
30		
3		

3) Multiply the bits round the edge to fill each square.

	40	8
30	1200 (40 × 30)	240 (8 × 30)
3	120 (40 × 3)	24 (8 × 3)

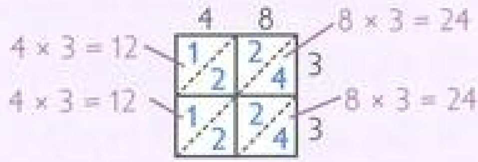
4) Finally, add up the numbers in the squares.

```
  1200
   240
   120
 +  24
  1584
```

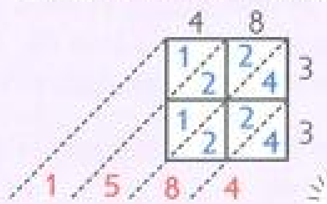
The 'Gelosia' Method:

This method's got lots of different names — you might know it as lattice multiplication or Chinese multiplication.

1) Arrange the calculation as shown and do 4 easy multiplications to fill up the grid.



2) Then just add up along the diagonals (going right to left) to get the answer.



You might need to carry a number over when adding up.

All that multiplication and not a calculator in sight...

Three different multiplication methods — try them out on these questions to see which you prefer.

1) Work out a) 55 × 18 b) 150 × 22 c) 502 × 35

Dividing Without a Calculator

OK, time for some dividing without a calculator — ready for another challenge?

Dividing Whole Numbers

There are two common ways to do division — long division and short division.
Here are some examples of both methods at work. Learn the method you find easier.

Short Division

You'll find it helpful to write out the first few multiples of the number you're dividing by.

EXAMPLE: What is 468 ÷ 9?

Multiples of 9:
$9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$
$9 \times 4 = 36$
$9 \times 5 = 45$

1) Set out the division as shown.

   9 | 4 6 8

2) Look at the first digit under the line. 4 doesn't divide by 9, so put a zero above and look at the next digit.

   0
   9 | 4 6 8

3) $9 \times 5 = 45$, so 9 into 46 goes 5 times, with a remainder of $46 - 45 = 1$.

   0 5
   9 | 4 6 ^{1}8   carry the remainder

4) 9 into 18 goes 2 times exactly.

   0 5 2   the top line has the final answer
   9 | 4 6 ^{1}8

So $468 \div 9 = 52$

Long Division

EXAMPLE: What is 354 ÷ 8?

Multiples of 8:
$8 \times 1 = 8$
$8 \times 2 = 16$
$8 \times 3 = 24$
$8 \times 4 = 32$
$8 \times 5 = 40$

1) Set out the division as shown.

   8 | 3 5 4

2) 3 doesn't divide by 8. Write a zero above the 3 and look at the next digit.
3) 8 into 35 goes 4 times, so put a 4 above the 5.
4) Take away $8 \times 4 = 32$ from 35. Write the answer underneath, and move the next digit after the 35 down.
5) 8 into 34 goes 4 times, so put a 4 above the 4. Take away $8 \times 4 = 32$ from 34. That leaves 2 and there are no more digits to bring down.

```
   0 4 4
8 | 3 5 4
  - 3 2
      3 4
    - 3 2
        2
```

6) You are left with a remainder so give that in your final answer.

So $354 \div 8 = 44$ remainder 2

When I'm dividing cake, I always keep the remainder...

Think you know how to tackle division questions? OK, let's see what you're made of...

1) Work out a) $128 \div 8$ b) $504 \div 7$ c) $550 \div 12$
2) Paul has a 200 cm piece of string. He cuts it into 18 cm pieces. What length of string will he have left over?

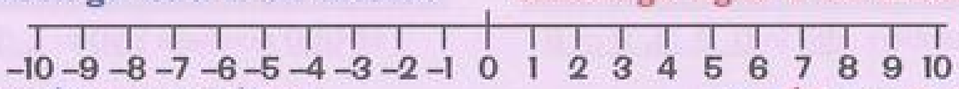
Negative Numbers

Numbers less than zero are negative. You can add, subtract, multiply and divide with them.

Adding and *Subtracting* with Negative Numbers

Use the number line for addition and subtraction involving negative numbers:

← numbers get lower in this direction | numbers get higher in this direction →



← move this way to subtract | move this way to add →

EXAMPLES:

What is –3 + 5? Start at –3 and move 5 places in the positive direction:

So –3 + 5 = 2

Work out 3 – 6 Start at 3 and move 6 places in the negative direction:

So 3 – 6 = –3

Find –1 – 5 Start at –1 and move 5 places in the negative direction:

So –1 – 5 = –6

Use These Rules for *Combining Signs*

+	+	makes	+
+	–	makes	–
–	+	makes	–
–	–	makes	+

These rules are ONLY TO BE USED WHEN:

1) Multiplying or dividing

EXAMPLES:

(invisible + sign)

Find: a) –3 × 5 — – + makes – so –3 × 5 = –15

b) –18 ÷ –3 — – – makes + so –18 ÷ –3 = 6

2) Two signs appear next to each other

EXAMPLES:

Work out: a) 3 – –9 — – – makes + so 3 – –9 = 3 + 9 = 12

b) 2 – –8 + –12 — – – makes + and + – makes –

so 2 – –8 + –12 = 2 + 8 – 12 = –2

Are you positive you know about negative numbers?

Combining signs — if they're the same it makes + and if they're different it makes –.

1) Work out: a) –6 + 11 b) –5 – 10 c) –3 × –6 d) 21 ÷ –7

2) On Friday the temperature in Negaton was –5 °C and the temperature in Tiverville was –19 °C. What was the difference in temperature between Negaton and Tiverville?

Special Types of Number

Whole numbers can be put into special groups with different properties. Here are a few of them.

Even and Odd Numbers

EVEN numbers all divide by 2

2 4 6 8 10 12 14 16 18 20 ...

All EVEN numbers END in 0, 2, 4, 6 or 8

ODD numbers don't divide by 2

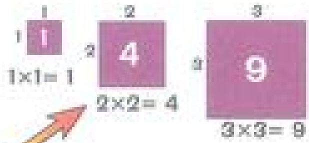
1 3 5 7 9 11 13 15 17 19 21 ...

All ODD numbers END in 1, 3, 5, 7 or 9

These rules for adding, subtracting and multiplying odd and even numbers are always true. Try them out with some random odd or even numbers. The answer will give you the rule.

Adding	Subtracting	Multiplying
odd + odd = even	odd – odd = even	odd × odd = odd
even + even = even	even – even = even	even × even = even
odd + even = odd	odd – even = odd	odd × even = even
	even – odd = odd	

Square Numbers



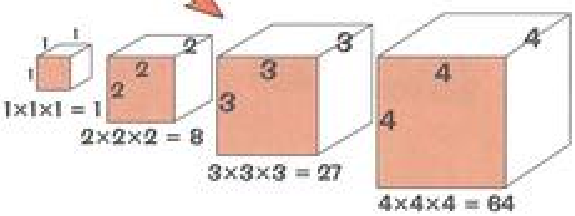
1) When you multiply a whole number by itself, you get a square number.
2) They're called square numbers because they're like the areas of squares.
3) You only have to know the first few by heart.

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
1	4	9	16	25	36	49	64	81	100	121	144
(1×1)	(2×2)	(3×3)	(4×4)	(5×5)	(6×6)	(7×7)	(8×8)	(9×9)	(10×10)	(11×11)	(12×12)

Cube Numbers

1) When you multiply a whole number by itself, then by itself again, you get a cube number.
2) They're called cube numbers because they're like the volumes of cubes.
3) It's useful to know these cube numbers by heart.

1^3	2^3	3^3	4^3	5^3	10^3
1	8	27	64	125	1000
(1×1×1)	(2×2×2)	(3×3×3)	(4×4×4)	(5×5×5)	(10×10×10)



Numbers love it when you tell them they're special...

If you've managed to get through all those special numbers, have a go at these questions.

1) Which of the following numbers are odd: 53, 61, 123, 74, 90, 872, 712, 7305, 5012
2) Find the first two numbers that are both square numbers and cube numbers.

Prime Numbers

There's one more special type of number you need to know about — the prime numbers...

PRIME Numbers Don't Divide by Anything

Prime numbers are all the numbers that only come up in their own times table:

2 3 5 7 11 13 17 19 23 29 31 37 ...

The only way to get ANY PRIME NUMBER is: 1 × ITSELF

E.g. The only numbers that multiply to give 19 are 1 × 19

EXAMPLE: Show that 18 is not a prime number.

Just find another way to make 18 other than 1 × 18: 3 × 6 = 18

18 divides by other numbers apart from 1 and 18, so it isn't a prime number.

Five Important Facts

1) 1 is NOT a prime number.
2) 2 is the ONLY even prime number.
3) The first four prime numbers are 2, 3, 5 and 7.
4) Prime numbers end in 1, 3, 7 or 9 (2 and 5 are the only exceptions to this rule).
5) But NOT ALL numbers ending in 1, 3, 7 or 9 are primes, as shown here: (Only the circled ones are primes.)

2	3	5	7
11	13	17	19
21	23	27	29
31	33	37	39
41	43	47	49
51	53	57	59
61	63	67	69

How to FIND Prime Numbers — a very simple method

1) All primes (above 5) end in 1, 3, 7 or 9 — ignore any numbers that don't end in one of those.
2) To find which of them ACTUALLY ARE primes you only need to divide each one by 3 and 7. If it doesn't divide exactly by either 3 or 7 then it's a prime.

This works for primes up to 120.

EXAMPLE: Find all the prime numbers in this list: 52, 53, 54, 55, 56, 57, 58, 59

1. Get rid of anything that doesn't end in 1, 3, 7 or 9: ~~52~~, 53, ~~54~~, ~~55~~, ~~56~~, 57, ~~58~~, 59
2. Now try dividing 53, 57 and 59 by 3 and 7:

53 ÷ 3 = 17.666... and 53 ÷ 7 = 7.571... so 53 is a prime number

57 ÷ 3 = 19 so 57 is NOT a prime number

59 ÷ 3 = 19.666... and 59 ÷ 7 = 8.428... so 59 is a prime number

So the prime numbers in the list are 53 and 59.

I hope you're all primed and ready for some maths...

Primes aren't too difficult to find once you know what to look for. Have a go at this question:

1) Write down all the prime numbers from this list: 49, 63, 38, 73, 77, 18, 39, 83

Multiples, Factors and Prime Factors

Ah, welcome to the lovely world of factors — grab a drink, pull up a seat and get ready to learn.

Multiples and Factors

The MULTIPLES of a number are just the values in its times table.

EXAMPLE: Find the first 5 multiples of 12.

You just need to find the first 5 numbers in the 12 times table: 12 24 36 48 60

The FACTORS of a number are all the numbers that divide into it exactly. Here's how to find them:

EXAMPLE: Find all the factors of 28.

Increasing by 1 each time

1 × 28
2 × 14
~~3 ×~~
4 × 7
~~5 ×~~
~~6 ×~~
7 × 4

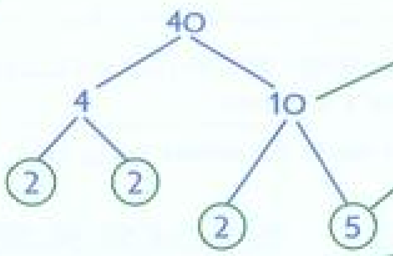
So the factors of 28 are:
1, 2, 4, 7, 14, 28

1. Start off with 1 × the number itself, then try 2 ×, then 3 × and so on, listing the pairs in rows.
2. Try each one in turn. Cross out the row if it doesn't divide exactly.
3. Eventually, when you get a number repeated, stop.
4. The factors are the numbers you haven't crossed out.

Finding Prime Factors — The Factor Tree

Any number can be written as a string of prime numbers all multiplied together — this is called a prime factorisation. The easiest way to find a prime factorisation is using a factor tree.

EXAMPLE: Find the prime factorisation of 40.



So $40 = 2 \times 2 \times 2 \times 5$
$= 2^3 \times 5$

You could split 40 into 2 and 20 or 5 and 8 instead — you'll always get the same prime factorisation.

1) Start with the number at the top, and split it into factors as shown.
2) Every time you get a prime, ring it.
3) Keep going until you can't go further (i.e. you're just left with primes), then write the primes out in order.
4) (Optional) Group numbers that are the same into powers. (i.e. $2 \times 2 \times 2 = 2^3$)

1) The prime factorisation of a number is always the same, no matter how you split it up.
2) Every number has a unique prime factorisation — no two are the same.

I hate 'truth or dare' — I much prefer 'fact or tree'...

Make sure you learn the factor tree method — it'll come in handy with question 2 below:

1) Find: a) The first 8 multiples of 9. b) All the factors of 36.
2) Express 60 as a product of its prime factors.

LCM and HCF

Here are two big fancy names for you — but don't be put off, they're both easy.

LCM — 'Lowest Common Multiple'

'Lowest Common Multiple' sounds a bit complicated, but all it means is this:

The SMALLEST number that will DIVIDE BY ALL the numbers in question.

METHOD:
1) LIST the MULTIPLES of ALL the numbers.
2) Find the SMALLEST one that's in ALL the lists.
3) Easy peasy innit?

EXAMPLE: Find the lowest common multiple (LCM) of:

a) 3 and 4

Multiples of 3: 3, 6, 9, 12, 15, ...

Multiples of 4: 4, 8, 12, 16, ...

So the LCM of 3 and 4 is 12.

b) 9 and 12

Multiples of 9: 9, 18, 27, 36, 45, 54, 63, ...

Multiples of 12: 12, 24, 36, 48, 60, 72, ...

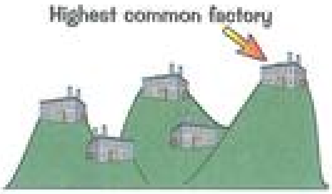
So the LCM of 9 and 12 is 36.

HCF — 'Highest Common Factor'

'Highest Common Factor' — all it means is this:

The BIGGEST number that will DIVIDE INTO ALL the numbers in the question.

METHOD:
1) LIST the FACTORS of ALL the numbers.
2) Find the BIGGEST one that's in ALL the lists.
3) Easy peasy innit?



EXAMPLE: Find the highest common factor (HCF) of 12 and 30.

Factors of 12 are: 1, 2, 3, 4, 6, 12

Factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30

So the highest common factor (HCF) of 12 and 30 is 6.

Just take care listing the factors — make sure you use the proper method (as shown on p.16) or you'll miss one and blow the whole thing out of the water.

Royalty never associate with such common numbers...

There are a couple of cracking methods for finding LCMs and HCFs on this page. Use them wisely.

1) Find the lowest common multiple (LCM) of 3 and 7.
2) Find the highest common factor (HCF) of 32 and 48.

Fractions, Decimals and Percentages

Fractions, decimals and percentages are three different ways of describing when you've got part of a whole thing. They're closely related and you can convert between them. This table shows the really common conversions which you ought to know straight off:

Fractions with a 1 on the top (e.g. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.) are called unit fractions.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.333333...	$33\frac{1}{3}$%
$\frac{2}{3}$	0.666666...	$66\frac{2}{3}$%
$\frac{1}{10}$	0.1	10%
$\frac{2}{10}$	0.2	20%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%

0.3333... and 0.6666... are known as 'recurring' decimals — the same pattern of numbers carries on repeating itself forever.

The more of those conversions you learn, the better — but for those that you don't know, you must also learn how to convert between the three types. These are the methods:

Fraction $\xrightarrow{\text{Divide}}$ Decimal $\xrightarrow{\times \text{ by } 100}$ Percentage

E.g. $\frac{7}{20}$ is $7 \div 20$ = 0.35 e.g. 0.35×100 = 35%

Fraction $\xleftarrow{\text{The awkward one}}$ Decimal $\xleftarrow{\div \text{ by } 100}$ Percentage

Converting decimals to fractions is a bit more awkward.
The digits after the decimal point go on the top, and a power of 10 on the bottom — with the same number of zeros as there were decimal places.

$0.6 = \frac{6}{10}$ $\quad 0.3 = \frac{3}{10}$ $\quad 0.7 = \frac{7}{10}$ etc.

$0.12 = \frac{12}{100}$ $\quad 0.78 = \frac{78}{100}$ $\quad 0.05 = \frac{5}{100}$ etc.

These can often be cancelled down — see p.19.

Converting decimals is easy — they'll believe anything...

Learn the stuff in the top table and the 4 conversion processes. Then it's time for some questions.

1) Turn the following decimals into fractions and reduce them to their simplest form.
a) 0.8 b) 0.3 c) 0.04 d) 0.15

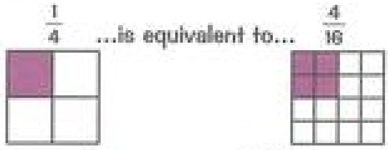
2) Which is greater: a) $\frac{1}{4}$ or 30%, b) 50% or $\frac{2}{5}$, c) 0.6 or $\frac{17}{25}$?

Fractions

This page tells you how to deal with fractions without your calculator.

Equivalent Fractions

1) Equivalent fractions are equal in size...
2) ...but the numbers on the top and bottom are different.
3) To get from one fraction to an equivalent one — MULTIPLY top and bottom by the SAME NUMBER:



$$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \qquad \frac{3}{4} \overset{\times 5}{=} \frac{15}{20} \qquad \frac{1}{5} \overset{\times 100}{=} \frac{100}{500}$$

Cancelling Down

1) You sometimes need to simplify a fraction by 'cancelling down'.
2) This means DIVIDING top and bottom by the SAME NUMBER.
3) To get the fraction as simple as possible, you might have to do this more than once:

$$\frac{3}{15} \overset{\div 3}{=} \frac{1}{5}$$

$$\frac{20}{40} \overset{\div 10}{=} \frac{2}{4} \overset{\div 2}{=} \frac{1}{2}$$

Ordering Fractions

E.g. Which is bigger, $\frac{2}{3}$ or $\frac{3}{4}$?

$$\frac{2}{3} \overset{\times 4}{=} \frac{8}{12} \qquad \frac{3}{4} \overset{\times 3}{=} \frac{9}{12}$$

1) Look at the bottom numbers ('denominators') of the fractions: 3 and 4.
2) Think of a number they will both go into — try 12.
3) Change each fraction (make equivalent fractions) so the bottom number is 12.
4) Now check which is bigger by looking at their top numbers ('numerators').
5) 9 is bigger than 8, so $\frac{3}{4}$ is bigger than $\frac{2}{3}$.

Mixed Numbers

Mixed numbers have an integer part and a fraction part.
Improper fractions are where the top number is bigger than the bottom number.

EXAMPLES:

1. Write $3\frac{4}{5}$ as an improper fraction.

1) Think of the mixed number as an addition:
$3\frac{4}{5} = 3 + \frac{4}{5}$

2) Turn the integer part into a fraction:
$3 + \frac{4}{5} = \frac{15}{5} + \frac{4}{5} = \frac{15+4}{5} = \frac{19}{5}$

2. Write $\frac{23}{3}$ as a mixed number.

Divide the top number by the bottom.

1) The answer gives the whole number part.
2) The remainder goes on top of the fraction.

$23 \div 3 = 7$ remainder 2 so $\frac{23}{3} = 7\frac{2}{3}$

When you're out for a meal, never order fractions...

Have a go at cancelling down and ordering the fractions below — and no cheating with a calculator.

1) Cancel these down as far as possible: a) $\frac{20}{28}$ b) $\frac{9}{36}$ 2) Which is bigger, $\frac{4}{5}$ or $\frac{2}{3}$?

Fractions

When dealing with fractions, always simplify your answer as much as possible.

Multiplying

1) Multiply the top numbers to find the numerator...
2) ...and multiply the bottom numbers to find the denominator.

E.g. $\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$

Dividing

1) Turn the 2nd fraction UPSIDE DOWN...
2) ...and then multiply, as shown above.

E.g. $\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{3 \times 3}{4 \times 1} = \frac{9}{4}$

Adding and Subtracting

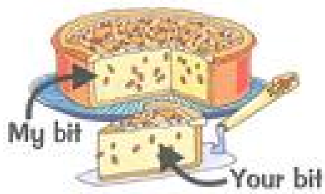
1) If the bottom numbers are the same, add or subtract the TOP NUMBERS ONLY, leaving the bottom number as it is.

E.g. adding: $\frac{2}{6} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$ and subtracting: $\frac{5}{7} - \frac{3}{7} = \frac{5-3}{7} = \frac{2}{7}$

2) If the bottom numbers are different, you have to make them the same using equivalent fractions (see previous page).

E.g. to do $\frac{1}{4} + \frac{5}{12}$, turn the $\frac{1}{4}$ into $\frac{3}{12}$ and then add as usual: $\frac{3}{12} + \frac{5}{12} = \frac{3+5}{12} = \frac{8}{12} = \frac{2}{3}$

Finding a Fraction of Something



1) Multiply the 'something' by the TOP of the fraction...
2) ...then divide it by the BOTTOM.

E.g. $\frac{9}{20}$ of £360 = £360 × 9 ÷ 20 = £162

In other words, £162 is $\frac{9}{20}$ of £360.

3) You can write one number as a fraction of another number just by putting the first number over the second and cancelling down. This works if the first number is bigger than the second number too — you'll just end up with a fraction greater than 1.

E.g. 32 is $\frac{32}{44} = \frac{8}{11}$ of 44, and 44 is $\frac{44}{32} = \frac{11}{8}$ of 32.

What fraction of this page are you going to remember?

Now you've learnt these useful tips, it's time to have a go at these below — without a calculator.

1) a) $\frac{2}{5} \times \frac{1}{6}$ b) $\frac{3}{7} \div \frac{5}{8}$ c) $\frac{1}{8} + \frac{9}{8}$ d) $\frac{11}{5} - \frac{7}{5}$ e) $\frac{3}{8} + \frac{1}{2}$ f) $\frac{1}{3} + \frac{1}{4}$

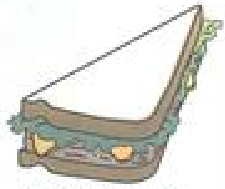
Percentages

These simple percentage questions shouldn't give you much trouble. Especially if you remember:

1) 'Per cent' means 'out of 100', so 20% means '20 out of 100' = $\frac{20}{100}$.
2) If a question asks you to work out the percentage OF something you can replace the word OF with a multiplication (×).

'Percentage Of' Questions

If you have a calculator, these questions are easy.



Find the other 50% of my sandwich

EXAMPLE: Find 18% of £4.

1) Translate the question into maths 18% of £4 = $\frac{18}{100} \times 4$
2) Work it out. 18 ÷ 100 × 4 = £0.72 = 72p

If you don't have a calculator, you're going to have to do a bit more work.

EXAMPLE: Find 35% of 120 kg.

1) First find 10% by dividing by 10. 120 ÷ 10 = 12
2) Then find 5% by dividing 10% by 2. 12 ÷ 2 = 6
3) Make 35% by adding up 10% and 5%. 35% = 10% + 10% + 10% + 5%
   = 12 + 12 + 12 + 6 = 42 kg

Writing One Number as a Percentage of Another

To write one number as a percentage of another, divide the first number by the second, then multiply by 100.

EXAMPLE: Give 36p as a percentage of 80p.

Divide 36p by 80p, then multiply by 100: (36 ÷ 80) × 100 = 45%

EXAMPLE: Farmer Littlewood measured the width of his prized pumpkin at the start and end of the month. At the start of the month it was 80 cm wide and at the end of the month it was 1.4 m wide. Give the width at the end of the month as a percentage of the width at the start.

1) Make sure both amounts are in the same units — convert 1.4 m to cm. 1.4 m = 140 cm
2) Divide 140 cm by 80 cm, then multiply by 100:
   (140 ÷ 80) × 100 = 175%

Express your joy as a % of maths lessons you have today...

That wasn't too bad, was it? A couple of new methods for you, so here are a couple of questions:

1) Find: a) 39% of 505 b) 15% of 340
2) Give: a) 18 m as a percentage of 60 m b) 1.2 km as a percentage of 480 m

Rounding Numbers

You need to be able to use 3 different rounding methods.
We'll do decimal places first, but there's the same basic idea behind all three.

Decimal Places (d.p.)

To round to a given number of decimal places:

If you're rounding to 2 d.p. the last digit is the second digit after the decimal point.

1. Identify the position of the 'last digit' from the number of decimal places.
2. Then look at the next digit to the right — called the decider.
3. If the decider is 5 or more, then round up the last digit.
   If the decider is 4 or less, then leave the last digit as it is.
4. There must be no more digits after the last digit (not even zeros).

EXAMPLE: What is 21.84 correct to 1 decimal place?

21.84 = 21.8

LAST DIGIT to be written (1st decimal place because we're rounding to 1 d.p.)

DECIDER

The LAST DIGIT stays the same because the DECIDER is 4 or less.

EXAMPLE: What is 39.7392739 to 2 decimal places?

39.7392739 = 39.74

LAST DIGIT to be written (2nd decimal place because we're rounding to 2 d.p.)

DECIDER

The LAST DIGIT rounds UP because the DECIDER is 5 or more.

Watch Out for Pesky Nines

If you have to round up a 9 (to 10), replace the 9 with 0, and add 1 to digit on the left.

EXAMPLE: Round 48.897 to 2 d.p:

9 0

48.897 ⟶ 48.89 ⟶ 48.90 to 2 d.p.

LAST DIGIT DECIDER

The question asks for 2 d.p. so you must put 48.90 not 48.9.

A decimal palace — where the richest decimals live...

This is important stuff, so learn the steps of the basic method and then have a crack at these:

1) Give: a) 16.765 correct to 1 d.p. b) 6.647895 correct to 3 d.p.
2) Give: a) 7.696 correct to 2 d.p. b) 11.7998 correct to 3 d.p.

Rounding Numbers

Here are the other two rounding methods — they're each slightly different in their own way.

Significant Figures (s.f.)

The 1st significant figure of any number is the first digit which isn't a zero.

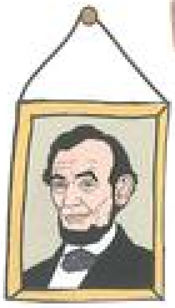
The 2nd, 3rd, etc. significant figures follow straight after the 1st — they're allowed to be zeros.

0.002309 506.07

SIG. FIGS: 1st 2nd 3rd 4th 1st 2nd 3rd 4th

To round to a given number of significant figures:

1. Find the last digit — if you're rounding to, say, 3 s.f., then the 3rd significant figure is the last digit.
2. Use the digit to the right of it as the decider, just like for d.p.
3. Once you've rounded, fill up with zeros, up to but not beyond the decimal point.



EXAMPLE: Round 1276.7 to 2 significant figures.

LAST DIGIT is the 2nd sig. fig.

Need two zeros to fill up to decimal point.

1276.7 = 1300

DECIDER is 5 or more → Last digit rounds up

To the Nearest Whole Number, Ten, Hundred, etc.

You might be asked to round to the nearest whole number, ten, hundred, thousand or million.

1. Identify the last digit, e.g. for the nearest whole number it's the units position, and for the 'nearest ten' it's the tens position, etc.
2. Round the last digit and fill in with zeros up to the decimal point.

EXAMPLE: Round 61729 to the nearest thousand.

LAST DIGIT is in the 'thousands' position

Fill in 3 zeros up to decimal point.

61729 = 62000

DECIDER is 5 or more → Last digit rounds up.

Have you figured out how significant this page is?

Lots of rounding for you there. Learn what's on the page, then have a crack at these questions:

1) Round: a) 2548 to 1 s.f. b) 36.542 to 2 s.f. c) 0.05575 to 3 s.f.

2) Give: a) 17.548 to the nearest whole number. b) 64550 to the nearest hundred.

Accuracy and Estimating

"Estimate" doesn't mean "take a wild guess", so don't just make something up...

Errors in Rounding

When you round a number using any of the methods on the previous two pages, your answer won't be exact — it will have some amount of error.

The error when rounding is given by ROUNDED VALUE – ACTUAL VALUE.

EXAMPLE: What is the error when 478 is given to 1 significant figure?

1) Round the number to 1 sig. fig. — 478 = 500 (1 sig. fig.)
2) Subtract the actual value from the rounded value. — 500 – 478 = 22

EXAMPLE: What is the error when 3.7643 is given to 2 decimal places?

1) Round the number to 2 d.p. — 3.7643 = 3.76 (2 d.p.)
2) Subtract the actual value from the rounded value. — 3.76 – 3.7643 = –0.0043

Estimating

1. Round everything off (e.g. to 1 significant figure).
2. Then work out the answer using these nice easy numbers.
3. Show all your working.

EXAMPLE: Estimate the value of 47 × 22.

$\approx$ means 'approximately equal to'.

1) Round each number to 1 s.f. — $47 \times 22 \approx 50 \times 20$
2) Do the calculation with the rounded numbers. — $50 \times 20 = 1000$

EXAMPLE: Estimate the value of $\frac{63.2 \times 13}{17}$.

1) Round each number to 1 s.f. — $\frac{63.2 \times 13}{17} \approx \frac{60 \times 10}{20}$
2) Do the calculation with the rounded numbers. — $= \frac{600}{20} = 30$

Rounding — the only time when errors are a good thing...

Make sure you can round to 1 significant figure (see p.23) or you'll struggle with all this estimating.

1) Find the error in rounding when: a) 5.352 is given to 1 d.p. b) 5230 is given to 1 sig. fig.
2) Kate buys 78 kg of chocolate cake at a cost of £11.93 per kg. Estimate the total cost.

Powers

You've already seen a few powers on page 14 — well, that's just the tip of the iceberg.

Powers are a very Useful Shorthand

1. Powers are 'numbers multiplied by themselves so many times':

$2 \times 2 \times 2 \times 2 = 2^4$ ('two to the power 4')
$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$ ('four to the power 6')
$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^8$ ('eight to the power 8')

2. The powers of ten are really easy — the power tells you the number of zeros:

$10^1 = 10$ $\quad 10^2 = 100$ $\quad 10^3 = 1000$ $\quad 10^4 = 10\,000$

3. Use the $x^\blacksquare$ button on your calculator to find powers — press 9 $x^\blacksquare$ 4 = to get $9^4 = 6561$.

4. Anything to the power 1 is just itself, e.g. $4^1 = 4$, $1726^1 = 1726$

5. Anything to the power 0 is 1, e.g $5^0 = 1$, $102738^0 = 1$

6. 1 to any power is still 1, e.g. $1^{457} = 1$.

The Power Rules

1) When MULTIPLYING, you ADD the powers.

EXAMPLES:

1. Simplify $6^4 \times 6^6$
$6^4 \times 6^6 = 6^{4+6}$
$= 6^{10}$

2. Simplify $2^2 \times 2^3$
$2^2 \times 2^3 = 2^{2+3}$
$= 2^5 = 32$

The two rules only work for powers of the same number.

2) When DIVIDING, you SUBTRACT the powers.

EXAMPLES:

1. Simplify $4^{11} \div 4^4$
$4^{11} \div 4^4 = 4^{11-4}$
$= 4^7$

2. Simplify $y^8 \div y^5$
$y^8 \div y^5 = y^{8-5}$
$= y^3$

Don't be put off by letters — they obey the same rules.

Don't let all these powers go to your head...

Learn this page off by heart, then cover it up and have a go at these...

1) Work out the following calculations without using a calculator: a) $2^3 + 8^2$ b) $3^4 - 5^2$
2) Use the power rules above to simplify: a) $4^5 \times 4^{11}$ b) $7^9 \div 7^7$
3) Using a combination of the power rules above, simplify $(7^5 \times 7^{13}) \div 7^8$.

Square Roots and Cube Roots

Take a deep breath, and get ready to tackle this page. Good luck with it, I'll be rootin' for ya...

Square Roots

'Squared' means 'multiplied by itself': $6^2 = 6 \times 6 = 36$

SQUARE ROOT $\sqrt{}$ is the reverse process: $\sqrt{36} = 6$

The best way to think of it is: **'Square Root' means 'What Number Times by Itself gives...'**

EXAMPLES:

1. What is $\sqrt{81}$?

   9 times by itself gives 81: $81 = 9 \times 9$

   So $\sqrt{81} = 9$

2. What is $\sqrt{7.84}$?

   Press: √ 7.84 = 2.8

3. Find both square roots of 100.

   $10 \times 10 = 100$, so positive square root $= 10$

   $-10 \times -10 = 100$, so negative square root $= -10$

All numbers also have a NEGATIVE SQUARE ROOT — it's just the '–' version of the normal positive one.

Cube Roots

'Cubed' means 'multiplied by itself and then by itself again': $2^3 = 2 \times 2 \times 2 = 8$

CUBE ROOT $\sqrt[3]{}$ is the reverse process: $\sqrt[3]{8} = 2$

'Cube Root' means 'What Number Times by Itself and then by Itself Again gives...'

EXAMPLES:

1. What is $\sqrt[3]{27}$?

   3 times by itself and then by itself again gives 27: $27 = 3 \times 3 \times 3$

   So $\sqrt[3]{27} = 3$

2. What is $\sqrt[3]{4913}$?

   Press: $\sqrt[3]{}$ 4913 = 17

Unlike square roots there is only one answer. Work it out in your head or use a calculator.

Higher roots are found in a similar way — $\sqrt[4]{}$ and $\sqrt[5]{}$ are the reverse processes of 'to the power 4' and 'to the power 5'.

Look at all those roots — it's a hairdresser's paradise...

These roots can be a bit tricky, but just remember, they are the opposite of powers.

1) Without using a calculator find both square roots of: a) 9 b) 121 c) 169

2) Work out the following to 2 d.p. a) $\sqrt{19}$ b) $\sqrt[3]{643}$ c) $\sqrt[5]{1729}$

Revision Summary for Section 1

Well, that's section 1 done — have a go at these questions to see how much you can remember.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Only use your calculator when the question tells you to

Ordering Numbers and Arithmetic (p1-13)

1) Find the value of: a) $4 + 10 \div 2$ b) $12 \div 3 \times 2$ c) $(8 \times 5) \div 2^2$
2) Write these numbers in words: a) 1 645 100 b) 8 007 182
3) Order these whole numbers from smallest to largest: 12, 564, 874, 911, 19, 87, 81, 98
4) Order these decimals from smallest to largest: 0.02, 1.8, 2.91, 0.09, 0.001, 0.51, 0.9
5) Find the missing numbers: a) $43 + 128 = 50 + \ldots$ b) $20 \times 18 = 2 \times \ldots$
6) Work out: a) $417 + 194$ b) $753 - 157$ c) $(2.3 + 1.123) - 0.75$
7) Find: a) 1.223×100 b) 15.12×1000 c) $6.75 \div 10$ d) $1.24 \div 200$
8) Work out: a) 23×18 b) $306 \div 9$ c) 131×19 d) $672 \div 7$
9) Work out: a) $-8 + 6$ b) $-4 - 10$ c) -7×-8 d) $81 \div -9$

Types of Number, Factors and Multiples (p14-17)

10) Define: a) even numbers b) odd numbers c) square numbers d) cube numbers
11) Find all the prime numbers between: a) 40 and 50 b) 80 and 90
12) Find: a) the first 5 multiples of 11 b) all the factors of 60
13) Express the following numbers as the product of prime factors: a) 24 b) 50 c) 90
14) Find: a) the LCM of 6 and 8 b) the HCF of 80 and 48

Fractions, Decimals and Percentages (p18-21)

15) Write: a) 0.6 as a fraction and a percentage b) 35% as a fraction and a decimal
16) a) Give two fractions equivalent to $\frac{3}{5}$. b) Simplify $\frac{8}{60}$. c) Which is bigger, $\frac{3}{10}$ or $\frac{1}{3}$?
17) Work out: a) $\frac{1}{3} + \frac{5}{9}$ b) $\frac{7}{12} - \frac{1}{2}$ c) $\frac{2}{11} \div \frac{3}{10}$ d) $\frac{7}{10} \times \frac{5}{6}$
18) Calculate: a) $\frac{2}{9}$ of 540 b) $\frac{3}{7}$ of 490
19) Use a calculator to find: a) 15% of 78 b) 154% of £86 c) 99% of £99
20) Use a calculator to give 79p as a percentage of £17.89 to 2 d.p.

Rounding and Estimating (p22-24)

21) Round: a) 164.353 to 1 d.p. b) 76 233 to 2 s.f. c) 765 444 to the nearest ten
22) What is the error when 945 is rounded to the nearest hundred?
23) Estimate the value of a) 38×31 b) $(62 \times 13) + 98$ c) $\frac{22.3 \times 54.3}{19.5}$

Powers and Roots (p25-26)

24) Without using a calculator, work out: a) 10^6 b) 555^1 c) $9^2 - 2^5$
25) Use the power rules to simplify: a) $6^2 \times 6^{11}$ b) $3^9 \div 3^5$ c) $2^{10} \times 2^8$
26) Use a calculator to find: a) $\sqrt{256}$ b) $\sqrt[3]{5.56}$ to 2 d.p. c) $\sqrt[4]{256}$

Algebra — Simplifying

Algebra really terrifies so many people. But honestly, it's not that bad.
Make sure you understand and learn these basic rules for dealing with algebraic expressions.

Terms

Before you can do anything else with algebra, you must understand what a term is:

A TERM IS A COLLECTION OF NUMBERS, LETTERS AND BRACKETS, ALL MULTIPLIED/DIVIDED TOGETHER

Terms are separated by + and – signs. Every term has a + or – attached to the front of it.

If there's no sign in front of the first term, it means there's an invisible + sign.

$4x^2$ $+ 5x$ $- 2y$ $+ 6y^2$ $+ 4$

'x^2' term 'x' term 'y' term 'y^2' term 'number' term

Simplifying or 'Collecting Like Terms'

To simplify an algebraic expression made up of all the same terms, just add or subtract them.

EXAMPLES:

1. Simplify r + r + r + r
   Just add up all the r's:
   r + r + r + r = 4r

'r' just means '1r'.

2. Simplify 2s + 3s – s
   Again, just combine the terms — don't forget there's a '–' before the last s:
   2s + 3s – s = 4s

If you have a mixture of different letters, or letters and numbers, it's a bit more tricky.
To simplify an algebraic expression like this, you combine 'like terms'
(e.g. all the x terms, all the y terms, all the number terms etc.).

EXAMPLE: Simplify $7x + 3 - x - 2$

Invisible + sign; number terms; x-terms

$$7x \quad +3 \quad -x \quad -2 \quad = \quad \underbrace{+7x \quad -x}_{6x} \quad \underbrace{+3 \quad -2}_{+1} \quad = 6x + 1$$

1) Put bubbles round each term — be sure you capture the +/– sign in front of each.
2) Then you can move the bubbles into the best order so that like terms are together.
3) Combine like terms.

EXAMPLE: Simplify $4x + y - x + 3y$

Invisible + sign; y-terms; x-terms

$$4x \quad +y \quad -x \quad +3y \quad = \quad \underbrace{+4x \quad -x}_{3x} \quad \underbrace{+y \quad +3y}_{+4y} \quad = 3x + 4y$$

You've won a boat — terms and conditions apply...

1) Simplify a) a + a + a b) 3d + 7d – 2d c) –f + f + 3f
2) Simplify a) 8x – y – 2x + 3y b) –3x + 2y – 5x – 7y

Algebra — Multiplying

Multiplying algebra is a lot like multiplying numbers — here are a few rules to get you started.

Letters Multiplied Together

Watch out for these combinations of letters in algebra that regularly catch people out:

1) abc means $a \times b \times c$ and 3a means $3 \times a$. The ×'s are often left out to make it clearer.
2) gn^2 means $g \times n \times n$. Note that only the n is squared, not the g as well.
3) $(gn)^2$ means $g \times g \times n \times n$. The brackets mean that BOTH letters are squared.
4) Powers tell you how many letters are multiplied together — so $r^6 = r \times r \times r \times r \times r \times r$.

EXAMPLES:

Careful — h times itself 3 times is h^3, not 3h (3h means $h + h + h$ or $3 \times h$).

1. Simplify $h \times h \times h$

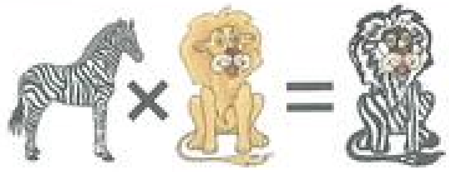
You have 3 h's multiplied together:

$h \times h \times h = h^3$

2. Simplify $3r \times 2s \times 2$

Multiply the numbers together, then the letters together:

$3r \times 2s \times 2 = 3 \times 2 \times 2 \times r \times s = 12rs$



Multiplying Brackets

There are a few key things to remember before you start multiplying out brackets:

1) The thing outside the brackets multiplies each separate term inside the brackets.
2) When letters are multiplied together, they are just written next to each other, e.g. pq.

EXAMPLE: Expand the following:

a) $2(x + 3)$
$= (2 \times x) + (2 \times 3)$
$= 2x + 6$

b) $3(2y + 5)$
$= (3 \times 2y) + (3 \times 5)$
$= 6y + 15$

c) $t(t - 6)$
$= (t \times t) + (t \times -6)$
$= t^2 - 6t$

EXAMPLE: Expand $x(x + 6) + y(2y + 3) + x(x + 3)$

1) Expand each bracket separately. $x(x + 6) + y(2y + 3) + x(x + 3)$
$= x^2 + 6x + 2y^2 + 3y + x^2 + 3x$

2) Group together like terms. $= x^2 + x^2 + 6x + 3x + 2y^2 + 3y$

3) Simplify the expression. $= 2x^2 + 9x + 2y^2 + 3y$

Now I get it, algebra just means al(ge × br × a)...

There's lots to take in here — hopefully a bit of practice on these questions will help you out.

1) Simplify: a) $d \times d \times d \times d \times d$ b) $2e \times 8f$
2) Expand: a) $3(x + 5)$ b) $x(2x + 3)$ c) $x(x + 4) - 3(x + 5)$

Formulas

A formula is a way of giving instructions without using loads of words.
So instead of saying "Square a number, times it by 7 and take away 11", say $N = 7x^2 - 11$. Nice.

Substituting Numbers into Expressions

EXPRESSION — a collection of terms (see p.28). Expressions DON'T have an = sign in them.

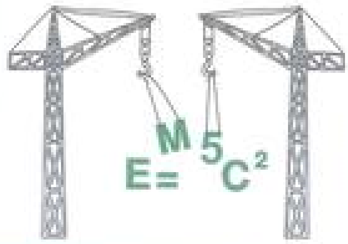
Sometimes you'll be given an expression and you'll be asked to substitute some values into it.

EXAMPLE: Substitute $x = 3$ into the expression $10x + 5$.

Think about what the expression is telling you to do. — Times x by 10 and add 5

Put the number ($x = 3$) in place of the letter and work out the value. — $3 \times 10 + 5 = 30 + 5$
$= 35$

Use BODMAS (p2) to work things out in the right order.



Substituting Numbers into Formulas

FORMULA — a rule that helps you work something out (it will have an = sign in it).

Substituting numbers into formulas is just like substituting numbers into expressions.

EXAMPLE: The formula for converting from kilometres (K) to miles (M) is $M = \frac{5}{8}K$.
Use this formula to convert 16 kilometres ($K = 16$) into miles.

1) Write out the formula in full. $M = \frac{5}{8} \times K$
2) Put the number (16) in place of the letter (K). $M = \frac{5}{8} \times 16$
3) Work out the multiplication. $M = 10$ so 16 kilometres = 10 miles

EXAMPLE: The formula for converting between Celsius (C) and Fahrenheit (F) is $F = \frac{9}{5}C + 32$.
If the temperature is 25 °C ($C = 25$), find the temperature in Fahrenheit.

1) Write out the formula in full. $F = \frac{9}{5} \times C + 32$
2) Put the number (25) in place of the letter (C). $F = \frac{9}{5} \times 25 + 32$
3) Work it out in stages. $F = 45 + 32$
$F = 77$
So 25 °C is equal to 77 °F

Substitute x = amazing into "formulas are x"...

There's no getting out of it, you'll have to learn everything on this page. Then have a crack at these:

1) Substitute $x = 4$ into these expressions: a) $3x + 3$ b) $5x - 7$ c) $\frac{1}{2}x - 1$
2) Use the formula $R = 7S - 6$ to find R when $S = 2$.

Making Formulas from Words

I'll let you in on some "tricks of the trade" for making formulas from words. There are two types:

Writing Formulas from Instructions

If you're given instructions about what to do with a number you'll need to be able to write them as a formula. The instructions can be any one of these (where 'x' stands for a number):

1) Multiply x 2) Divide x 3) Square or square root x (x^2 or $\sqrt{x}$) 4) Cube x (x^3) 5) Add or subtract a number

EXAMPLE: To find y:

a) add eight to x.

Think about what the instructions are telling you — underline important bits if you need to.

$x \longrightarrow x + 8 \longrightarrow y = x + 8$

Start with x. Add 8. Set the expression equal to y.

b) divide x by three and then add four.

$x \longrightarrow \frac{x}{3} \longrightarrow \frac{x}{3} + 4 \longrightarrow y = \frac{x}{3} + 4$

Start with x. Divide x by 3. Add 4. Set the expression equal to y.

c) square x and then subtract two.

$x \longrightarrow x^2 \longrightarrow x^2 - 2 \longrightarrow y = x^2 - 2$

Start with x. Square x. Subtract 2. Set the expression equal to y.

Making Formulas from Words

Here you'll have to make up a formula by labelling things with letters, e.g. 'c' for 'cost'.

EXAMPLE: To find the age (a) of a barrowbeetle, you divide the number of stripes (s) by two and then subtract one. Write a formula for the age of a barrowbeetle.

Pick out the important words — underline them if you need to.

$s \longrightarrow \frac{s}{2} \longrightarrow \frac{s}{2} - 1 \longrightarrow a = \frac{s}{2} - 1$

Start with s. Divide s by 2. Subtract 1. Set the expression equal to a.

Cuteness of animal = size of eyes × furriness

There's a real skill to picking out the bits of information you need. Have a go at these questions:

1) To find y: a) Multiply x by 5 and then subtract 3. b) Take the square root of x and then add 1.
2) An ice cream costs 40p for the cone plus 50p for each scoop of ice cream you have on it. Write a formula for the cost of the ice cream (c pence) in terms of the number of scoops (s).

Solving Equations

To solve equations, you must find the value of x (or any given letter) that makes the equation true.

The 'Common Sense' Approach

The trick here is to realise that the unknown quantity 'x' is just a number and the 'equation' is a cryptic clue to help you find it.

EXAMPLE: Solve the equation $x + 2 = 22$.

This just means 'find the value of x'.

This is what you should say to yourself:

'Something + 2 = 22', hmmm, so that 'something' must be 20. $x = 20$

In other words don't think of it as algebra, but as 'find the mystery number'.

The 'Proper' Way

The 'proper' way to solve equations is to keep rearranging them until you end up with '$x =$ ' on one side. There are a few important points to remember when rearranging:

Golden Rules

1) Always do the SAME thing to both sides of the equation.
2) To get rid of something, do the opposite.
   The opposite of + is – and the opposite of – is +.
   The opposite of × is ÷ and the opposite of ÷ is ×.
3) Keep going until you have a letter on its own.

EXAMPLES:

1. Solve $x + 3 = 7$.

The opposite of +3 is –3

This means 'take away 3 from both sides'.

$x + 3 = 7$

(–3) $x + 3 - 3 = 7 - 3$

$x = 4$

2. Solve $x - 2 = 3$.

The opposite of –2 is +2

$x - 2 = 3$

(+2) $x - 2 + 2 = 3 + 2$

$x = 5$

3. Solve $2x = 10$.

2x means 2 × x, so do the opposite — divide both sides by 2

$2x = 10$

(÷2) $2x \div 2 = 10 \div 2$

$x = 5$

4. Solve $\frac{x}{2} = 4$.

$\frac{x}{2}$ means x ÷ 2, so do the opposite — multiply both sides by 2

$\frac{x}{2} = 4$

(×2) $\frac{x}{2} \times 2 = 4 \times 2$

$x = 8$

Use your common sense and learn the 'proper' way...

It's a good idea to write down what you're doing at every stage — put it in brackets next to the equation (like in the examples above). Try it out on these questions.

1) Solve these equations:
   a) $x + 6 = 9$ b) $x - 2 = 9$
   c) $9x = 27$ d) $\frac{x}{3} = 7$

Solving Equations

You're not done with solving equations yet. Grab a cup of tea because things are about to get fun.

Solving Two-Step Equations

If you come across an equation like $8x - 2 = 14$ (where there's an x-term and a number on the same side), use the same method as before — just do it in two steps:

1) Add or subtract the number first.
2) Multiply or divide to get 'x = '.

EXAMPLE: Solve the equation $3x + 2 = 11$.

$$3x + 2 = 11$$

The opposite of +2 is −2, so subtract 2 from both sides.

$$(-2) \quad 3x + 2 - 2 = 11 - 2$$

$$3x = 9$$

The opposite of ×3 is ÷3, so divide both sides by 3.

$$(\div 3) \quad 3x \div 3 = 9 \div 3$$

$$x = 3$$

EXAMPLE: Solve the equation $\frac{x}{2} - 3 = 4$.

$$\frac{x}{2} - 3 = 4$$

The opposite of −3 is +3, so add 3 to both sides.

$$(+3) \quad \frac{x}{2} - 3 + 3 = 4 + 3$$

$$\frac{x}{2} = 7$$

The opposite of ÷2 is ×2, so multiply both sides by 2.

$$(\times 2) \quad \frac{x}{2} \times 2 = 7 \times 2$$

$$x = 14$$

Equations with an 'x' on Both Sides

For equations like $3x + 1 = x - 7$ (where there's an x-term on each side), you have to:

1) Get all the x's on one side and all the numbers on the other.
2) If you need to, multiply or divide to get 'x = '.

EXAMPLE: Solve the equation $2x - 7 = x + 3$.

$$2x - 7 = x + 3$$

To get the x's on only one side, subtract x from each side.

$$(-x) \quad 2x - 7 - x = x + 3 - x$$

$$x - 7 = 3$$

Now add 7 to get the numbers on the other side.

$$(+7) \quad x - 7 + 7 = 3 + 7$$

$$x = 10$$

Trying to find x? It's always in the last place you look...

You can always check if you've got these questions right. Just pop the value you've found back into the original equation, and check that it works. Give it a go on these questions.

1) Solve these two-step equations: a) $2x - 9 = 3$ b) $4x + 7 = 23$
2) Solve: a) $4x - 5 = 3x + 5$ b) $2x - 3 = 3x + 2$ c) $6y - 2 = 4y + 6$

Number Patterns and Sequences

Sequences are just patterns of numbers or shapes that follow a rule.
You need to be able to spot what the rule is.

Finding Number Patterns

The trick to finding the rule for number patterns is to write down what you have to do to get from one number to the next in the gaps between the numbers.
There are 2 main types to look out for:

1) Arithmetic sequences — Add or subtract the same number each time

E.g. 2, 5, 8, 11, 14 ... (+3, +3, +3, +3, +3)   30, 24, 18, 12 ... (−6, −6, −6, −6)

The RULE: 'Add 3 to the previous term'   'Subtract 6 from the previous term'

2) Geometric sequences — Multiply or divide by the same number each time

E.g. 2, 6, 18, 54 ... (×3, ×3, ×3, ×3)   40 000, 4000, 400, 40 ... (÷10, ÷10, ÷10, ÷10)

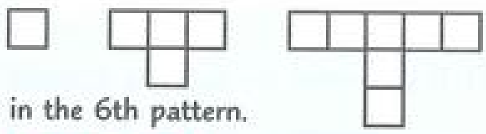
The RULE: 'Multiply the previous term by 3'   'Divide the previous term by 10'

You might get number patterns that follow a different rule — for example, you might have to add or subtract a changing number each time, or add together the two previous terms.
You just need to describe the pattern and use your rule to find the next term.

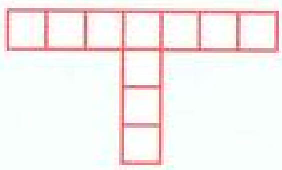
Shape Patterns

If you have a pattern of shapes, you need to be able to continue the pattern. You might also have to find the rule for the pattern to work out how many shapes there'll be in a later pattern.

EXAMPLE: Here are some patterns made of squares.
a) Draw the next pattern in the sequence.
b) Work out how many squares there will be in the 6th pattern.



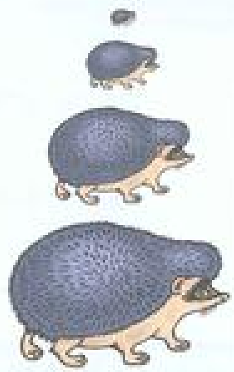
a) Just continue the pattern — add an extra square to each of the three legs.



b) Set up a table to find the rule:

Pattern number	1	2	3	4	5	6
Number of squares	1	4	7	10	13	16

The rule is 'add 3 to the previous term'. So just keep on adding 3 to extend the table until you get to the 6th term — which is 16.



Who is Pat anyway? And why does he keep turning?...

Remember, you always need to work out how to get from one term to the next — that's the rule.

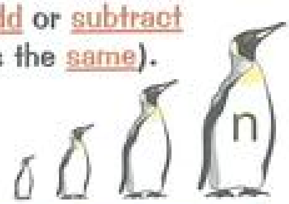
1) A sequence starts 38, 32, 26, 20. Write down the next term in the sequence and find the rule.

Number Patterns and Sequences

You might have to "find an expression for the nth term of a sequence" — this is a rule with n in, like $5n - 3$. It gives every term in the sequence when you put in different values for n.

Finding the nth Term of a Sequence

This method works for sequences with a common difference — where you add or subtract the same number each time (i.e. the difference between each pair of terms is the same).



EXAMPLE: Find an expression for the nth term of the sequence that starts 2, 8, 14, 20, ...

n:	1	2	3	4
Sequence:	2	8	14	20
	+6	+6	+6	
6n:	6	12	18	24
	−4	−4	−4	−4
term:	2	8	14	20

So the expression for the nth term is $6n - 4$

$n = 1$ gives $6n - 4 = 6 - 4 = 2$ ✓
$n = 2$ gives $6n - 4 = 12 - 4 = 8$ ✓
$n = 3$ gives $6n - 4 = 18 - 4 = 14$ ✓

1. Find the common difference. It's 6 — this tells you that the sequence is just like the six times table but with some addition or subtraction.
2. The six times table is the same as 6n.
3. Work out what you have to add or subtract to get from 6n to each term. So it's −4.
4. Put '6n' and '−4' together.
5. Check your formula by putting the first few values of n back in.

When the sequence is decreasing the common difference is negative.

EXAMPLE: A sequence starts 1, −2, −5, −8...

a) Find an expression for the nth term of the sequence.

n:	1	2	3	4
Sequence:	1	−2	−5	−8
	−3	−3	−3	
−3n:	−3	−6	−9	−12
	+4	+4	+4	+4
term:	1	−2	−5	−8

So the expression for the nth term is $-3n + 4$

The common difference is negative — so instead of being +3 it's −3 and the expression has $-3n$ in it.

To get from $-3n$ to the nth term in the sequence you need to +4.

Put $-3n$ and $+4$ together.

b) Find the 13th term in the sequence.

$n = 13$ gives $-3n + 4 = (-3 \times 13) + 4$
$= -39 + 4 = -35$

Put $n = 13$ into the expression for the nth term.

Ooo, sparkly shiny sequences...

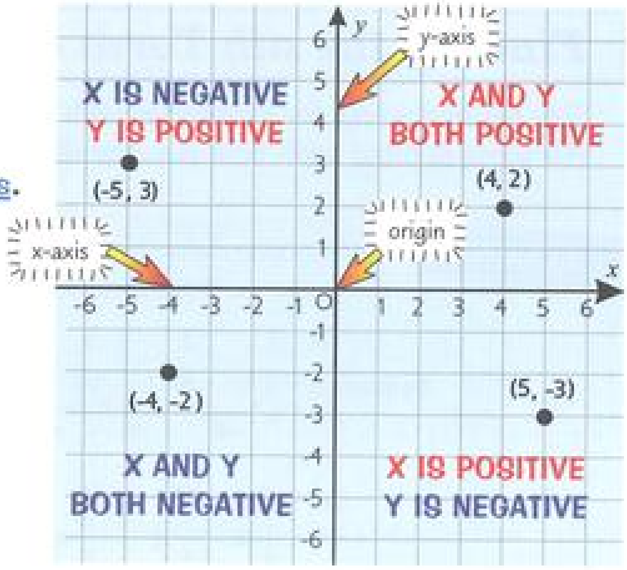
1) Find an expression for the nth term of the following sequences: a) 5, 8, 11, 14, ... b) 3, −2, −7, −12 ...

X and Y Coordinates

Graph questions can be fun. OK, maybe not fun, but better than being out in a hail storm in just your pants. First, you need to know the basics...

Plot Coordinates on a Grid

1) You draw graphs on a grid, a bit like this one.
2) It's made by two lines crossing — called the axes.
3) The y-axis goes from bottom to top, and the x-axis goes from left to right.
4) They meet at the point with coordinates (0, 0) — this is called the origin.



X, Y Coordinates — Getting them in the Right Order

1) You must always give coordinates in brackets like this: (x, y)
2) And you always have to be really careful to get them the right way round — x first, then y.
3) Here are three ways you could remember:

(x , y)

1) The two coordinates are always in ALPHABETICAL ORDER, x then y.
2) x is always the flat axis going ACROSS the page. In other words 'x is a...cross' Get it — x is a '×'. (Hilarious isn't it)
3) Remember it's always IN THE HOUSE (→) and then UP THE STAIRS (↑), so it's ALONG first and then UP, i.e. x-coordinate first, and then y-coordinate.

EXAMPLE:

What are the coordinates of points A, B and C on this graph?

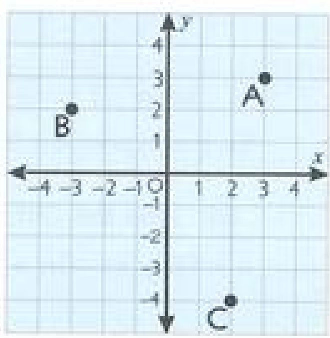
A = (3, 3)

B = (–3, 2)

C = (2, –4)

You need to read off the x-axis to find the first coordinate.

You need to read off the y-axis to find the second coordinate.



In the house and up the... oh wait, I'm in a bungalow...

Getting the coordinates the right way round is the first step — find the method that works for you.

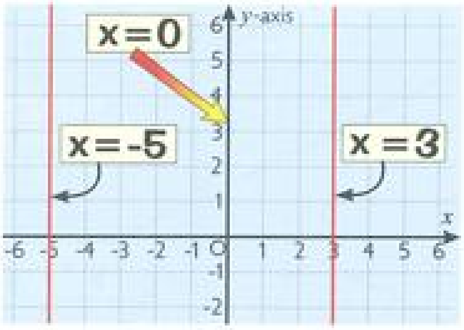
1) Copy the grid from the example above and plot these points: a) (0, 4) b) (–2, –4)

Straight Line Graphs

Over the next couple of pages you'll get to see all sorts of straight lines. You're in for a treat.

Horizontal and *Vertical* Lines: E.g. $x = 3$ and $y = -2$

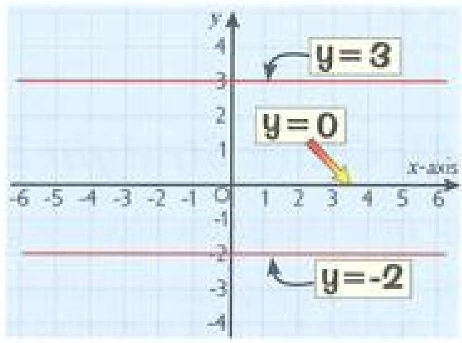
Vertical lines are always "x = a number"



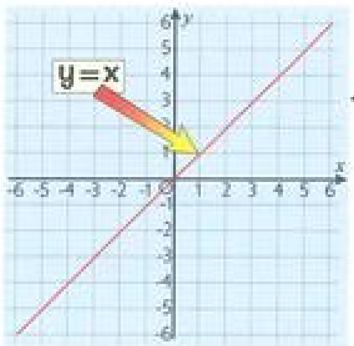
1) E.g. $x = -5$ is a vertical line through '-5' on the x-axis.
2) The y-axis is also the line $x = 0$.

Horizontal lines are always "y = a number"

1) E.g. $y = 3$ is a horizontal line through '3' on the y-axis.
2) The x-axis is also the line $y = 0$.



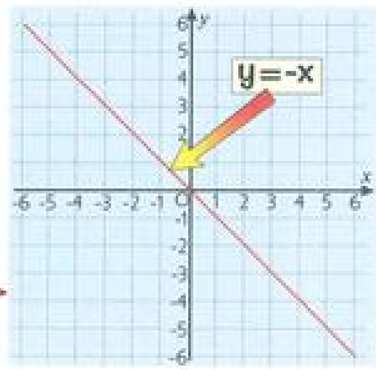
The *Main Diagonals*: "$y = x$" and "$y = -x$"



'$y = x$' is the main diagonal that goes UPHILL from left to right.

Both these lines go through the origin (0, 0).

'$y = -x$' is the main diagonal that goes DOWNHILL from left to right.



Horizzzontal lines are always lying down...

It's definitely worth learning all the graphs on this page. Now try this question:

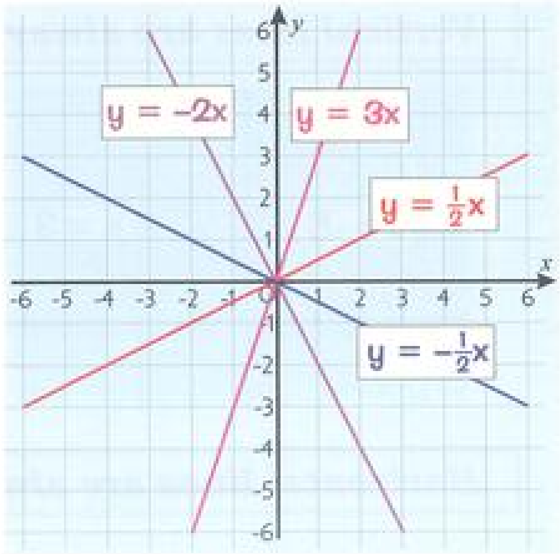
1) a) Draw a grid with the x-axis going from -6 to 6 and the y-axis going from -6 to 6.
   b) Plot these lines on the grid you've drawn: a) $y = 2$ b) $x = 1$ c) $-y = x$.

Straight Line Graphs

Joy of joys, here are some more straight line graphs for you to enjoy...

Other Lines Through the Origin: 'y = ax' and 'y = –ax'

$y = ax$ and $y = -ax$ are equations for **SLOPING LINES THROUGH THE ORIGIN** (where 'a' is just a number).



1) The value of 'a' (known as the gradient) tells you the steepness of the line.
2) The bigger 'a' is, the steeper the slope. E.g. the line $y = 3x$ on the right is steeper than $y = \frac{1}{2}x$, because 3 is bigger than ½.
3) A MINUS SIGN tells you it slopes DOWNHILL. E.g. $y = -2x$ and $y = -\frac{1}{2}x$ both slope downhill.

All Other Straight Lines

Straight lines won't have any other bits than these.

1) Other straight line equations are a little more tricky.
2) The next page shows you how to draw them, but the first step is spotting them in the first place.
3) STRAIGHT LINE EQUATIONS just have 'SOMETHING X, SOMETHING Y, AND A NUMBER'.
4) If an equation has things like x^2 (or other powers), xy or $\frac{1}{x}$, then it's NOT A STRAIGHT LINE.

EXAMPLE:

Which of these equations are straight line equations?

$y = xy + 3$ $\quad$ $3y + 3x = 12$ $\quad$ $y = x^2 + 8$ $\quad$ $y = 3x + 2$ $\quad$ $y - x - 3 = 2$

$\cancel{y = xy + 3}$	Contains an xy term so isn't a straight line equation.
$3y + 3x = 12$	Only contains y, x and a number so is a straight line equation.
$\cancel{y = x^2 + 8}$	Contains an x^2 term so isn't a straight line equation.
$y = 3x + 2$	Only contains y, x and a number so is a straight line equation.
$y - x - 3 = 2$	Only contains y, x and numbers so is a straight line equation.

Who's up for a game of 'spot the straight line graph'?

It's worth learning all this stuff because it will come in handy later in this section.

1) Decide which of these equations are straight line equations:

a) $-y = 2x$ $\quad$ b) $y + 3x = 0$ $\quad$ c) $xy + x = 5$ $\quad$ d) $y = \frac{1}{x} + 5$ $\quad$ e) $y + 2x = x$

Plotting Straight Line Graphs

On this page you get to unleash your drawing skills on the world — shame it's only straight line graphs though. They can be difficult to get right but luckily this method will lead you to the correct answer every time:

1) Choose at least 3 values of x and draw up a wee table.
2) Work out the corresponding y-values.
3) Plot the coordinates, and draw the line.

Doing the 'Table of Values'

EXAMPLE: Draw the graph of $y = 2x + 1$ for values of x from –3 to 2.

1) Choose 3 easy x-values for your table:
Use x-values from the grid you're given.
Avoid negative ones if you can.

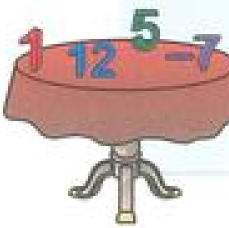
x	0	1	2
y			

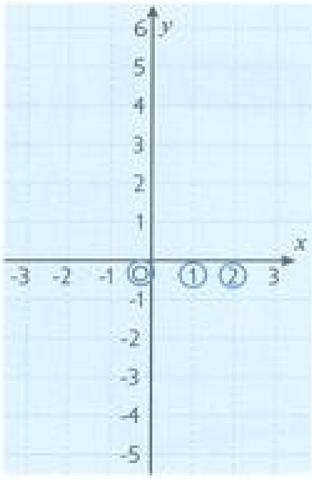
2) Find the y-values by putting each x-value into the equation:

x	0	1	2
y	1	3	5

When $x = 0$,
$y = 2x + 1$
$= (2 \times 0) + 1 = 1$

When $x = 2$,
$y = 2x + 1$
$= (2 \times 2) + 1 = 5$





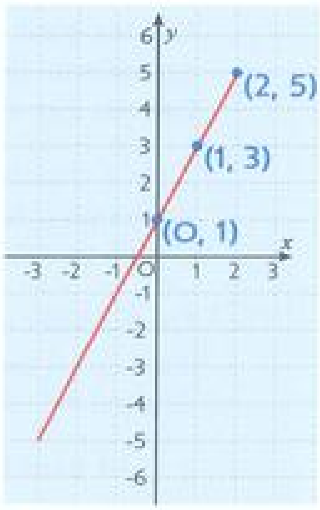
Plotting the Points and Drawing the Graph

EXAMPLE: ...continued from above.

3) Plot each pair of x- and y-values from your table.
The table gives the coordinates:
(0, 1), (1, 3) and (2, 5).

4) Now draw a straight line through your points — remember to extend the line through all the x-values given in the question.

If one point looks a bit wacky, check 2 things:
– the y-value you worked out in the table
– that you've plotted it properly.



No, not potting the plants, plotting the points...

If you have to plot an equation like $3x + y = 5$, you'll have to do a bit of rearranging before you can find the y-values (this equation would become $y = 5 - 3x$).

1) Draw the graph of $y = x - 3$ for values of x from 0 to 6.
2) Draw the graph of $y + 2x = 4$ for values of x from –2 to 2.

Reading Off Graphs

Here's a nice little page for you on reading graphs. It doesn't matter what type of graph you're reading off, you just use the same method every time.

Getting Answers from a Graph

FOR A SINGLE CURVE OR LINE, you **ALWAYS** get the answer by:
1) drawing a straight line to the graph from one axis,
2) and then down or across to the other axis.

EXAMPLE:

a) Find the value of x when $y = 5$.

Draw a line across from 5 on the y-axis to the graph, then down to the x-axis. Read off the x-value.

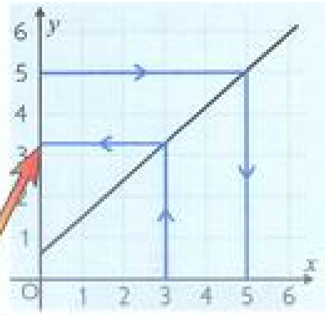
$x = 5$

b) Find the value of y when $x = 3$.

Draw a line up from 3 on the x-axis to the graph and then across to the y-axis. Read off the y-value.

$y \approx 3.2$

Sometimes the answer won't be a whole number and you'll have to estimate — $\approx$ means 'is approximately equal to'.



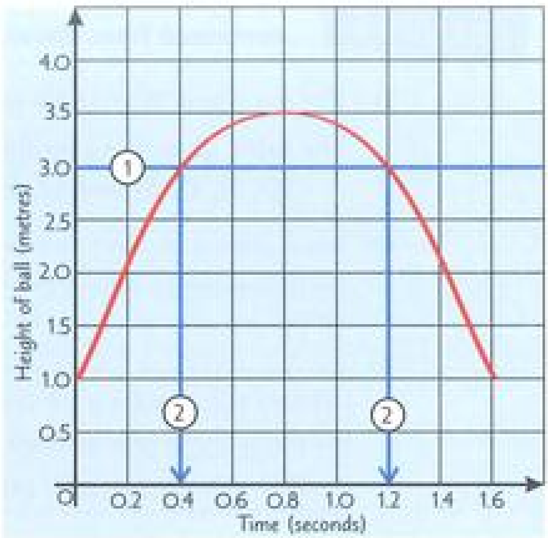
EXAMPLE: The graph shows the height of a ball as it is thrown up in the air.

Use the graph to find the approximate times when the ball was 3 m above the ground.

① Draw a line straight across from 3 on the 'height of ball' axis to the graph. Careful — this line crosses the graph twice.

② Draw straight lines down to the 'time' axis at each of these points. Read off the values.

0.4 seconds and 1.2 seconds



I can read 300 graphs per minute, what about you...

Reading graphs is easy once you know how. Use the graph above to answer these questions:

1) Find the height of the ball after 0.8 seconds.

2) Find the approximate times when the ball was 1.5 metres above the ground.

Travel Graphs

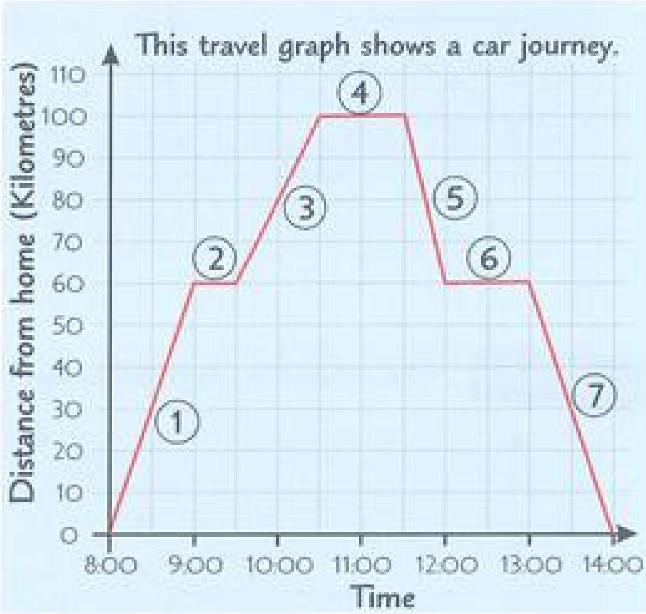
Yay, just what you wanted to see — more graphs... (what did you expect in a section on graphs?)

Know What Travel Graphs Show

1) A <u>TRAVEL GRAPH</u> is always <u>DISTANCE</u> (↑) against <u>TIME</u> (→)
2) <u>FLAT SECTIONS</u> are where it's <u>STOPPED</u>.
3) The <u>STEEPER</u> the graph the <u>FASTER</u> it's going.
4) The graph <u>GOING UP</u> means it's travelling <u>AWAY</u>.
5) The graph <u>COMING DOWN</u> means it's <u>COMING BACK AGAIN</u>.

Travel Graph Example

EXAMPLE: Explain what is happening at each stage of the car journey.



(1) The car travelled for <u>60 km</u> at a <u>steady speed</u> for <u>1 hour</u>.

(2) The car <u>stopped</u> for <u>30 minutes</u> (flat sections mean 'stopped').

(3) The car travelled away from home for another <u>40 km</u> at a <u>steady speed</u> for <u>1 hour</u>. It is now <u>100 km</u> away from home.

(4) The car <u>stopped</u> again but this time for <u>an hour</u>.

(5) The car travelled <u>40 km</u> back towards home at a <u>steady speed</u> for <u>30 minutes</u>. This is the <u>quickest</u> the car travels as it's the <u>steepest part</u> of the graph.

(6) The car <u>stopped</u> for another <u>hour</u> — 60 km away from home.

(7) The car travelled the last <u>60 km</u> back home at a <u>steady speed</u> for an <u>hour</u>.

Don't graph and drive...

Once you've explained the journey of a travel graph it's easy to answer questions on it.

1) On the travel graph above: a) How many times was the car stationary?
b) What was the total time that the car was moving?

Conversion Graphs

Conversion graphs help you switch between different units, e.g. from £ to dollars.

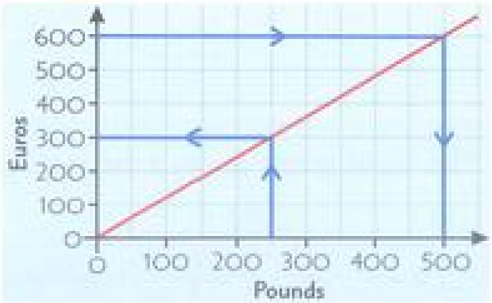
Reading off Conversion Graphs

Reading off conversion graphs is the same as reading off any other graph.
You use the same method as you saw on page 40. Here's a reminder of how it's done:

1) Draw a straight line from the value you know on one axis to the graph.
2) Change direction and draw a line straight to the other axis. Then read off the value.

Conversion Graph Examples

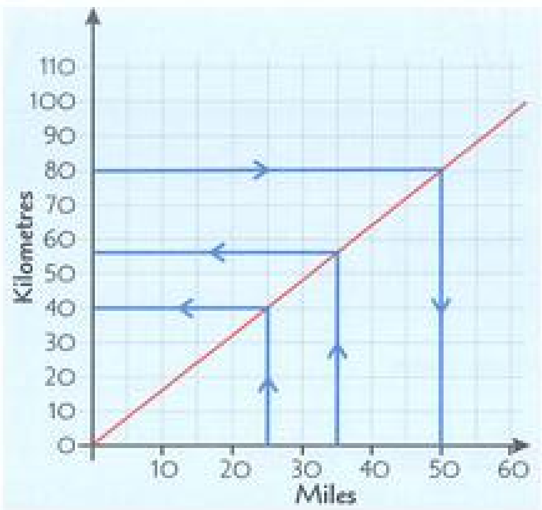
EXAMPLE: The conversion graph below can be used to convert between pounds and euros.



a) Convert 250 pounds into euros.
Draw a line up from 250 on the pounds axis to the graph.
Then draw a line straight across to the euros axis.
300 euros

b) Convert 600 euros into pounds.
Draw a line across from 600 on the euros axis to the graph.
Then draw a line straight down to the pounds axis.
500 pounds

EXAMPLE: The conversion graph below can be used to convert between miles and kilometres.



a) Convert 25 miles into kilometres.
Draw a line up from 25 on the miles axis 'til it hits the line, then go across to the km axis.
40 km

b) Convert 80 kilometres into miles.
Draw a line across from 80 on the km axis 'til it hits the line, then go down to the miles axis.
50 miles

c) Estimate how many kilometres are equal to 35 miles.
Sometimes you won't be able to find an exact answer and you'll have to estimate.
56 km

Have you been converted to a graph lover yet?

With conversion graphs it's a simple matter of reading values from a graph — nothing too tricky.
Use the two conversion graphs above to answer these questions:
1) Estimate: a) the number of euros in 400 pounds. b) how many km are equal to 20 miles.

Revision Summary for Section 2

Well, that wraps up Section 2 — time to test yourself and find out how much you really know.
Try these questions and tick off each one when you get it right.
When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Algebra (p28-29)

1) Simplify: a) $a + a + a + a + a$ b) $3b + 8b - 2b$
2) Simplify: a) $d + 3e + 5d - 2e$ b) $9f + 2 - 11f + 7$
3) Simplify: a) $g \times g \times g \times g$ b) $m \times n \times 9$
4) Expand: a) $3(v + 8)$ b) $-7(2w + 5)$
5) Expand and simplify: a) $x(3x + 4) + 6x$ b) $y(7y + 5) + 3(y - 5)$

Formulas (p30-31)

6) Use the formula $P = 3Q + 8$ to find P when: a) $Q = 7$ b) $Q = -3$
7) The formula for converting from Celsius (C) to Fahrenheit (F) is $F = \frac{9}{5}C + 32$.
Use the formula to convert –20 °C into Fahrenheit.
8) Lucian is organising a camping trip and buys s sleeping bags and t tents.
Sleeping bags cost £8 each and tents cost £15 each. He spends £P in total.
Write a formula for P in terms of s and t.

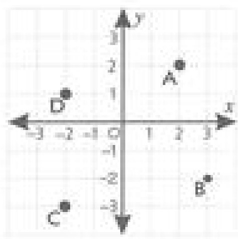
Solving Equations (p32-33)

9) Solve: a) $x + 12 = 19$ b) $x - 6 = 16$ c) $3x = 36$ d) $\frac{x}{4} = 20$
10) Solve: a) $3x + 5 = 14$ b) $9x - 11 = 25$ c) $9x - 6 = x + 10$

Number Patterns and Sequences (p34-35)

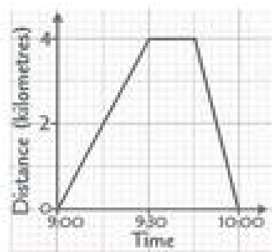
11) For each of the following sequences, find the next term and write down the rule you used.
a) 2, 8, 14, 20, ... b) 3, 9, 27, 81, ... c) 2, 3, 5, 8, 13, ...
12) Find an expression for the nth term of the sequence that starts 5, 7, 9, 11, ...

Coordinates and Straight Line Graphs (p36-39)



13) Give the coordinates of points A to D in the diagram on the right.
14) Say which of these equations are straight line equations:
a) $y = -2x^2$ b) $y = -3x$ c) $-y + x = 0$ d) $xy + 9 = 3$
15) Draw the graph of $y = 2x + 2$ for the values of x from –3 to 3.

Reading Off Graphs (p40-42)



16) The graph on the right shows Bob's bicycle journey to the shop and back.
a) Did he ride faster on his way to the shop or on his way home?
b) How long did he spend in the shop?
17) Explain how to convert units using a conversion graph.

Ratios

I can tell you're going to love this section. Let's start off with a few ratios.

Reducing Ratios to Their Simplest Form

To reduce a ratio to a simpler form, divide all the numbers in the ratio by the same thing (like cancelling down fractions on p.19).
It's in the simplest form when there's nothing left you can divide by.

EXAMPLE: Write the ratio 10:15 in its simplest form.

Both numbers have a factor of 5, so divide them by 5. $\quad 10:15 \;(\div 5)$

You can't reduce this any further. So this is the simplest form. $\quad = 2:3$

Scaling Up Ratios

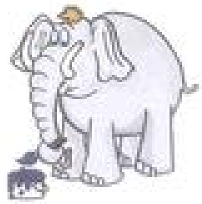
If you know the ratio between parts and the actual size of one part, you can scale the ratio up to find the other parts.

EXAMPLE: Purple paint is made from red paint and blue paint in the ratio 5:4. If 20 pots of red paint are used, how much blue paint is needed?

You need to multiply by 4 to go from 5 to 20 on the left-hand side (LHS) — so do that to both sides:

red paint : blue paint
= 5:4
$\times 4$
= 20:16

So 16 pots of blue paint are needed.



Proportional Division

In a proportional division question a TOTAL AMOUNT is split into parts in a certain ratio.
The key word here is PARTS — concentrate on 'parts' and it all becomes quite painless.

EXAMPLE: Kim and Chris share £200 in the ratio 3:7. How much does Chris get?

1) ADD UP THE PARTS:
   The ratio 3:7 means there will be a total of 10 parts: $3 + 7 = 10$ parts
2) DIVIDE TO FIND ONE "PART":
   Just divide the total amount by the number of parts: £200 $\div$ 10 = £20 (= 1 part)
3) MULTIPLY TO FIND THE AMOUNTS:
   We want to know Chris's share, which is 7 parts: 7 parts = 7 $\times$ £20 = £140

I always share my money — in the ratio 1:0...

Make sure you know how to scale up ratios, and the three steps for proportional division.

1) Give the following ratios in their simplest form: a) 7:21 b) 12:10 c) 8:28
2) 40 litres of fruit punch is made from orange juice and apple juice in the ratio 3:5.
   a) How much orange juice is used? b) How much apple juice is used?

Proportion Problems

Proportion problems all involve amounts that increase or decrease together. Awww.

The Golden Rule for Proportion Questions

You can solve lots of different proportion questions using the same method.
All you have to do is remember this golden rule:

DIVIDE FOR ONE, THEN TIMES FOR ALL

EXAMPLE: 3 painters can paint 9 rooms per day.
How many rooms per day could 7 painters paint?

Start by dividing by 3 to find how many rooms 1 painter could paint per day. $9 \div 3 = 3$ rooms per day

Then multiply by 7 to find how many rooms 7 painters could paint per day. $3 \times 7 = 21$ rooms per day

Scaling Recipes Up or Down

Scaling recipes is a useful real-life skill — have a look at this example of how it's done.

EXAMPLE: Ruth is making some white bread using the recipe shown on the right. She wants to make enough to serve 30 people.
How much of each ingredient will Ruth need?

White Bread (serves 4)
320 g bread flour
40 g soft butter
16 g yeast
200 ml water

Use the GOLDEN RULE again: **DIVIDE FOR ONE, THEN TIMES FOR ALL**

which means: Divide each amount by 4 to find how much FOR ONE PERSON, then multiply by 30 to find how much FOR 30 PEOPLE.

So for 1 person you need:		And for 30 people you need:
320 g ÷ 4 = 80 g bread flour	⇒	30 × 80 g = 2400 g bread flour
40 g ÷ 4 = 10 g soft butter	⇒	30 × 10 g = 300 g soft butter
16 g ÷ 4 = 4 g yeast	⇒	30 × 4 g = 120 g yeast
200 ml ÷ 4 = 50 ml water	⇒	30 × 50 ml = 1500 ml water

Sometimes you can just multiply — e.g. in the example above, if you wanted to know the ingredients for 16 servings of bread, you could just times everything in the recipe by 4.
If you're a bit unsure of what to do, just remember — the GOLDEN RULE always works...

Never kill the goose that laid this golden rule...

Proportions are easy — remember the golden rule and you'll never go wrong.

1) 4 lumberjacks chop 12 trees in a day. How many trees could 13 lumberjacks chop in a day?
2) Use the recipe above to work out how much of each ingredient you need to serve 20 people.

Proportion Problems

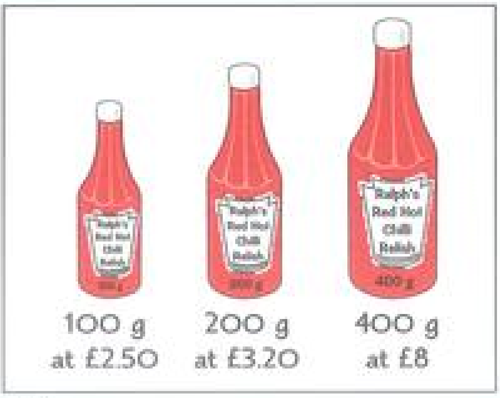
These questions are all about working out which product is the best value for money.

Best Buy Questions — Find the Amount per Penny

These are similar to the proportion questions you saw on p.45, but here you're comparing the 'value for money' of 2 or 3 similar items. For these, follow this GOLDEN RULE...

Divide by the PRICE in pence (to get the amount per penny)

EXAMPLE: Ralph's Red Hot Chilli Relish comes in three different sizes, as shown below. Which of these represents the best value for money?



The GOLDEN RULE says:

DIVIDE BY THE PRICE IN PENCE TO GET THE AMOUNT PER PENNY

In the 100 g jar you get 100 g ÷ 250p = 0.4 g per penny
In the 200 g jar you get 200 g ÷ 320p = 0.625 g per penny
In the 400 g jar you get 400 g ÷ 800p = 0.5 g per penny

The 200 g jar is the best value for money, because you get the most relish per penny.

With any question comparing 'value for money', DIVIDE BY THE PRICE (in pence) and it will always be the BIGGEST ANSWER that is the BEST VALUE FOR MONEY.

...or Find the Price per Unit

For some questions, the numbers mean it's easier to divide by the amount to get the cost per unit (e.g. per gram, per litre, etc.). In that case, the best buy is the smallest answer — the lowest cost per unit. Doing the example above in this way, you'd get:

The relish in the 100 g jar costs 250p ÷ 100 g = 2.5p per gram
The relish in the 200 g jar costs 320p ÷ 200 g = 1.6p per gram
The relish in the 400 g jar costs 800p ÷ 400 g = 2p per gram

The 200 g jar is the best value for money, because it's the cheapest per gram.

My best buy? It has to be this book...

All you need to know is that the best buy is the biggest amount per penny or the lowest cost per unit.

1) Which is the better buy: 400 ml of juice for £1.60, or 1 litre of the same juice for £3?

Percentage Increase and Decrease

There are two different ways of finding the new amount after a percentage increase or decrease:

Method 1 — Find the % then Add or Subtract

EXAMPLE: A dress has increased in price by 30%. It originally cost £40. What is the new price of the dress?

1) Divide by 100 to turn the percentage into a decimal. — $30\% = 30 \div 100 = 0.3$
2) Multiply the original amount by the decimal. — $0.3 \times £40 = £12$ — This is 30% of £40.
3) Add this onto (or subtract from) the original value. — $£40 + £12 = £52$ — It's an increase so add it on.

Method 2 — The Multiplier Method

Find the multiplier — the decimal that represents the percentage change.
E.g. 5% increase is 1.05 (= 1 + 0.05) and 5% decrease is 0.95 (= 1 – 0.05).

EXAMPLE: A hat is reduced in price by 20% in the sales. It originally cost £12. What is the new price of the hat?

1) Find the multiplier. — $20\%\text{ decrease} = 1 - 0.20 = 0.8$ — A % decrease has a multiplier less than 1.
2) Multiply the original amount by the multiplier. — $£12 \times 0.8 = £9.60$

Simple Interest

Simple interest means a certain percentage of the original amount is paid at regular intervals (usually once a year). The amount of interest is the same every time it's paid.

EXAMPLE: Elsa invests £1000 in an account which pays 2% simple interest each year. How much interest will she earn in 5 years?

1) Work out the amount of interest earned in one year:

$2\% = 2 \div 100 = 0.02$
$2\%\text{ of }£1000 = 0.02 \times £1000 = £20$

2) Multiply by 5 to get the total interest for 5 years:

$5 \times £20 = £100$

Interested in percentages? They're simple...

There are two methods for percentage increase/decrease here — pick your favourite and try these:

1) **A unicorn costing £4000 is reduced by 15% in a sale. What is its new price?**
2) **An account pays 5% simple interest each year. In 4 years how much interest would £80 make?**

Metric and Imperial Units

There's nothing too bad on this page — just some facts to learn.

Metric Units

1) Length mm, cm, m, km
2) Area mm^2, cm^2, m^2, km^2,
3) Volume mm^3, cm^3, m^3, ml, litres
4) Mass g, kg, tonnes
5) Speed km/h, m/s

'Weight' is often used instead of 'mass' in everyday language.

MEMORISE THESE KEY FACTS:

1 cm = 10 mm	1 tonne = 1000 kg
1 m = 100 cm	1 litre = 1000 ml
1 km = 1000 m	1 litre = 1000 cm^3
1 kg = 1000 g	1 cm^3 = 1 ml

Imperial Units

1) Length inches, feet, yards, miles
2) Area square inches, square feet, square miles
3) Volume cubic inches, cubic feet, gallons, pints
4) Mass ounces, pounds, stones, tons
5) Speed mph

IMPERIAL UNIT CONVERSIONS

1 foot = 12 inches
1 yard = 3 feet
1 gallon = 8 pints
1 stone = 14 pounds (lb)
1 pound = 16 ounces (oz)

Metric-Imperial Conversions

Learn these approximate conversions to help you change from metric units to imperial units:

APPROXIMATE CONVERSIONS

1 inch ≈ 2.5 cm
1 kg ≈ 2.2 pounds (lb)
1 foot ≈ 30 cm
1 litre ≈ 1.75 pints
1 gallon ≈ 4.5 litres
1 mile ≈ 1.6 km (or 5 miles ≈ 8 km)

'≈' means 'approximately equal to'.

Units make my heart 0.45 kg...

There's nothing too tricky here, just lots of learning. You'll need to know these conversions to understand the next couple of pages — but don't panic, you can always flick back to them.

1) Use the conversion tables above to find the following:
   a) 1 metre in millimetres b) 1 yard in inches c) 1 stone in ounces

Conversion Factors

A conversion factor is a number that tells you how many times bigger or smaller one thing is compared to another. You can use them to change between different units.
E.g. a kilogram (kg) is 1000 times bigger than a gram (g), so the conversion factor is 1000.

3-Step Method for Converting

1. Find the conversion factor (always easy).
2. Multiply AND divide by it
3. Choose the common-sense answer.

Examples of Converting

EXAMPLE: A zoo has a miniature gorilla called Augustus, who weighs 3000 g. What is his weight in kg?

1) Find the conversion factor
   1 kg = 1000 g
   Conversion factor = 1000
2) Multiply AND divide by it
   3000 × 1000 = 3 000 000 kg — ridiculous
   3000 ÷ 1000 = 3 kg — makes sense
3) Choose the sensible answer
   3000 g = 3 kg

EXAMPLE: Nicholas is in a golf competition and has a 30 foot putt to win the match.

a) How far is this in yards?
   1) Find the conversion factor
      1 yard = 3 feet
      Conversion factor = 3
   2) Multiply AND divide by it
      30 × 3 = 90 yards
      30 ÷ 3 = 10 yards
   3) Choose the sensible answer — there should be fewer yards than feet
      30 feet = 10 yards

b) How far is this in inches?
   1) Find the conversion factor
      1 foot = 12 inches
      Conversion factor = 12
   2) Multiply AND divide by it
      30 × 12 = 360 inches
      30 ÷ 12 = 2.5 inches
   3) Choose the sensible answer — there should be more inches than feet
      30 feet = 360 inches

Conversation factor — blah, blah, blah...

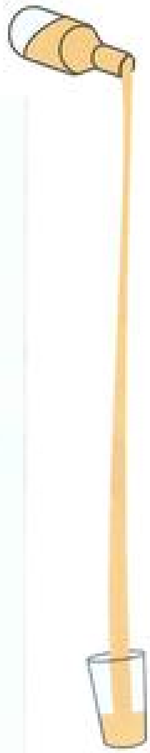
You always use the same method when converting — the trick is to remember the unit conversions.
1) Convert the following: a) 5600 cm^3 into litres b) 7 stone into pounds
2) A goat eats 80 ounces of food in a day. How many pounds of food does the goat eat in a day?

Conversion Factors

Here are some trickier examples of conversion factors. Don't fret, just use the same method as you saw on the previous page and you'll sail straight to the answer.

Converting Between Metric and Imperial

Converting from metric units to imperial units looks worse than it is.
If you need a reminder about conversion factors, have another quick look at page 49.



EXAMPLE: Mrs Watson has made 36 litres of orange squash for the school sports day.

a) How many gallons of orange squash has she made?

1) Find the conversion factor — 1 gallon = 4.5 litres
Conversion factor = 4.5

2) Multiply AND divide by it — $36 \times 4.5 = 162$ gallons
$36 \div 4.5 = 8$ gallons

3) Choose the sensible answer — there should be fewer gallons than litres — 36 litres = 8 gallons

b) If she poured the squash into pint glasses, how many glasses could she fill?

1) Find the conversion factor — 1 litre = 1.75 pints
Conversion factor = 1.75

2) Multiply AND divide by it — $36 \times 1.75 = 63$ pints
$36 \div 1.75 = 20.571...$ pints

3) Choose the sensible answer — there should be more pints than litres — 36 litres will fill 63 pint glasses

EXAMPLE: Write the following measurements in order of size from smallest to largest:
8500 cm³, 6.5 litres, 14 pints

1) First, write all three measurements in the same unit — I'm going to choose litres

1 litre = 1000 cm³
Conversion factor = 1000
$8500 \times 1000 = 8500000$ litres ✗
$8500 \div 1000 = 8.5$ litres ✓

1 litre = 1.75 pints
Conversion factor = 1.75
$14 \times 1.75 = 24.5$ litres ✗
$14 \div 1.75 = 8$ litres ✓

2) Write them out in order — 6.5 litres, 8 litres, 8.5 litres

3) Convert back to the original units — 6.5 litres, 14 pints, 8500 cm³

Here is a fancy squirrel to brighten up a dull page...

Always choose the sensible answer — if you end up with one ridiculously big or ridiculously small answer, it'll be the other one.

1) An elephant statue weighs 660 pounds. How much does it weigh in kilograms?
2) Write the following measurements in order of size: 2 metres, 7 feet, 180 cm

Reading Timetables

I'm sure you're a dab hand at reading clocks, but here's a quick reminder...

am means morning.
pm means afternoon or evening.

12 am (00:00) means midnight.
12 pm (12:00) means noon.

12-hour clock	24-hour clock
12.00 am	00:00
1.12 am	01:12
12.15 pm	12:15
1.47 pm	13:47
11.32 pm	23:32

The hours on 12- and 24- hour clocks are different after 1 pm. To go from 12-hour to 24-hour add 12 hours, and subtract 12 go the other way.

$$3.24 \text{ pm} \underset{-12 \text{ h}}{\overset{+12 \text{ h}}{\rightleftarrows}} 15:24$$

Do Time Calculations in Stages

EXAMPLE: How many minutes are there between 7.20 pm and 10.05 pm?

1) Split the time between 7.20 pm and 10.05 pm into simple stages.
   7.20 pm $\xrightarrow{+ 2 \text{ hours}}$ 9.20 pm $\xrightarrow{+ 40 \text{ minutes}}$ 10.00 pm $\xrightarrow{+ 5 \text{ minutes}}$ 10.05 pm
2) Convert the hours to minutes. 2 hours = 2 × 60 = 120 minutes
3) Add to get the total minutes. 120 + 40 + 5 = 165 minutes

Avoid calculators — the decimal answers they give are confusing, e.g. 2.5 hours = 2 hours 30 mins, NOT 2 hours 50 mins.

Using Timetables

If you've ever been to a bus or train station you'll have seen lots of timetables. Reading a timetable to find what bus or train to catch is a pretty important skill.

EXAMPLE: Look at the timetable below. What is the time of the latest train leaving Cramford that would get you to Cloudy Lane before 3.00 pm?

1) Work out the time you need to be at Cloudy Lane using the 24-hour clock.
   3:00 + 12:00 = 15:00 (or 1500)

Train Timetable

Ashingtown	1315	1332	1349	1406
Cramford	1330	1347	1404	1421
Newpeth	1345	1402	1419	1436
Bedcastle	1400	1417	1434	1451
Cloudy Lane	1415	1432	1449	1506

2) Look across the row for the Cloudy Lane times and find the latest train that arrives before 1500.
   1415, 1432, 1449 and 1506
3) Look up that column until you get to the Cramford row, and read off the time that train leaves Cramford. Change your answer back into the 12-hour clock.
   1404 (or 14:04), so in the 12-hour clock 14:04 – 12:00 = 2.04 pm

Learn this page and you'll never be late again...

Think of all the timetables out there just waiting for you to go and read them.

1) Using the timetable above, what is the latest train you could get from Ashingtown that would get you to Bedcastle before 2.30 pm? Give your answer using the 12-hour clock.

Maps and Scale Drawings

Scales tell you what a distance on a map or drawing represents in real life. They can be written in different ways, but they all say something like "1 cm represents 5 km".

Map Scales

1 cm = 3 km — "1 cm represents 3 km"

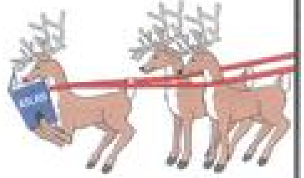
1 : 2000 — 1 cm on the map means 2000 cm in real life. Converting to m gives "1 cm represents 20 m".

0 km 1 — Use a ruler — the line's 2 cm long, so 2 cm means 1 km. Dividing by 2 gives "1 cm represents 0.5 km".

See p.49 for a reminder about conversions.

To convert between maps and real life, learn these rules:

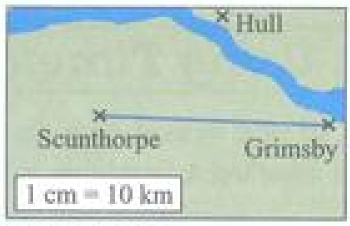
- Make sure your scale is of the form "1 cm = ..."
- To find REAL-LIFE distances, MULTIPLY by the SCALE.
- To find MAP distances, DIVIDE by the SCALE.
- Always check your answer looks sensible.



Converting Between Map Distances and Real Life

To convert a distance on a map to a real-life distance you always multiply.

EXAMPLE: This map shows three places in the UK. Work out the distance from Grimsby to Scunthorpe in km.



1. Measure with a ruler: Distance on map = 3 cm
2. Read off the scale: Scale is 1 cm = 10 km
3. For real life, multiply: Real distance is: $3 \times 10 = 30$ km

This looks sensible. ✓

To convert a real-life distance to a distance on a map you always divide.

EXAMPLE: The distance between New Garlington and Jordstone is 6 km. Work out how far apart they would be on maps with the following scales:

a) 1 cm = 0.5 km

Divide the real-life distance by the scale to find the map distance.

Real-life distance = 6 km   Scale is 1 cm = 0.5 km

$6 \div 0.5 = 12$ cm

b) 1 : 100000

Work out the scale in cm : km. 1 cm : 100000 cm = 1 cm : 1000 m = 1 cm : 1 km

Divide the real-life distance by the scale. $6 \div 1 = 6$ cm

I've been to Grimsby but I've never been to Scunthorpe...

Learn the rules for converting between maps and real life, then have a go at these questions.

1) Use the map above to work out the real-life distance from Hull to Grimsby.

2) Piel and Tusk are 21 km apart. How far apart would they be on a map with scale 1 cm = 3 km?

Maps and Scale Drawings

You get to do some drawing on this page so prepare yourself for some serious amounts of fun.

Scale Drawings

Scale drawings work just like maps. You'll have to use the rules on page 52 to convert between real life and scale drawings. If you're lucky you might get to do some drawing.

EXAMPLE:

This is a scale drawing of Josephine's garden.
1 cm represents 2 m.

a) Find the real length and width of the patio in m.

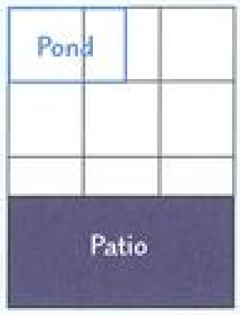
Scale drawings will often be shown on a grid.

1. Measure with a ruler. Length on drawing = 3 cm
   Width on drawing = 1.5 cm
2. Multiply to get real-life length. Real length = 3 × 2 = 6 m
   Real width = 1.5 × 2 = 3 m

Real-life units are in m.

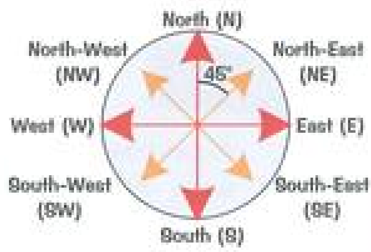
b) Josephine's pond is 3 m long and 2 m wide. Draw the pond on the scale drawing.

1. Divide to get the scale drawing length and width.
   Length on drawing = 3 ÷ 2 = 1.5 cm
   Width on drawing = 2 ÷ 2 = 1 cm
2. Draw the pond using a ruler in any sensible position and label it.



Compass Directions

Compass points describe the direction of something.
You'll have seen a compass before — make sure you know all 8 directions.



EXAMPLE: Puddleton is 10 km east of Muddleton.

a) How far apart would they be on this map?

Real-life distance = 10 km

Scale is 1 cm = 5 km

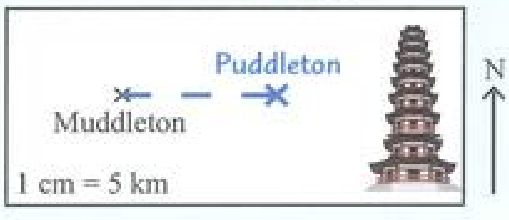
Divide for a map distance.

Distance on map = 10 ÷ 5 = 2 cm

This looks sensible. ✓

b) Mark Puddleton on the map.

Measure 2 cm to the east (right) of Muddleton:



I got so many maps, I keep some in my aunt's house...

Learn the rules for converting between maps and real life, then have a go at this question.

1) Josephine's flower bed measures 2 m by 6 m. Draw it on a copy of the scale drawing above.

Speed

Learn the formula triangle on this page and you'll be ready to tackle any questions on speed...

Speed = Distance ÷ Time

Speed is the distance travelled per unit time — the number of km per hour or metres per second.

This is the basic formula for calculating speed from distance and time:

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

You also need to be able to find distance from speed and time, and time from speed and distance. Luckily there's no need for any algebra — it all becomes simple if you use the formula triangle...

Using the Formula Triangle

A formula triangle is a handy tool for remembering formulas.

Here's the formula triangle for speed — use the words SaD Times to help you remember the order of the letters (S^DT).
So if it's a question on speed, distance and time, just say SAD TIMES.

To use the formula triangle, cover up the thing you want to find and write down what's left.

To find SPEED — cover S:

$\longrightarrow S = \frac{D}{T}$

To find DISTANCE — cover D:

$\longrightarrow D = S \times T$

To find TIME — cover T:

$\longrightarrow T = \frac{D}{S}$

EXAMPLES:

1. A car travels 30 km at 60 km per hour. How long does this take?

1) Write down the formula triangle.

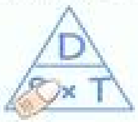
2) You want time so covering T gives: $T = \frac{D}{S}$

3) Put in the numbers. $T = \frac{30\text{ km}}{60\text{ km/h}}$

4) Give the units. $= 0.5$ hours
$= 30$ minutes

2. A sled travelled 800 m in 40 seconds. What was the average speed of the sled?

1) Write down the formula triangle.



2) You want speed so covering S gives: $S = \frac{D}{T}$

3) Put in the numbers. $S = \frac{800\text{ metres}}{40\text{ seconds}}$

4) Give the units. $= 20$ m/s

I've got a letter here for Mula Triangle...

You only have to remember one formula triangle to remember 3 formulas. That's value for money.

1) A cannonball travelled at an average speed of 20 m/s for 10 seconds. How far did it travel?

2) Shaun runs 5 km in 25 minutes. What is his speed in km/h?

Revision Summary for Section 3

Just when you thought you were done with section 3, some sneaky revision questions appeared.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Ratios and Proportion (p44-46)

1) Reduce these ratios to their simplest form: a) 2:10 b) 14:16 c) 27:18
2) A tub contains screws and nails in the ratio 3:7.
   If there are 9 screws, how many nails are there?
3) Emma and Lauren split a 400 g cake in the ratio 5:3. How much cake does Emma get?
4) 4 boys can wash 20 cars in a day. How many cars could 6 boys wash in a day?
5) 20 sweets cost £1.40. How much would 12 sweets cost?
6) A recipe for 4 people requires 20 olives. How many olives are needed for 18 people?
7) A shop sells three different sizes of cheese: 300 g for £1.50, 450 g for £2 and 750 g for £3. Which is the best buy?

Percentage Increase and Decrease (p47)

8) A shop increases its prices by 15%. How much does a £6 mug cost after the increase?
9) The price of a £1200 bike has decreased by 35%. How much is it worth now?
10) Kamil invests £150 in an account that pays 1% simple interest each year.
    How much will there be in the account after 4 years?

Units, Conversions and Time (p48-51)

11) From memory, write down all the metric unit conversions and all the imperial unit conversions from page 48. Now do the same for the metric-imperial conversions.
12) Convert 6000 mm into cm.
13) Convert 360 inches into yards.
14) A giant watermelon weighs 10 kg. What is this in pounds?
15) Convert the following: a) 80 km to miles b) 2.5 feet to cm
16) A film starts at 6.40 pm and finishes at 8.20 pm. How long is the film in minutes?

Maps and Scale Drawings (p52-53)

17) The distance between two towns is 20 miles.
    How far apart would they be on a map with a scale of 1 cm : 5 miles?
18) On a scale drawing, the dimensions of a car park are 5 cm by 2.5 cm.
    The scale is 1 cm = 10 m. What are the real-life dimensions of the car park?
19) Phillip starts at his home and walks 2 km south to the bus station. He then takes the bus 4 km west to the cinema. Make a scale drawing of his journey with scale 1 cm = 2 km.

Speed (p54)

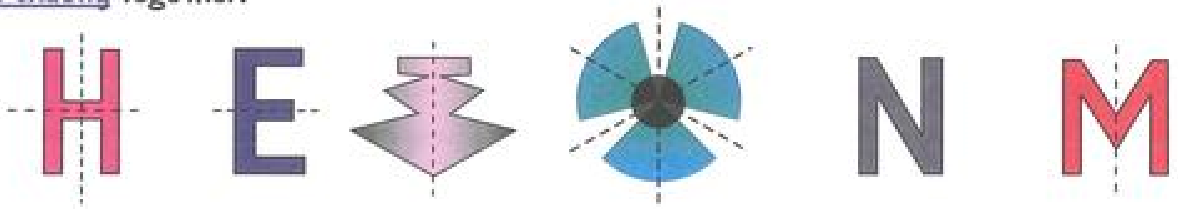
20) A runner ran a 1500 m race in 4 minutes 10 seconds. What was her average speed in m/s?
21) Ramin cycles for 3 hours at a speed of 25 km/h. How far does he cycle?

Symmetry

There are two types of symmetry you need to know — line symmetry and rotational symmetry.

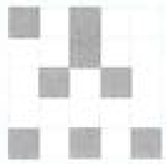
Line Symmetry

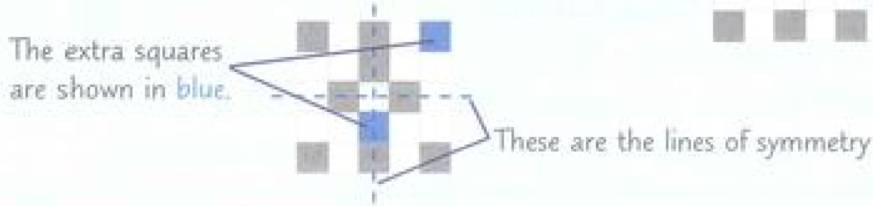
This is where you draw one or more MIRROR LINES across a shape and both sides will fold exactly together.



2 LINES OF SYMMETRY | 1 LINE OF SYMMETRY | 1 LINE OF SYMMETRY | 3 LINES OF SYMMETRY | NO LINES OF SYMMETRY | 1 LINE OF SYMMETRY

EXAMPLE: Shade two squares on the pattern on the right to make a pattern with two lines of symmetry, and draw on the lines of symmetry.





Rotational Symmetry

This is where you can rotate the shape into different positions that look exactly the same.

Order 1 | Order 2 | Order 2 | Order 3 | Order 4

The ORDER OF ROTATIONAL SYMMETRY is the posh way of saying: 'how many different positions look the same'. You should say the Z-shape above has 'rotational symmetry of order 2'.

When a shape has only 1 position you can either say that it has 'rotational symmetry of order 1' or that it has 'NO rotational symmetry'.

The new funfair attraction — the hall of mirror lines...

Make sure you know what the different types of symmetry are. You can use a mirror to help you check line symmetry, but don't get too carried away admiring your reflection. You look lovely.

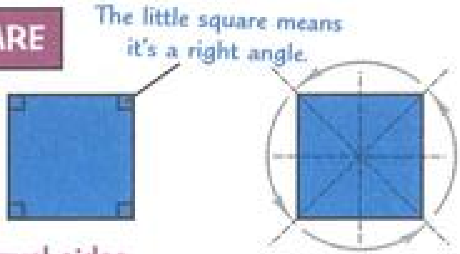
1) How many lines of symmetry does the T-shape above have?
2) What is the order of rotational symmetry of the H-shape above?

Quadrilaterals

There are lots of nice juicy facts about quadrilaterals on this page — but sadly that means that you have to learn them all.

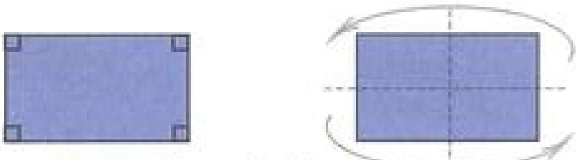
Quadrilaterals (Four-Sided Shapes)

SQUARE



4 equal sides
4 equal angles of 90° (right angles).
4 lines of symmetry,
rotational symmetry of order 4.
Diagonals cross at right angles.

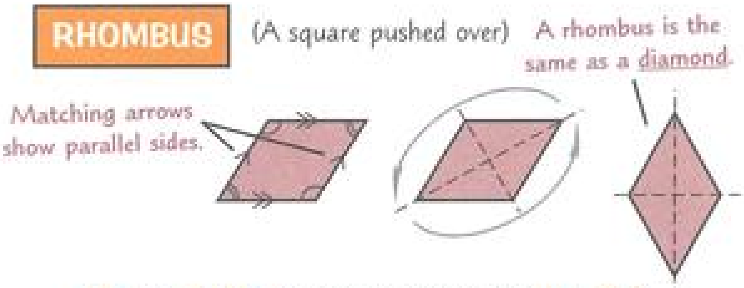
RECTANGLE



2 pairs of equal sides
(opposite sides are equal).
4 equal angles of 90° (right angles).
2 lines of symmetry,
rotational symmetry of order 2.

RHOMBUS

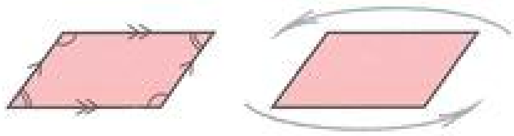
(A square pushed over)



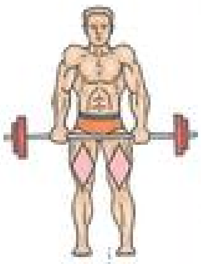
4 equal sides (opposite sides are parallel).
2 pairs of equal angles.
2 lines of symmetry,
rotational symmetry of order 2.
Diagonals cross at right angles.

PARALLELOGRAM

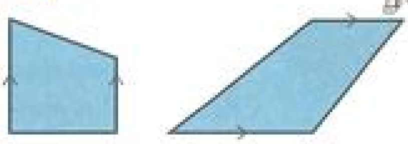
(A rectangle pushed over)



2 pairs of equal sides
(each pair are parallel).
2 pairs of equal angles.
NO lines of symmetry,
rotational symmetry
of order 2.

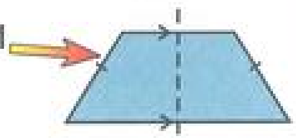


TRAPEZIUM



1 pair of parallel sides.
NO lines of symmetry*.
No rotational symmetry.

*except for an isosceles trapezium (a trapezium where the non-parallel sides are the same length), which has 1 line of symmetry.



KITE

2 pairs of equal sides.
1 pair of equal angles.
1 line of symmetry.
No rotational symmetry.
Diagonals cross at right angles.

A rhombus engagement ring isn't quite as appealing...

Learn the names of all the shapes and make sure you know how to spell them (parallellallellogram is a tricky one). Then learn the properties of each shape, and have a go at this question.

1) I am thinking of a shape with four sides. It has 2 pairs of equal sides and its diagonals cross at right angles. It has no rotational symmetry. What is the name of the shape I'm thinking of?

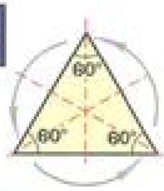
Triangles and Regular Polygons

There are some more 2D shapes coming up on this page — let's start off with triangles to ease you in gently, then build up to shapes with lots of sides (ooooo).

Triangles (Three-Sided Shapes)

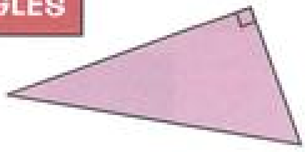
EQUILATERAL TRIANGLES

3 equal sides and
3 equal angles of 60°.
3 lines of symmetry,
rotational symmetry of order 3 (see p.56).



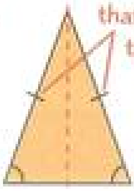
RIGHT-ANGLED TRIANGLES

1 right angle (90°).
No lines of symmetry.



ISOSCELES TRIANGLES

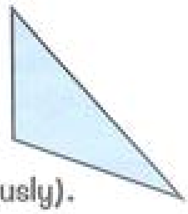
2 sides the same.
2 angles the same.
1 line of symmetry.
No rotational symmetry.



These dashes mean that the two sides are the same length.

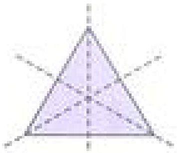
SCALENE TRIANGLES

All three sides different.
All three angles different.
No symmetry (pretty obviously).

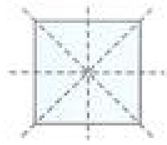


Regular Polygons

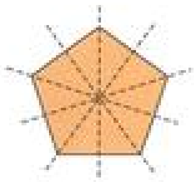
A polygon is a many-sided shape. A regular polygon is one where all the sides and angles are the same. The regular polygons are a never-ending series of shapes with some fancy features.



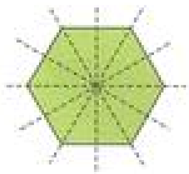
EQUILATERAL TRIANGLE
3 sides
3 lines of symmetry
Rotational symmetry of order 3



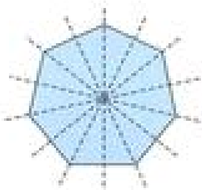
SQUARE
4 sides
4 lines of symmetry
Rotational symmetry of order 4



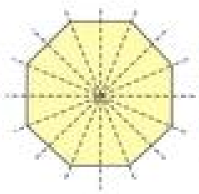
REGULAR PENTAGON
5 sides
5 lines of symmetry
Rotational symmetry of order 5



REGULAR HEXAGON
6 sides
6 lines of symmetry
Rotational symmetry of order 6



REGULAR HEPTAGON
7 sides
7 lines of symmetry
Rotational symmetry of order 7
(A 50p piece is like a heptagon.)



REGULAR OCTAGON
8 sides
8 lines of symmetry
Rotational symmetry of order 8

A regular octopus has 8 equal-length legs...

There's nothing too tricky on this page — once you know the number of sides of a regular polygon, you also know the number of lines of symmetry and its order of rotational symmetry. Sorted.

1) A regular polygon has 20 sides. How many lines of symmetry does it have?
What is its order of rotational symmetry?

Congruence and Similarity

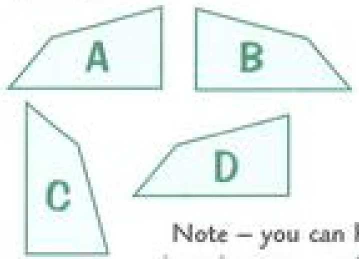
There are two fancy words for you to learn on this page — similar and congruent.
They're not that difficult really — just make sure you don't get them mixed up.

Congruent — Same Shape, Same Size

Congruence is another ridiculous maths word which sounds really complicated when it's not:

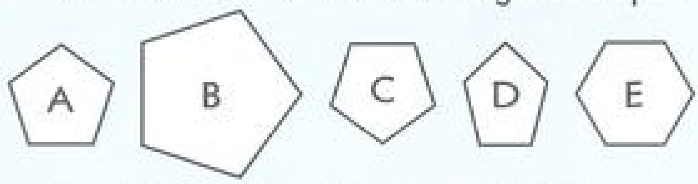
If two shapes are CONGRUENT, they are EXACTLY THE SAME — the SAME SIZE and the SAME SHAPE.

These shapes are all congruent:



Note – you can have mirror images or rotations.

EXAMPLE: Two of the shapes below are congruent.
Write down the letters of the congruent shapes.



Just pick out the two shapes that are exactly the same — remember that the shape might have been rotated or reflected.
By eye, you can see that the congruent shapes are A and C.

Similar — Same Shape, Different Size

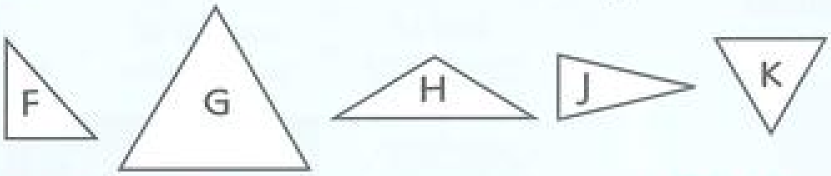
Similar has a special meaning in maths, but nothing complicated:

If two shapes are SIMILAR, they are exactly the SAME SHAPE but DIFFERENT SIZES.

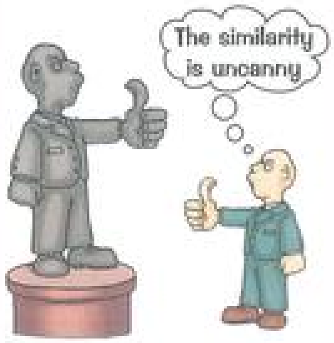
So all circles are similar. So are all equilateral triangles, all squares, all regular pentagons, etc.

When you have similar shapes, the angles are always the same and one shape is an enlargement of the other (see p.77).

EXAMPLE: Two of the shapes below are similar.
Write down the letters of the similar shapes.



Equilateral triangles are similar, so the similar shapes are G and K.



Maths and torture seem pretty similar to me...

To help remember the difference between similarity and congruence, think 'similar siblings, congruent clones' — siblings are alike but not exactly the same, clones are identical.

1) Draw a shape that is similar to shape E above.

Perimeter and Area

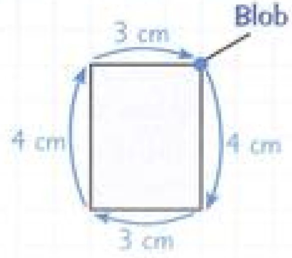
Perimeter is the distance all the way around the outside of a 2D shape.

Perimeter — Distance Around the Edge of a Shape

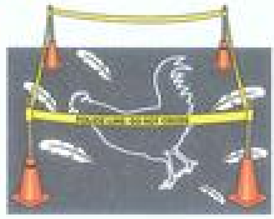
To find a perimeter, you add up the lengths of all the sides — here's the best way to do it:

1) Put a BIG BLOB at one corner and then go around the shape.
2) Write down the LENGTH of every side as you go along.
3) Even sides that seem to have NO LENGTH GIVEN — you must work them out.
4) Keep going until you get back to the BIG BLOB.
5) ADD UP all the lengths you've written down.

EXAMPLE: Find the perimeter of the shape drawn on the grid below. Each grid square represents 1 cm².



Perimeter = 4 + 3 + 4 + 3
= 14 cm

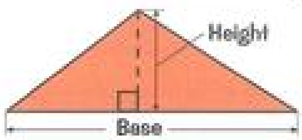


You Must Learn These Four Area Formulas

Area of RECTANGLE = length × width

$A = l \times w$

Area of TRIANGLE = $\frac{1}{2}$ × base × vertical height



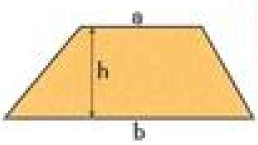
$A = \frac{1}{2} \times b \times h$

Note that the height must always be the vertical height, not the sloping height.

Area of PARALLELOGRAM = base × vertical height

$A = b \times h$

Area of TRAPEZIUM = average of parallel sides × distance between them



$A = \frac{1}{2} \times (a + b) \times h$

Clear the area — there's been a lobster spillage...

There are some examples of using the formulas on the next page so you can see how they work.

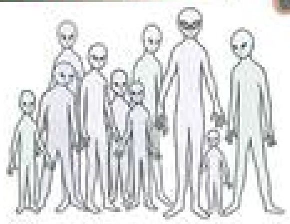
1) Find the perimeter of a square with sides of length 5 cm.

Areas

If you thought I'd just give you the area formulas and not show you how to use them, shame on you — I'd never be that mean. Except for that one time... but I probably shouldn't mention that.

Using the Area Formulas

Rectangle



EXAMPLE: Find the area of the rectangle on the right.

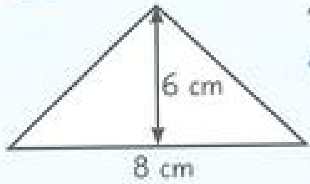
Just put the numbers into the formula:

area of rectangle = length × width
= 2 × 5
= 10 cm^2

Area is measured in square units (i.e. cm^2, m^2 etc.).

Triangle

EXAMPLE: Find the area of the triangle below.

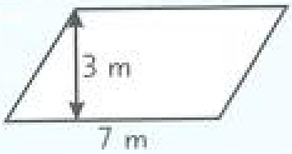


Again, put the numbers into the formula:

area of triangle = $\frac{1}{2}$ × base × vertical height
= $\frac{1}{2}$ × 8 × 6
= 24 cm^2

Parallelogram

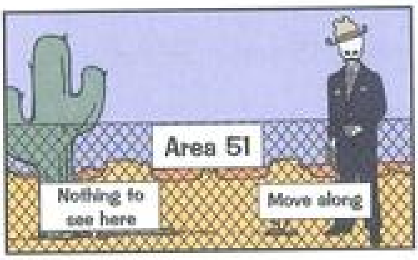
EXAMPLE: Find the area of the parallelogram below.



Use the formula for area:

area of parallelogram = base × vertical height
= 7 × 3
= 21 m^2

Trapezium



EXAMPLE: Find the area of the trapezium below.

Use the formula for area:

area of trapezium = $\frac{1}{2}$(a + b) × height
= $\frac{1}{2}$(3 + 11) × 4
= 7 × 4
= 28 cm^2

An 'amster is much 'airier than an 'edgehog...

Be careful with units in area questions — the answer should be in square units of whatever units were used in the question. So a shape measured in mm will have an area in mm^2, etc.

1) Find the area of a parallelogram with a base of 8 cm and a height of 5 cm.
2) The parallel sides of a trapezium measure 2 m and 6 m, and the distance between them is 7 m. Find the area of the trapezium.

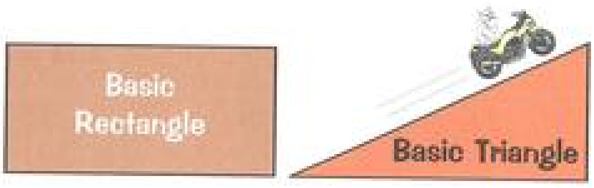
Area of Compound Shapes

Make sure you know the area formulas from page 60 — you need them again here.

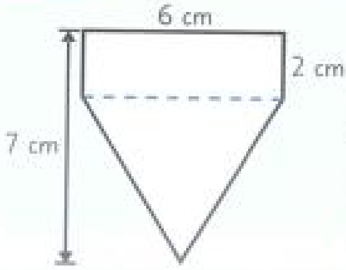
Areas of More Complicated Shapes

You sometimes have to find the area of strange-looking shapes. What you always find with these questions is that you can break the shape up into simpler ones that you can deal with.

1) **SPLIT THEM UP** into the basic shapes: **RECTANGLES**, **TRIANGLES**, etc.
2) Work out the area of each bit **SEPARATELY**.
3) Then **ADD THEM ALL TOGETHER**.



EXAMPLE: Find the area of the shape below.



Split the shape into a rectangle and triangle as shown and work out the area of each shape:

Area of rectangle = length × width = $6 \times 2 = 12$ cm^2

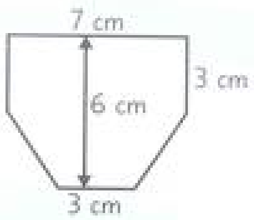
To find the height of the triangle, subtract the height of the rectangle from the total height of the shape (so $7 - 2 = 5$).

Area of triangle $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 6 \times 5 = 15$ cm^2

Total area of shape = $12 + 15 = 27$ cm^2

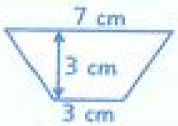
EXAMPLE: The shape of a school badge is shown on the right.

a) Find the area of the badge.



You need to work out the area of the badge — so split it into two shapes (a rectangle and a trapezium):

Find the area of the rectangle:
Area = l × w = $7 \times 3 = 21$ cm^2

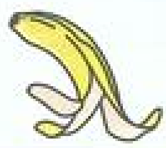


Find the area of the trapezium:
Area = $\frac{1}{2}(a + b) \times h = \frac{1}{2}(7 + 3) \times 3 = 15$ cm^2

So the total area of the badge is 21 cm^2 + 15 cm^2 = 36 cm^2

b) The material needed to make the badge costs 11p per cm^2. Work out the cost of the material needed for each badge.

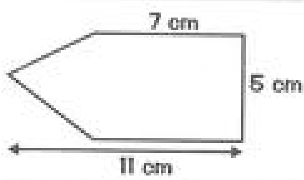
Just multiply the area by the cost per cm^2:
Cost = 36×11 = 396p = £3.96



Split the bill? I'd rather split the shape...

As long as you know the area formulas, there's nothing on this page to trip you up. Well, apart from that sneaky banana skin — watch out for that.

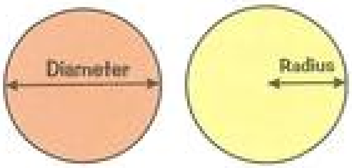
1) Find the area of the shape on the right.



Circles

Another page, another formula (or two) to learn. It's probably best to have a snack before starting this page. All the talk of pi can make you a bit peckish.

Radius and Diameter



The DIAMETER goes right across the circle, passing through the centre.
The RADIUS goes from the centre of the circle to any point on the edge.

The DIAMETER IS EXACTLY DOUBLE THE RADIUS

So if the radius is 4 cm, the diameter is 8 cm,
and if the diameter is 24 m, the radius is 12 m.

Area, Circumference and π

There are two more important formulas for you to learn — circumference and area of a circle.
The circumference is the distance round the outside of the circle (its perimeter).

1) CIRCUMFERENCE $= \pi \times$ diameter
$= \pi \times$ radius $\times 2$

$$C = \pi \times D \quad \text{or} \quad C = 2 \times \pi \times r$$

2) AREA $= \pi \times (\text{radius})^2$

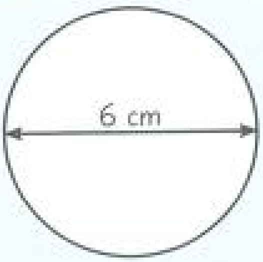
$$A = \pi \times r^2$$

$\pi = 3.141592.... = 3.142$ (approx)

The big thing to remember is that π (called "pi") is just an ordinary number (3.14159...) which is often rounded off to 3.142. You can just use the π button on your calculator (which is way more accurate).

EXAMPLE:

a) Find the circumference of the circle below. Give your answer to 1 d.p.



You're given the diameter, so use $C = \pi \times D$:

Circumference $= \pi \times$ diameter
$= \pi \times 6$
$= 18.849... = 18.8$ cm (1 d.p.)

b) Find the area of the circle. Give your answer to 1 d.p.

Here, you need the radius to find the area — so divide the diameter by 2:

Radius = diameter ÷ 2 = 6 ÷ 2 = 3 cm

Now put the numbers into the formula for area:

Area $= \pi \times (\text{radius})^2$
$= \pi \times 3^2$
$= 28.274... = 28.3$ cm^2 (1 d.p.)

But what if you have a square pie?

Learn the formulas for the circumference and area of a circle, then try this question.

1) Find the circumference and area of a circle with a diameter of 20 mm.

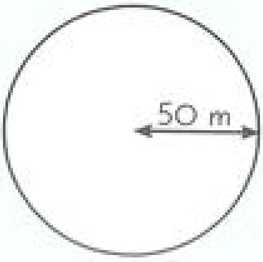
Circle Questions

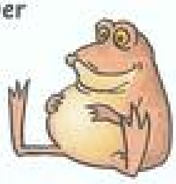
ALWAYS check that you're using the right value in circle formulas — don't get the radius and diameter muddled up. If you need the one you haven't been given, multiply or divide by 2.

Circumference Problems with Circles

There's a whole range of circle questions you could be asked — but if you learn the circle formulas you should be fine. Make sure you read the questions carefully to find out what you're being asked to do.

EXAMPLE: Giles is planning a fun run in a park around a circular lake with radius 50 m. If the run needs to be at least 5 km long, what is the smallest number of laps of the lake needed to cover this distance?





First find the diameter of the lake:
Diameter = radius × 2 = 50 × 2 = 100 m

Now find the circumference of the lake:
Circumference = π × diameter = π × 100 = 314.159... m

Convert the run distance to m (see p.49):
5 km = 5000 m

Now divide the run distance by the circumference to see how many laps will be needed:
5000 ÷ 314.159... = 15.915...

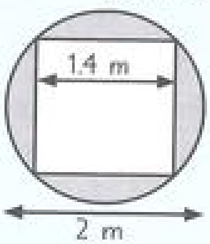
You need to round up as 15 laps wouldn't be far enough (it'd be less than 5 km).

So 16 laps are needed to cover at least 5 km.

Area Problems with Circles

With area questions, you'll sometimes have to subtract one area from another. It's always a good idea to draw a diagram — that way you can see which bit you need to subtract.

EXAMPLE: Molly has a circular tablecloth with diameter 2 m. She cuts out a square with side length 1.4 m from the tablecloth, and throws away the rest of the cloth. How much cloth does she throw away? Give your answer to 2 d.p.



You need to find the shaded area by taking the area of the square away from the area of the circle:

The radius of the circle is 2 ÷ 2 = 1 m

Area of circle = $\pi \times r^2 = \pi \times 1^2 = \pi \times 1$ = 3.141... m^2

Area of square = 1.4 × 1.4 = 1.96 m^2

Area thrown away = area of circle – area of square
= 3.141... – 1.96 = 1.181... = 1.18 m^2 (2 d.p.)

Do your homework outside — it's much airier...

Once you know how to find the circumference and area of a circle, you can tackle pretty much any circle question life throws at you — like this one here:

1) Arun has a circular lawn of radius 20 m with a circular fountain of radius 1 m in the middle of it. Find the area of the lawn, giving your answer to 2 d.p.

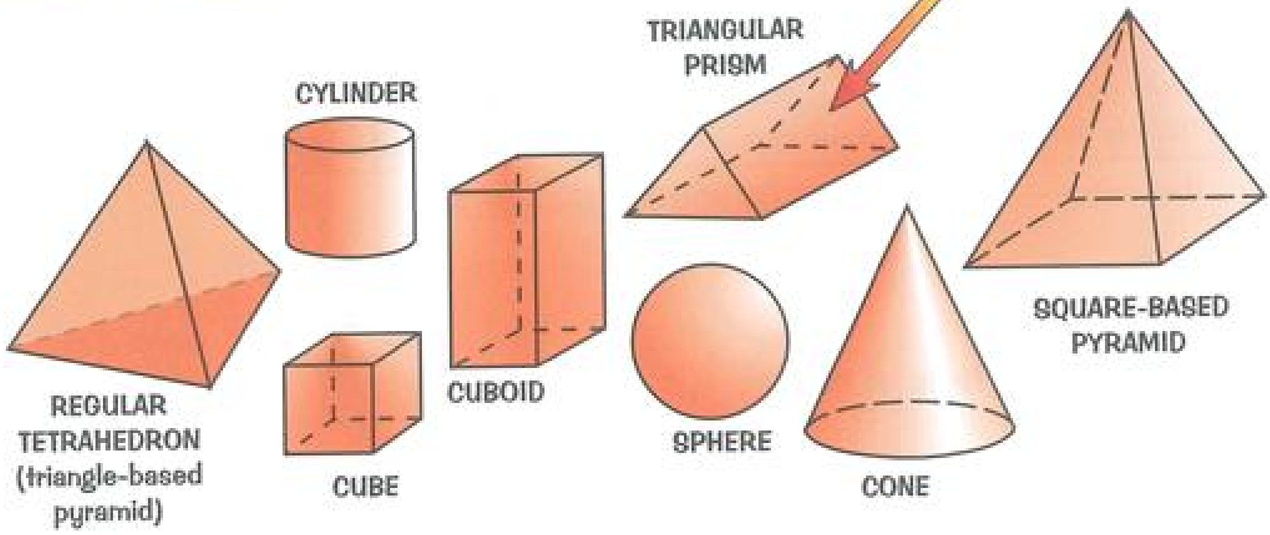
3D Shapes

I was going to make some pop-out 3D shapes to put on this page, but I couldn't find the scissors and sticky tape. Sorry. Still, you need to learn it all though — so chin up and learn the page.

Eight Solids to Learn

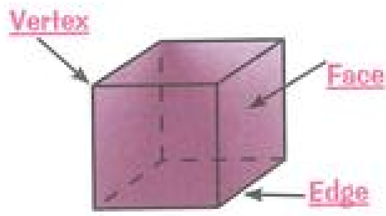
3D shapes are solid shapes. These are the ones you need to know:

There's more about prisms on p.67.



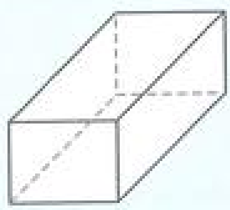
Different Parts of Solids

There are different parts of 3D shapes you need to be able to spot. These are vertices (corners), faces (the flat bits) and edges. You might be asked to find the number of vertices, faces and edges — just count them up, and don't forget the hidden ones.



EXAMPLE:

For the cuboid on the right, write down the number of faces, the number of edges and the number of vertices.



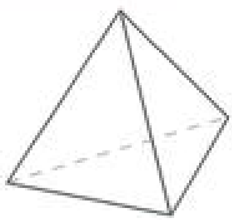
A cuboid has 6 faces (there's one on the bottom and two at the back that you can't see).

It has 12 edges (again, there are some hidden ones — the dotted lines in the diagram).

It has 8 vertices (one is hidden).

Well done — you've made it onto solids...

Remember, it's 1 vertex but 2 vertices (they're funny words — don't let them catch you out). Make sure you know the names of all the different 3D shapes, then have a go at this question.



1) a) Write down the name of the shape on the right.
   b) How many faces, edges and vertices does this shape have?

Nets and Surface Area

Pencils and rulers at the ready — you might get to do some drawing over the next two pages.

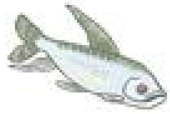
Nets and Surface Area

1) A NET is just a hollow 3D shape folded out flat.
2) SURFACE AREA only applies to 3D objects — it's the total area of all the faces added together.
3) There are two ways to find the surface area:

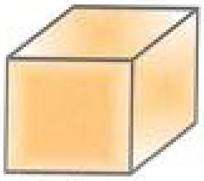
> 1) Work out the area of each face and add them all together (don't forget the hidden faces).
> 2) Sketch the net, then find the area of the net (this is the method we'll use on these pages).

Remember — SURFACE AREA OF SOLID = AREA OF NET.

Cubes

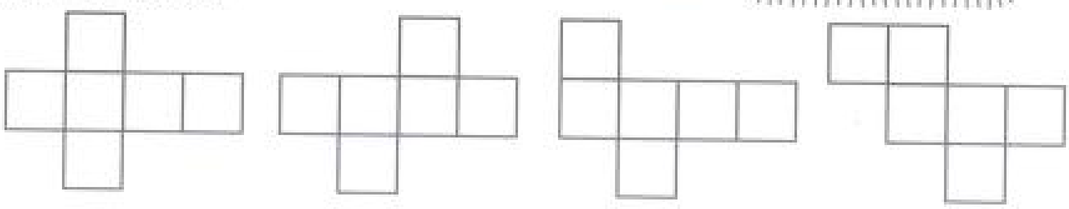


These are just some of the nets of a cube — there are lots more.

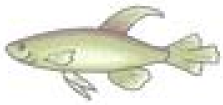


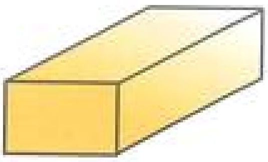
Cube

Nets of cubes

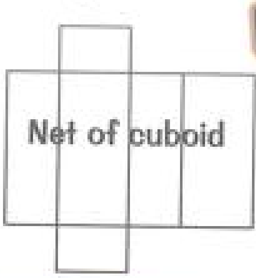


Cuboids



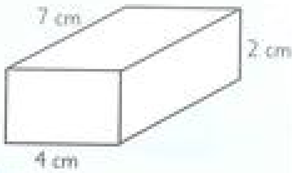


Cuboid

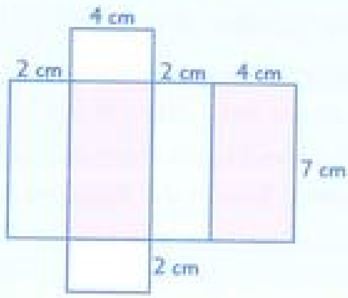


Notice that the net of a cuboid is made up of 3 different sized rectangles (there are 2 of each size). This is helpful when you're working out the surface area.

EXAMPLE: Find the surface area of this cuboid:



Sketch the net of the shape, and label all the measurements:



Then work out the area of each face and add them up (note there are 2 each of 3 different rectangles).

Surface area $= 2(2 \times 7) + 2(4 \times 7) + 2(4 \times 2)$

$= 28 + 56 + 16 = 100 \text{ cm}^2$

What's your net worth? About 25 cm²...

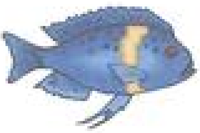
Even if you're not asked to draw the net of a shape, it's often a good idea to do a quick sketch and label it — it's really helpful for finding the surface area. Just make sure you get the labels right.

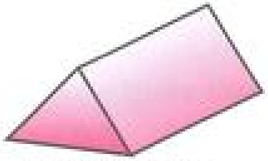
1) Find the surface area of a cube with side length 4 cm.

Nets and Surface Area

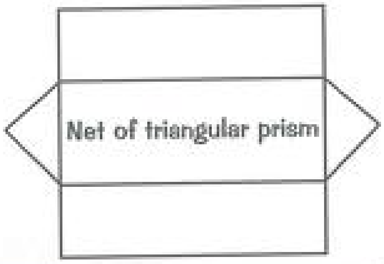
Another page on nets and surface area — and now things get really exciting.

Triangular Prisms



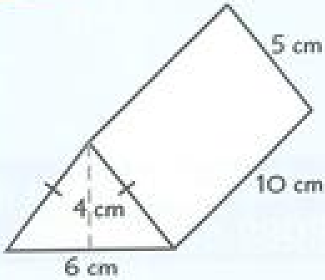


Triangular Prism



EXAMPLE: Find the surface area of the triangular prism below.

Have a look back at p60 for more on areas.



You can see from the net above that a triangular prism has 3 rectangular faces and 2 triangular faces. It's an isosceles triangle, so the rectangular faces will be of two different sizes.

Area of bottom rectangular face = $10 \times 6 = 60\ \text{cm}^2$

Area of side rectangular face = $10 \times 5 = 50\ \text{cm}^2$

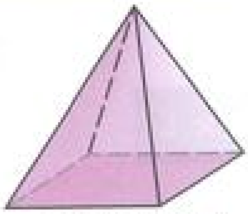
Area of triangular face = $\frac{1}{2} \times 6 \times 4 = 12\ \text{cm}^2$

Total surface area = $60 + (2 \times 50) + (2 \times 12)$

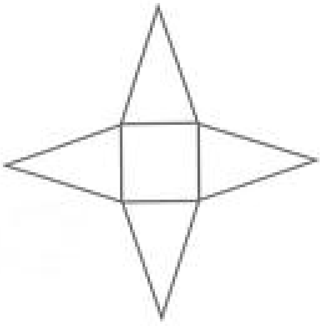
$= 60 + 100 + 24 = 184\ \text{cm}^2$

There are two faces of each of these sizes.

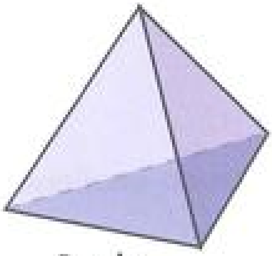
Pyramids



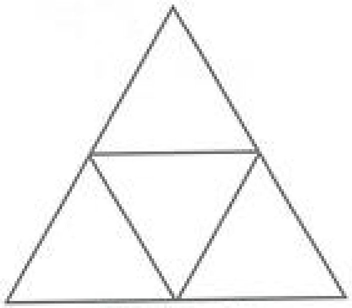
Square-Based Pyramid



Net of square-based pyramid

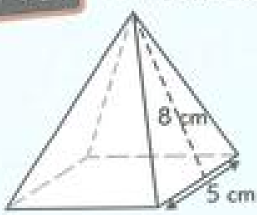


Regular Tetrahedron



Net of regular tetrahedron

EXAMPLE: Find the surface area of the square-based pyramid below.

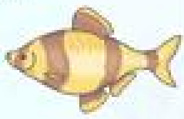


You can see from the net above that a square-based pyramid has 1 square face and 4 triangular faces.

Area of square face = $5 \times 5 = 25\ \text{cm}^2$

Area of triangular face = $\frac{1}{2} \times 5 \times 8 = 20\ \text{cm}^2$

Total surface area = $25 + (4 \times 20) = 25 + 80 = 105\ \text{cm}^2$



Which way to the surfers' area...

You have to be a bit careful when finding the surface area of a triangular prism — the rectangles will be different sizes (unless the triangle is equilateral), so don't get caught out.

1) Find the surface area of a square-based pyramid with base sides measuring 2 cm and triangular faces with vertical height 5 cm.

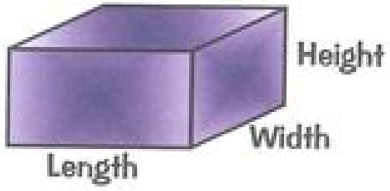
Volume

Now it's time to work out the volumes of 3D shapes.

LEARN these volume formulas...

Volumes of Cuboids

A cuboid is a rectangular block. Finding its volume is dead easy:



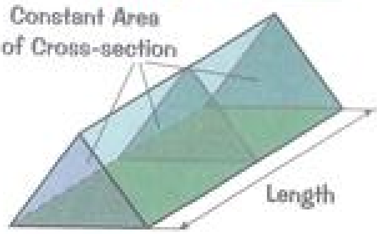
Volume of Cuboid = length × width × height

$V = L \times W \times H$

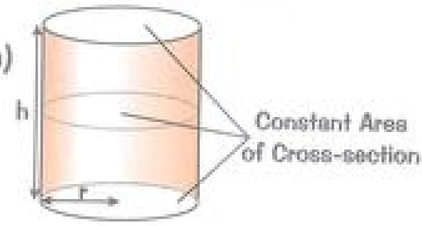
Volumes of Prisms

A PRISM is a solid (3D) object which is the same shape all the way through — i.e. it has a CONSTANT AREA OF CROSS-SECTION.

Triangular Prism



Cylinder (circular prism)



Volume of Prism = cross-sectional area × length $V = A \times L$

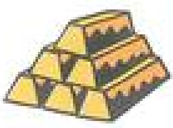
EXAMPLE: Find the volume of the triangular prism on the right.

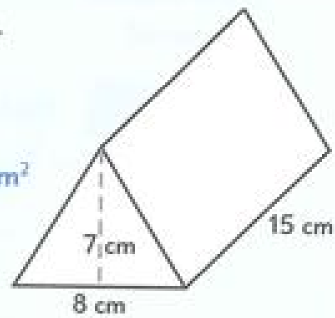
First find the area of the cross-section (using the formula for the area of a triangle from p.60):

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \times 7 = 28\text{ cm}^2$

Then put the numbers into the formula for volume:

$V = A \times L = 28 \times 15 = 420\text{ cm}^3$





Pump up the volume...

Two more formulas to learn on this page, but they're not too bad really. The prism formula works on any prism, no matter what the shape of the cross-section — even if it's something really weird.

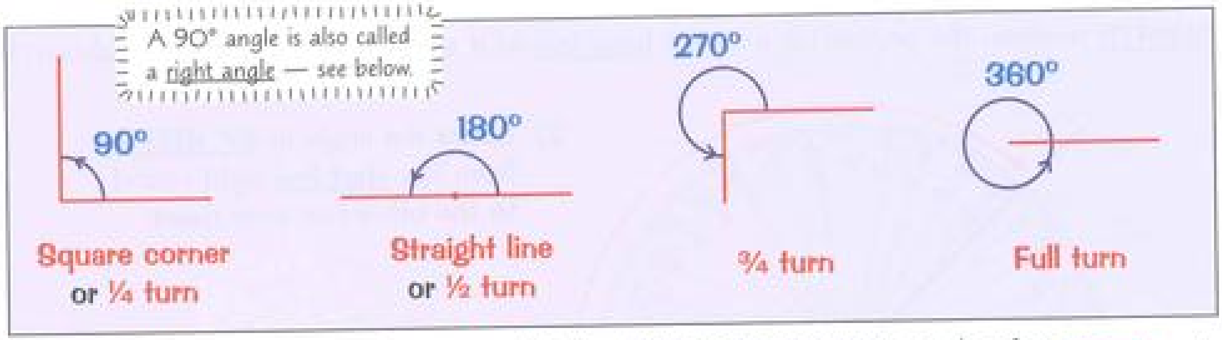
1) The cross-sectional area of an octagonal prism is 28 cm². The length of the prism is 6 cm. Find the volume of the prism.

Lines and Angles

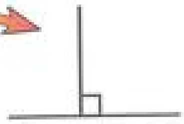
There's nothing too scary on this page — just some special angles and some fancy notation.

Four Special Angles

There are 360° in a full turn, and it can be divided into 4 special angles:



When two lines meet at 90° they are said to be PERPENDICULAR to each other.

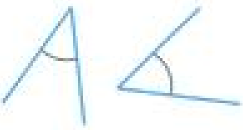


Fancy Angle Names

Some angles have special names which you need to know.

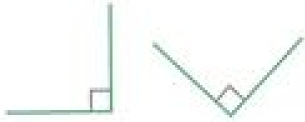
ACUTE angles

Sharp pointy ones (less than 90°)



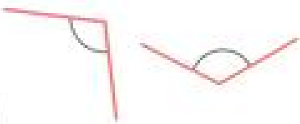
RIGHT angles

Square corners (exactly 90°)



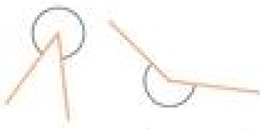
OBTUSE angles

Flatter ones (between 90° and 180°)



REFLEX angles

Ones that bend back on themselves (more than 180°)

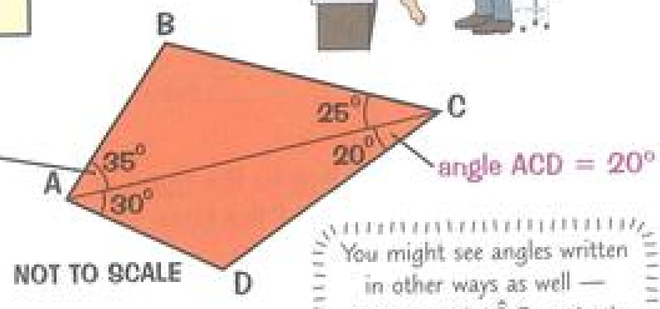


Three-Letter Angle Notation

The best way to say which angle you're talking about in a diagram is by using THREE letters.

For example in the diagram, angle BAC = 35°.

1) The middle letter is where the angle is.
2) The other two letters tell you which two lines enclose the angle.



You might see angles written in other ways as well — ∠ABC and $A\hat{B}C$ are both the same as angle ABC.

Don't be so obtuse...

Make sure you learn all the different names, and don't get caught out by three-letter angle notation.

1) Write down the names of the following angles: a) 98° b) 234° c) 11°.

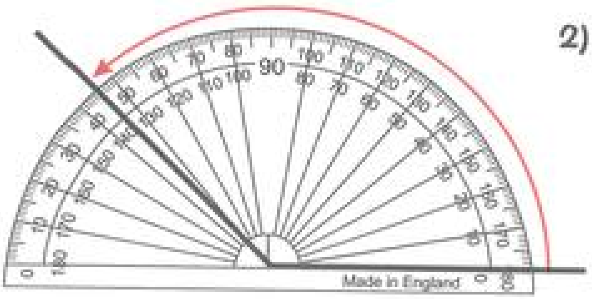
Measuring and Drawing Angles

The 2 big mistakes that people make with protractors:

1) Not putting the 0° line at the start position.
2) Reading from the WRONG SCALE.

Measuring Angles with a Protractor

1) ALWAYS position the protractor with the base line of it along one of the lines as shown here:

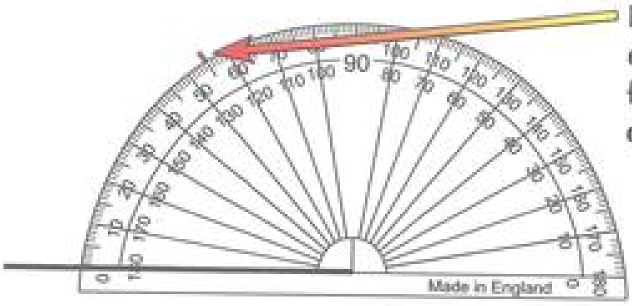


2) Count the angle in 10° STEPS from the start line right round to the other line over there.

DON'T JUST READ A NUMBER OFF THE SCALE — chances are it'll be the wrong one because there are TWO scales to choose from.
The answer here is 135° (NOT 45°) which you will only get right if you start counting 10°, 20°, 30°, 40° etc. from the start line until you reach the other line.

Drawing Angles with a Protractor

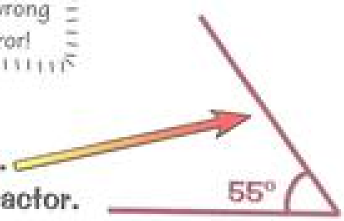
Draw a straight horizontal line to be your base line. Put the protractor on the line so that the middle of the protractor is on one end of the line as shown:



Draw a little line or dot next to the angle you're drawing (count up in tens from 0° to make sure you follow the right scale). Here, I'm drawing an angle of 55°, so I'm using the outside scale.

Be careful — reading from the wrong scale is a very very common error!

Then join your base line to the mark you've just made with a straight line. You must join the end of the base line that was in the middle of the protractor.



All the pros use tractors...

The best way to get to grips with the stuff on this page is to have a go yourself — get out your protractor, ruler and pencil and go crazy. Well, don't actually go crazy — just draw some angles.

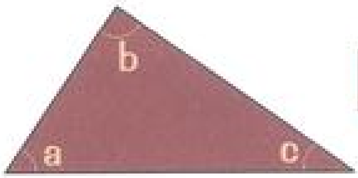
1) Draw an angle measuring: a) 45° b) 70° c) 150°.
2) Measure the angles you've just drawn to check that they're correct.

Five Angle Rules

The angle rules on this page are really important — they pop up all over the place, so make sure you learn them all. Then learn them again, just to make sure.

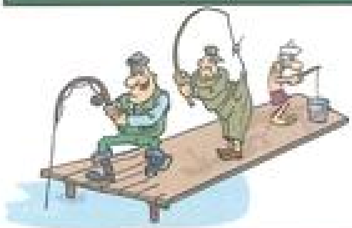
5 Simple Rules — That's All

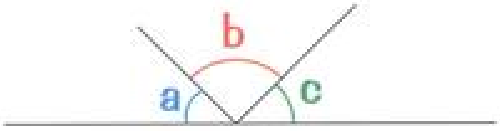
1) Angles in a triangle add up to 180°.



$a + b + c = 180°$

2) Angles on a straight line add up to 180°.

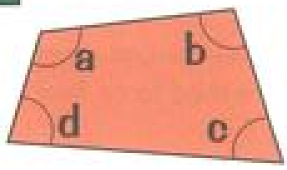




$a + b + c = 180°$

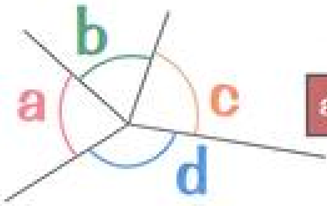
3) Angles in a quadrilateral add up to 360°.

Remember that a quadrilateral is a 4-sided shape.



$a + b + c + d = 360°$

4) Angles round a point add up to 360°.

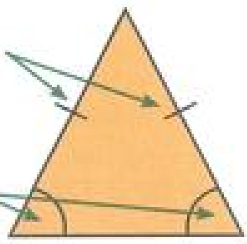


$a + b + c + d = 360°$

5) Isosceles triangles have 2 sides the same and 2 angles the same.

These dashes indicate two sides the same length.

These angles are the same.



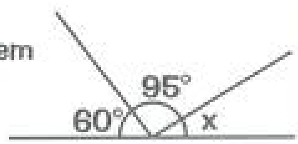
In an isosceles triangle, you only need to know one angle to be able to find the other two.

There are some examples of using these rules on the next page.

Visit the Angle of the North...

None of the rules here are particularly difficult, but make sure you don't get them mixed up. Once you're happy with them all, have a go at this question.

1) Find the size of angle x in the diagram on the right.

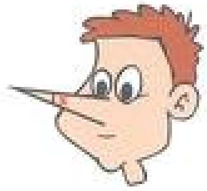


Five Angle Rules

It's time to see all the angle rules from the previous page in action. They do all their own stunts.

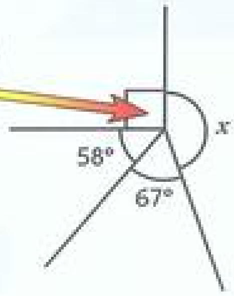
Using One Rule

It's a good idea to write down the rules you're using when finding missing angles — it helps you keep track of what you're doing.



EXAMPLE: Find the size of angle x.

Remember — this little square means that it's a right angle (90°).

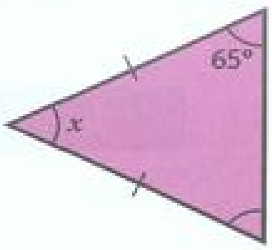


Use rule 4:
Angles round a point add up to 360°,
so $x + 90° + 58° + 67° = 360°$
$x = 360° - 90° - 58° - 67° = 145°$

Using More Than One Rule

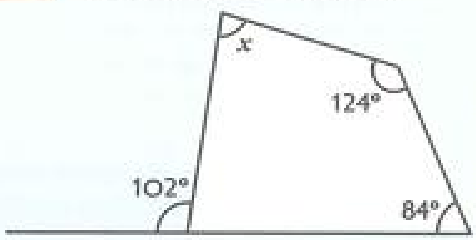
It's a bit trickier when you have to use more than one rule — but writing down the rules is a big help again. The best method is to find whatever angles you can until you can work out the ones you're looking for.

EXAMPLE: Find the size of angle x.



The dashes mean it's an isosceles triangle, so use rule 5:
Isosceles triangles have 2 sides and 2 angles the same.
So the angles on the right of the triangle are both 65°.
Now use rule 1:
Angles in a triangle add up to 180°, so $65° + 65° + x = 180°$
$x = 180° - 130°$
$x = 50°$

EXAMPLE: Find the size of angle x.



First use rule 2 to find the unknown angle at the bottom of the quadrilateral:
$180° - 102° = 78°$
Then use rule 3:
$x + 78° + 124° + 84° = 360°$
$x = 360° - 78° - 124° - 84° = 74°$

I've lost my favourite angle — will you help me find it...

If you're really stuck, just fill in any angles you can find and see where it gets you.

1) A quadrilateral has angles measuring 74°, 86° and 146°. Find the size of the other angle.

Parallel Lines

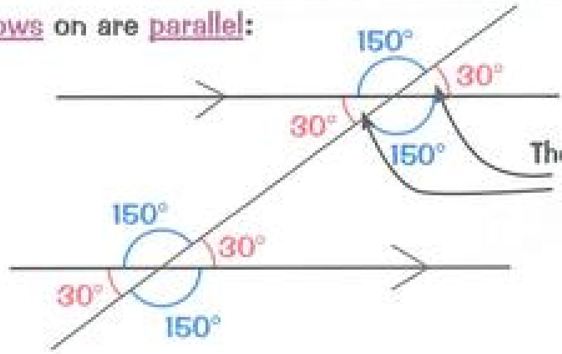
Parallel lines point in the same direction. They're always the same distance apart and never meet.

Angles Around Parallel Lines

When a line crosses two parallel lines...

1) The two bunches of angles are the same.
2) There are only two different angles: a smaller one and a bigger one.
3) These ALWAYS ADD UP TO 180°. E.g. 30° and 150° below.

The two lines with the arrows on are parallel:

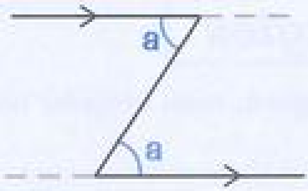


These are vertically opposite angles. They're equal to each other.

Alternate and Corresponding Angles

Watch out for these 'Z' and 'F' shapes popping up.
They're a dead giveaway that you've got a pair of parallel lines.

ALTERNATE ANGLES

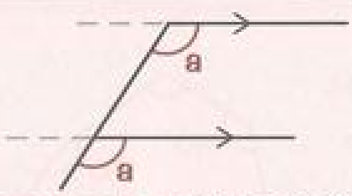


Alternate angles are the same.
They are found in a Z-shape.

EXAMPLE: Find the size of angle x.

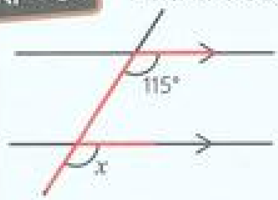
This diagram shows alternate angles (spot the backwards Z-shape).
Alternate angles are the same, so $x = 122°$

CORRESPONDING ANGLES



Corresponding angles are the same.
They are found in an F-shape.

EXAMPLE: Find the size of angle x.

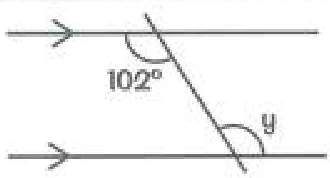


This diagram shows corresponding angles (spot the characteristic F-shape).
Corresponding angles are the same, so $x = 115°$

Keep your feet parallel and you'll be fine...

It's OK to use the letters Z and F to help you identify the angles, but make sure you know the proper names too.

1) Find the size of angle y in the diagram on the right.



Interior and Exterior Angles

You're not quite done with angles yet (sorry) — now it's time for angles in polygons.

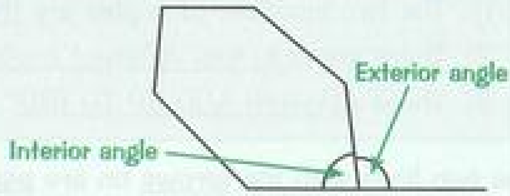
Exterior and *Interior* Angles

You need to know what exterior and interior angles are and how to find them.

For ANY POLYGON (regular or irregular):

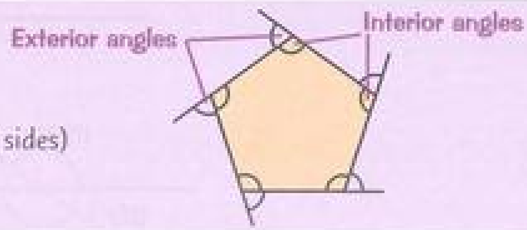
SUM OF EXTERIOR ANGLES = 360°

INTERIOR ANGLE = 180° – EXTERIOR ANGLE



For REGULAR POLYGONS only:

$$\text{EXTERIOR ANGLE} = \frac{360°}{n}$$ (n is the number of sides)



EXAMPLE:

Find the exterior and interior angles of a regular hexagon.

Hexagons have 6 sides: exterior angle $= \frac{360°}{n} = \frac{360°}{6} = 60°$

Use the exterior angle to find the interior angle: interior angle = 180° – exterior angle
= 180° – 60° = 120°

The Tricky One — Sum of Interior Angles

This formula for the sum of the interior angles works for ALL polygons, even irregular ones.

SUM OF INTERIOR ANGLES = (n – 2) × 180° (n is the number of sides)

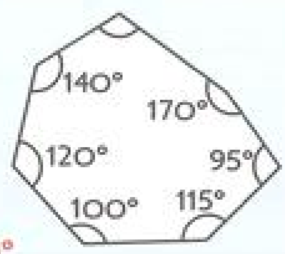
EXAMPLE:

Find the sum of the interior angles of the polygon on the right.

The polygon is a pentagon, so n = 5:
Sum of interior angles = (n – 2) × 180° = (5 – 2) × 180° = 540°

EXAMPLE:

Find the missing angle in the diagram on the right.



First, find the sum of the interior angles of the 7-sided shape:

Sum of interior angles = (n – 2) × 180°
= (7 – 2) × 180° = 900°

Subtract from 900° to find the missing angle:
900° – 170° – 95° – 115° – 100° – 120° – 140° = 160°

Octagon lottery winner: I'm still just a regular guy...

1) Work out the size of an exterior angle and an interior angle of a regular octagon.

Transformations

There are 4 transformations you need to know — translation, reflection, rotation and enlargement.

1) Translations

A translation is just a SLIDE around the page. When describing a translation, you must say how far along and how far up the shape moves using a vector.

> Vectors describing translations look like this. $\rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$
> x is the number of spaces right, y is the number of spaces up.

If the shape moves left x will be negative, and if it moves down y will be negative.

EXAMPLE: Describe the transformation that maps:

a) triangle A onto triangle B.

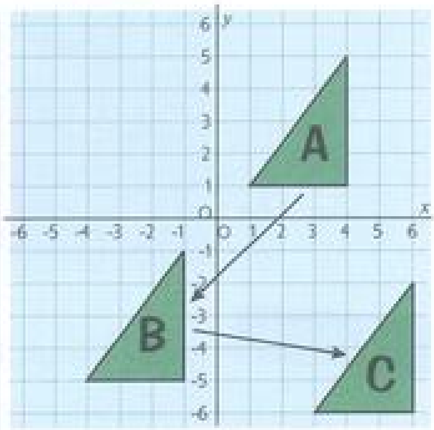
To get from triangle A to triangle B you need to move 5 units left and 6 units down, so it's:

A translation by the vector $\begin{pmatrix} -5 \\ -6 \end{pmatrix}$

b) triangle B onto triangle C.

It's a movement of 7 units right and 1 unit down, so it's:

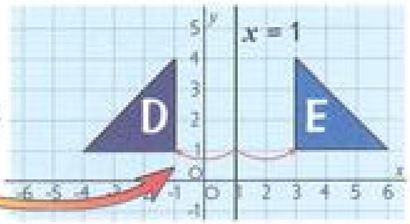
A translation by the vector $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$



2) Reflections

See p.37-38 for more on straight lines.

Triangle D is mapped onto triangle E by a reflection in the line $x = 1$.
Notice that the matching corners are equal distances from the mirror line.

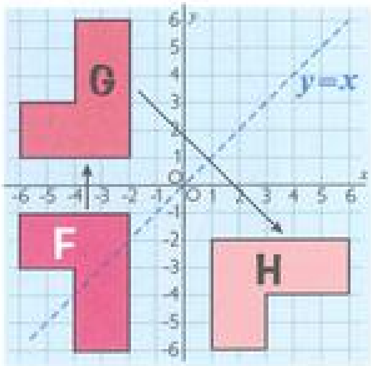


To describe a reflection, you must give the equation of the mirror line.

EXAMPLE: Describe the transformation that maps:

a) Shape F onto shape G.
A reflection in the x-axis

b) Shape G onto shape H.
A reflection in the line $y = x$



2 down, 2 to go...

Remember what information you have to give for each transformation — for a translation, you need to give the vector, and for a reflection, you need to give the equation of the mirror line.

1) Describe the transformation that maps triangle A onto triangle C in the diagram above.

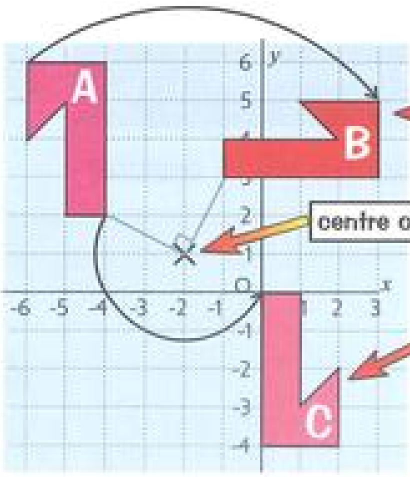
Transformations

Transformation number 3 coming up — rotation.

3) Rotations

To describe a rotation, you need 3 details:

1) The angle of rotation (usually 90° or 180°).
2) The direction of rotation (clockwise or anticlockwise).
3) The centre of rotation



Shape A is mapped onto Shape B by a rotation of 90° clockwise about point (–2, 1).

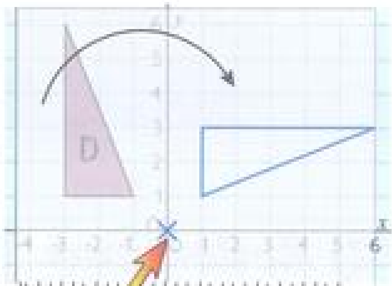
Shape A is mapped onto Shape C by a rotation of 180° about point (–2, 1).

For a rotation of 180°, it doesn't matter whether you go clockwise or anticlockwise.

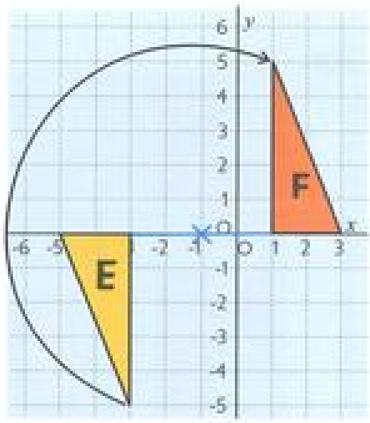
EXAMPLE: Rotate Triangle D 90° clockwise about (0, 0).

The best way to tackle this is with tracing paper:

1) Trace the shape and mark the centre of rotation at (0, 0).
2) Put your pencil point on the centre of rotation and rotate the tracing paper 90° clockwise. You'll know when you've gone far enough — the horizontal side will be vertical, and vice versa.
3) Mark the corners of the shape in their new positions on the grid, then draw the shape.



Hold the tracing paper down with your pencil point here.



EXAMPLE: Describe the transformation that maps Triangle E onto Triangle F.

A rotation of 180° about (–1, 0).

You can use tracing paper to help you find the centre of rotation. Trace the original shape and then try putting your pencil on different points until the traced shape rotates onto the image. When this happens your pencil must be on the centre of rotation.

Finding the centre of the Earth sounds more exciting...

This time, you have to give three details about the transformation — the angle, the direction and the centre of rotation. Don't forget any — otherwise you won't have fully described the transformation.

1) Triangle G has corners (–3, 2), (–3, 4) and (–8, 2). Draw triangles F (above) and G on a graph, and describe the rotation that maps triangle F onto triangle G.

Enlargements

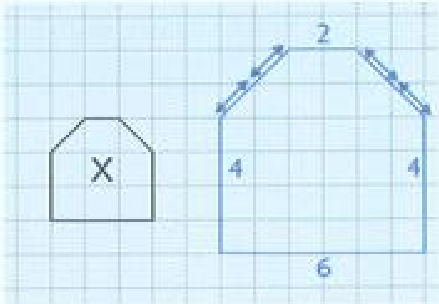
You've made it to the final transformation now — and it's a good one. Get ready for... enlargements.

4) Enlargements

The scale factor for an enlargement tells you how long the sides of the new shape are compared to the old shape. E.g. a scale factor of 3 means you multiply each side length by 3.

EXAMPLE: Enlarge shape X by a scale factor of 2.

Make each side twice as long as the matching side on shape X. Start with the horizontal and vertical sides. Take care with the sloping sides — they're much trickier.



Describing an Enlargement

For an enlargement, you must specify:

1) The scale factor.
2) The centre of enlargement.

There's a formula for the scale factor: $\text{scale factor} = \frac{\text{new length}}{\text{old length}}$

EXAMPLE: Describe the transformation that maps Triangle A onto Triangle B.

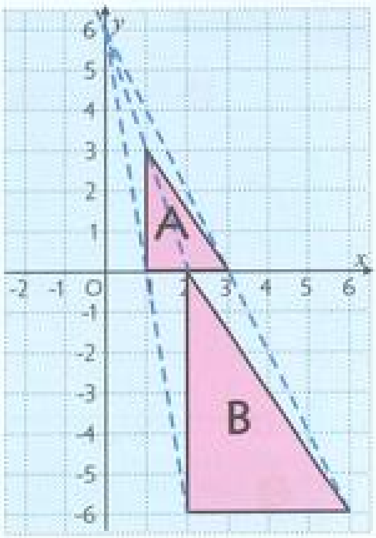
Use the formula to find the scale factor. (Just do this for one pair of sides.)

Old length of triangle base = 2 units
New length of triangle base = 4 units

$\text{scale factor} = \frac{\text{new length}}{\text{old length}} = \frac{4}{2} = 2$

To find the centre of enlargement, draw lines that go through matching corners of both shapes and see where they cross.

So the transformation is an enlargement of scale factor 2, centre (0, 6).



I think I have what it takes to find the Scale Factor...

Here, you need to give the scale factor and the centre of enlargement to describe the transformation.

1) Plot the triangles X (1, 2), (–1, 3), (–1, 5) and Y (7, –4), (1, –1), (1, 5) on a grid and find the scale factor for the enlargement that maps triangle X onto triangle Y.

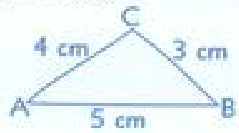
Triangle Construction

How you construct a triangle depends on what information you're given about the triangle...

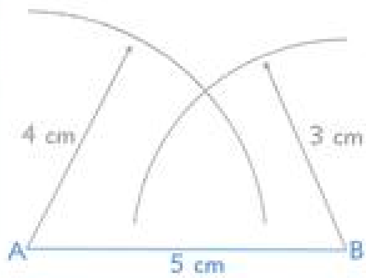
Three Sides — Use a Ruler and Compasses

EXAMPLE: Construct the triangle ABC where AB = 5 cm, BC = 3 cm, AC = 4 cm.

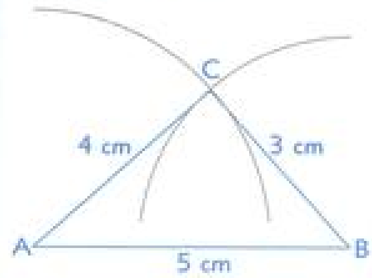
1. First, sketch and label a triangle so you know roughly what's needed. It doesn't matter which line you make the base line.



2. Draw the base line. Label the ends A and B.



3. For AC, set the compasses to 4 cm, put the point at A and draw an arc. For BC, set the compasses to 3 cm, put the point at B and draw an arc.



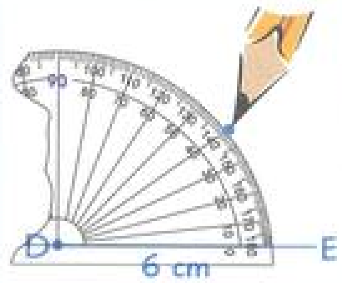
4. Where the arcs cross is point C. Now you can finish your triangle.

Sides and Angles — use a Ruler and Protractor

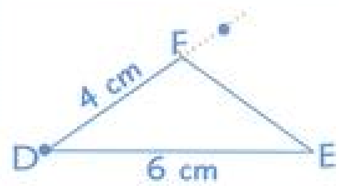
EXAMPLE: Construct triangle DEF. DE = 6 cm, DF = 4 cm, and angle EDF = 40°.

1. Roughly sketch and label the triangle.

2. Draw the base line.



3. Draw angle EDF (the angle at D) — place the centre of the protractor over D, measure 40° and put a dot.



4. Measure 4 cm towards the dot and label it F. Join up D and F. Now you've drawn the two sides and the angle. Just join up F and E to complete the triangle.

Warning — construction site...

Always do your constructions in pencil — that way, if it goes a bit wrong you can always rub it out.

1) Construct a triangle with sides 3 cm, 4 cm and 4 cm. Leave your construction marks showing.

Constructions

The best way to learn how to do these constructions is to have a go.

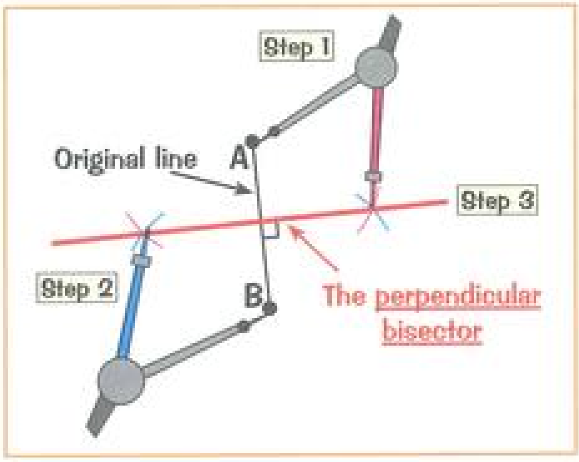
Two Constructions

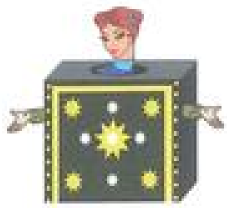
When you're doing these constructions, make sure you:

1) Keep the compass setting THE SAME while you make all the marks.
2) Make sure you leave your compass marks showing.

1 The Perpendicular Bisector of a Line

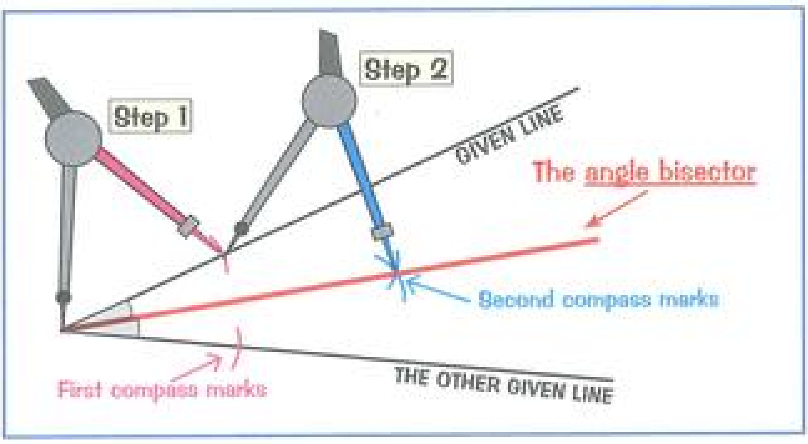
The perpendicular bisector of line segment AB is a line at right angles to AB, passing through the midpoint of AB. This is the method to use if you're asked to draw it.

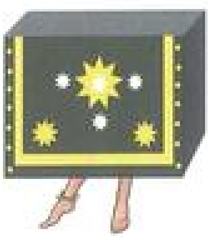




2 The Bisector of an Angle

The bisector of an angle divides an angle exactly in half — so you get two equal angles.





On your marks, get set, construct...

Don't rub out your compass marks when doing constructions — leave all your working showing.

1) Do each of the constructions on this page. Make sure you leave all your construction lines.

Revision Summary for Section 4

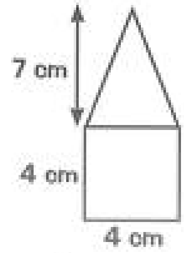
You've reached the end of the section — guess what you have to do now...

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

2D Shapes (p56-59)

1) Write down the number of lines of symmetry and order of rotational symmetry for a rhombus.
2) Name 2 quadrilaterals that have 2 pairs of equal angles.
3) Name the 4 different types of triangle.
4) A regular polygon has 7 sides. What is the name of this polygon?
5) What are congruent and similar shapes?

Perimeter and Area (p60-64)



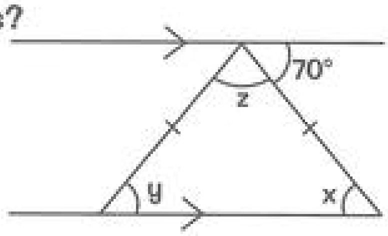
6) Find the perimeter and area of a rectangle that measures 11 cm by 5 cm.
7) Find the area of a triangle with base 12 cm and vertical height 8 cm.
8) Find the area of the shape on the right.
9) Find the circumference and area of a circle with radius 8 cm.
10) How many times must a wheel with diameter 10 cm turn to cover more than 1 m?

3D Shapes (p65-68)

11) Write down the number of faces, edges and vertices of a triangular prism.
12) Find the surface area and volume of a cuboid measuring 2 cm by 3 cm by 5 cm.
13) Sketch a net of a regular tetrahedron.
14) Find the volume of a prism with cross-sectional area 40 cm² and length 10 cm.

Angles (p69-74)

15) Give an example of a) an acute angle, b) an obtuse angle, c) a reflex angle.
16) Draw an angle that measures 35°.
17) What are the five angle rules?
18) What type of angles do you find in a Z-shape on parallel lines?
19) For the diagram on the right, find the size of:
    a) angle x b) angle y c) angle z.



20) Work out the size of an exterior angle and an interior angle of a regular decagon (a 10-sided shape).
21) What's the sum of the interior angles in a hexagon?

Transformations (p75-77)

22) What 3 details must you give when describing a rotation?
23) Draw a triangle with coordinates (1, 1), (4, 1) and (3, 4). Enlarge the triangle by a scale factor 3 and centre (–1, 0) and write down its coordinates after the enlargement.

Constructions (p78-79)

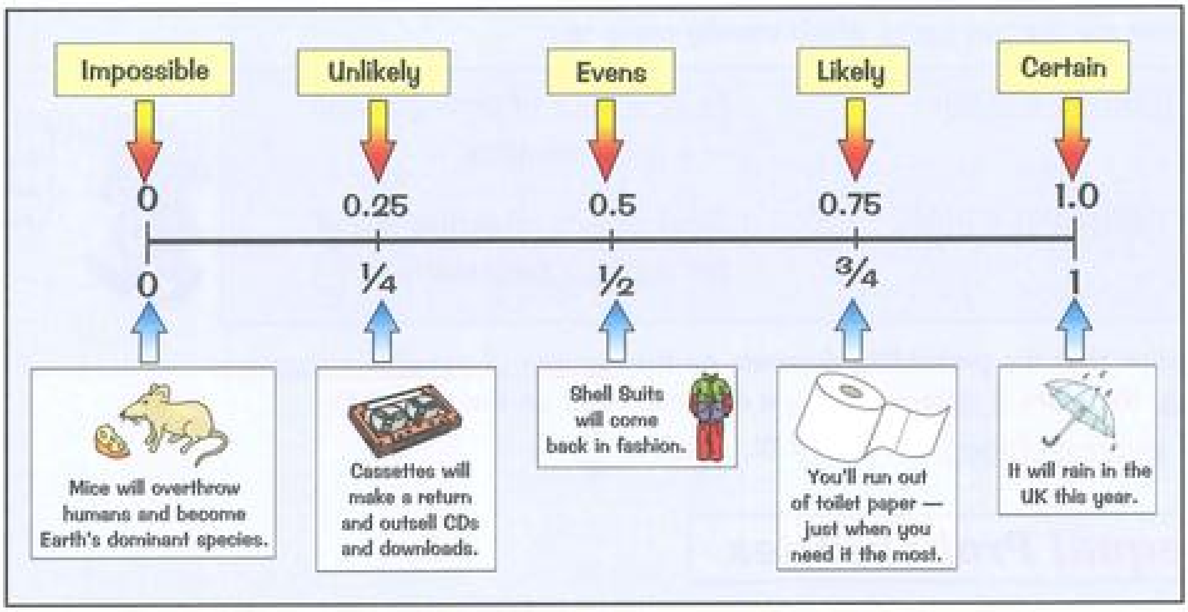
24) Construct triangle ABC, where AB = 3 cm, AC = 3.5 cm and CAB = 50°.
25) Draw an acute angle of any size, then bisect it, leaving your construction marks showing.

Probability

An '<u>event</u>' is just something that happens, and the <u>probability</u> of an event is <u>how likely</u> it is.

All Probabilities are Between 0 and 1

1) Probabilities can only have values <u>from 0 to 1</u> (including 0 and 1).
2) You should be able to put the probability of <u>any event</u> happening on this <u>scale of 0 to 1</u>.

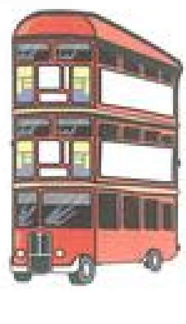


3) You can give probabilities using <u>FRACTIONS</u>, <u>DECIMALS</u> or <u>PERCENTAGES</u>.
   E.g. a <u>probability of ½</u> can also be written as <u>0.5</u> or <u>50%</u>.
4) To <u>save on words</u>, we can write e.g:
   'the probability of tossing a coin and getting a head is <u>0.5</u>' as: <u>P(H) = 0.5</u>.
   P is for probability
   H is for head

The Probability of Something Not Happening = 1 – P

Probabilities always ADD UP to 1.

1) If <u>only one</u> possible result can happen at a time, then the probabilities of <u>all</u> the results <u>add up to 1</u>.
2) If the probability of something happening is <u>0.3</u>, then the chance of it <u>NOT HAPPENING</u> is <u>1 – 0.3</u> = <u>0.7</u>.
3) It's what's left when you <u>subtract it from 1</u> (or 100% for percentages).



EXAMPLE: The probability that a bus will be late is 0.75. Find the probability that the bus won't be late.

P(not late) = 1 – P(late) = 1 – 0.75 = 0.25

What do you mean? Shell suits are always in fashion...

1) The probability that a netball team wins is 0.6. What is the probability that it doesn't win?

Equal and Unequal Probabilities

Probabilities can be given as fractions, decimals or percentages, so make sure you know how to convert between them (have a look back at p.18 if you're a bit wobbly).

Equal Probabilities

1) When the different results or 'outcomes' of something happening all have the same chance of happening, then the probabilities will be EQUAL.
2) These are the two cases which usually come up:

1) TOSSING A COIN:	Equal chance of getting a head or a tail (probability = $\frac{1}{2}$)
2) THROWING A DICE:	Equal chance of getting any of the numbers (probability = $\frac{1}{6}$)

3) Notice that the probability depends on the number of possible outcomes. E.g. there are 6 different ways a dice can land, so the probability of any one of them is '1 out of 6', which is $\frac{1}{6}$.

Unequal Probabilities

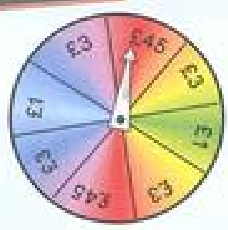
In most cases, different outcomes have different probabilities — some are more likely than others.

EXAMPLE: A ball is picked at random from a bag containing 7 blue balls, 8 red balls and 5 green balls. Find P(blue).

The chances of picking out the three colours are NOT EQUAL because there are different numbers of balls in each colour.
The probability of picking a blue is:

$$P(\text{blue}) = \frac{\text{number of blues}}{\text{total number of balls}} = \frac{7}{20}$$

EXAMPLE: What is the probability of winning £45 on this spinner?



The pointer has the same chance of stopping on every sector.
There are 2 out of 8 which say £45, so

$$P(£45) = \frac{2}{8} = \frac{1}{4} \quad \text{(or 0.25 or 25\%)}$$

The formula to use is:

$$\text{Probability} = \frac{\text{Number of ways for something to happen}}{\text{Total number of possible outcomes}}$$

P(green) depends on how old the cheese is...

You'll often have to add things up to find the number to divide by — like in the balls in a bag example.

1) A box of chocolates contains 4 dark chocolates, 8 milk chocolates and 3 white chocolates. What is the probability of picking a milk chocolate?

Listing Outcomes

Listing outcomes is quite easy — all you have to do is write down all the things that could happen.

Listing All Outcomes: Use a Sample Space

1) You might get asked to list all the possible outcomes for TWO THINGS HAPPENING TOGETHER. A simple question might be to list all the possible results from tossing two coins:

> The possible outcomes from TOSSING TWO COINS are:
>
> HH HT TH TT

'TH' means tails on the first coin and heads on the second.

2) You can work out probabilities from your list.
E.g. P(TT) = $\frac{1}{4}$, because there are 4 outcomes in total, and only 1 of these is TT.

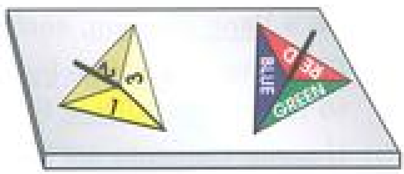
3) For harder questions, you're better off listing all the possible results in a sample space diagram — a posh name for a table.

4) This sample space diagram shows the possible outcomes for the spinners below:

	Red	Blue	Green
1	1R	1B	1G
2	2R	2B	2G
3	3R	3B	3G

3 different outcomes on colour spinner.

3 different outcomes on number spinner.



5) Any number on the number spinner could come up with any colour on the colour spinner. Spinning them together gives 3 × 3 = 9 different combinations altogether. The sample space is a list of these 9 outcomes.

6) The probability of spinning e.g. a 2 and a GREEN (2G) is 1 out of 9, so $\boxed{P(2G) = \frac{1}{9}}$.

7) If both spinners are number spinners, you can also fill in the sample space with the total of the two numbers:

EXAMPLE: Two spinners numbered 1-3 are spun and the scores added together. Fill in the sample space diagram, and use it to find the probability that the score is 5.

1st spinner \ 2nd spinner	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

These are the totals of the two spinners, e.g. 3 + 3 = 6.

There are 2 different ways of scoring 5 out of 9 possible outcomes, so

$P(5) = \frac{2}{9}$

I'd like to sample some ice cream...

Drawing a table is often the best way to make sure you don't miss any outcomes.

1) A dice is rolled and a coin is tossed. Draw a sample space diagram showing the possible outcomes.

Venn Diagrams

Venn diagrams are a way of displaying data in intersecting circles — they're very pretty.

A Set is a Collection of Objects

1) Sets are just collections of things (e.g. numbers).
2) A pair of curly brackets {} tell you it's a set.
3) The things in the set are called elements.

Here are some examples of sets:

Elements

Curly brackets show it's a set

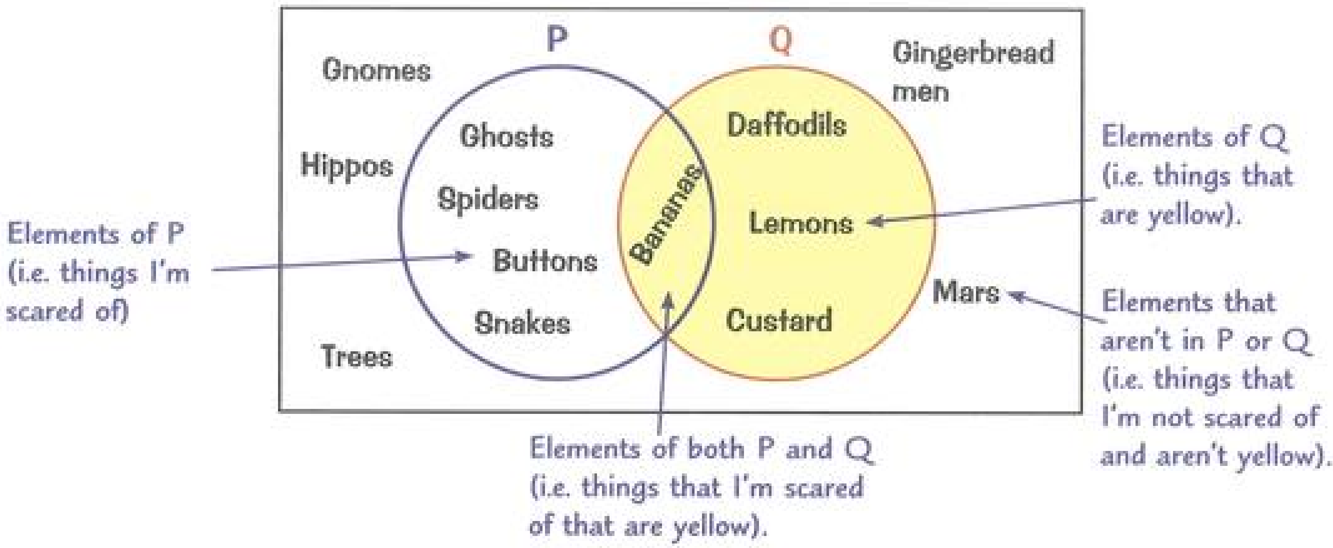
A = {2, 4, 6, 8}   B = {dog, cat, hamster, rabbit, goldfish}

C = {odd numbers}   D = {students in Year 9}

4) n(A) just means 'the number of elements in set A'. So here, n(A) = 4 and n(B) = 5.

Show Sets on Venn Diagrams

1) On a Venn diagram, each set is represented by a circle.
2) Elements of a set go inside its circle. Elements that don't belong to the set go outside the circle.
3) If some elements are in both sets, the circles overlap and these elements go in the overlap (this is called the intersection).
4) So if set P was {things that I'm scared of} and set Q was {things that are yellow}, the Venn diagram would look something like this:



Venn vill I see you again...

Venn diagrams are actually quite fun — have a go at drawing your own for this question.

1) A = {1, 3, 5, 7, 9} and B = {2, 3, 5, 7, 11}. Draw a Venn diagram to show A and B.
2) For the sets above, find n(P) and n(Q) (don't forget the element in the intersection).

Venn Diagrams

Another page on Venn diagrams now — and things get a little trickier here.

The Universal Set is the Set of Everything

1) The universal set can be a bit confusing. It's the group of things that the elements of a set are selected from — so if A = {even numbers}, the universal set might be {numbers from 1-20}, which means that A is all the even numbers from 1-20.
2) The universal set is shown by this funny symbol: ξ.
3) On a Venn diagram, the universal set is a rectangle that goes round the outside of the circles.

EXAMPLE: ξ = {numbers from 1 to 10}, A = {factors of 10} and B = {prime numbers}.

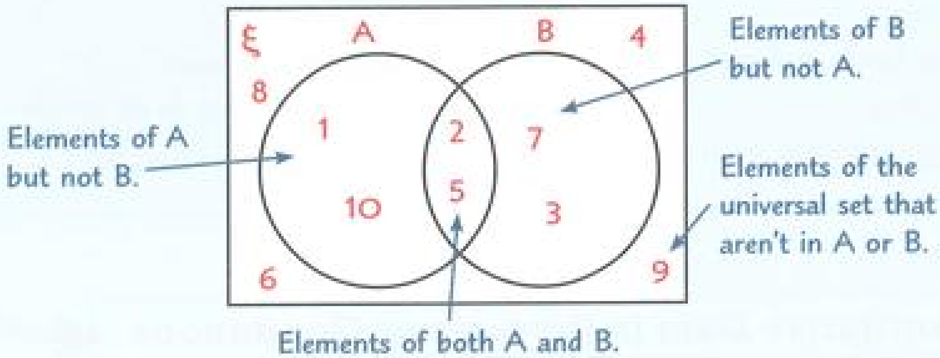
a) Find A and B.

Just write out the elements of each set:
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 2, 5, 10} and B = {2, 3, 5, 7}

It's a good idea to write out the universal set (as long as it's not too big) so you can see which elements you have to choose from.

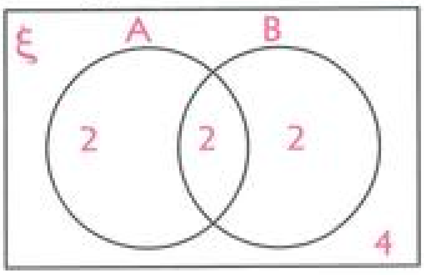
b) Show sets A and B on a Venn diagram.

Use the sets you've just found to put the numbers in the right places.



Show the Number of Elements on a Venn Diagram

You can also show the number of elements in each set on a Venn diagram — so for the example above, the Venn diagram would look like this:



My favourite element is Xenon...

Always check whether you have to put in the actual elements or just the number of elements.

1) ξ = {numbers from 1-20}, X = {odd numbers} and Y = {square numbers}. Find X and Y, then draw a Venn diagram showing the number of elements in each set.

Types of Data

Data is a fancy word for information. There are different types of data, which have fancy names...

Data can be Primary or Secondary

PRIMARY data is data YOU'VE collected.

There are two main ways you can get primary data:

- A SURVEY, e.g. a questionnaire.
- An EXPERIMENT (like you do in science lessons).

SECONDARY data is collected by SOMEONE ELSE.

There are lots of ways you can get secondary data, e.g. from:

- newspapers
- the internet
- databases
- historical records

Data can be Qualitative or Quantitative

Think 'quantity means numbers' as a way to remember which is which.

QUALITATIVE data is in WORD form.

For example:

- gender (male or female)
- eye colour
- favourite football team

QUANTITATIVE data is in NUMBER form.

For example:

- heights of people
- the time taken to do a task
- the weight of objects

Quantitative Data is Discrete or Continuous

DISCRETE DATA can be measured exactly — in whole numbers or certain values.

For example:

- the number of goals scored
- the number of people in a shop
- the number of pages in this book

CONTINUOUS DATA can take any value over a certain range.

For example:

- the height of this page (it's 297 mm to the nearest mm but that's not its exact height)
- the weight of a pumpkin
- the length of a carrot

Sorry, I can't date 'er — she's just not my type...

You might think this page is just boring definitions — well, I can't say it is but I'm not saying it isn't.

1) Say whether this data is qualitative, discrete or continuous:
   a) The number of spectators at a rugby match.
   b) The colours of pebbles on a beach.
   c) The number of books on a shelf.
   d) The length of my arm.

Line Graphs and Pictograms

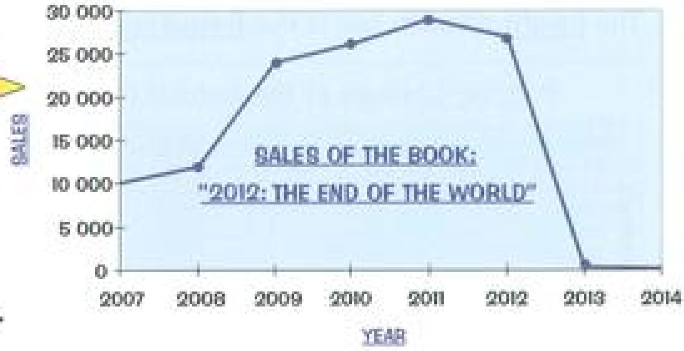
Data can be shown in different types of charts, tables and graphs.
Line graphs and pictograms make it easy to see what the data shows.

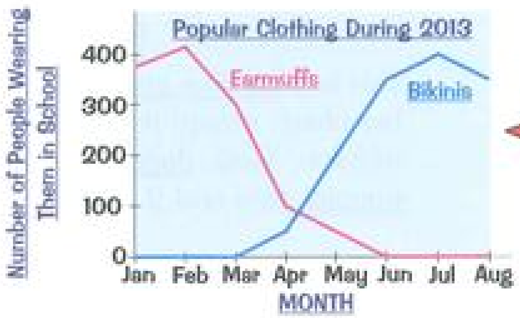
Line Graphs

1) A line graph is a set of points joined with straight lines.

2) They often have 'time' along the bottom to show how something changes over time:

3) You can draw two line graphs on the same grid to compare two things, as shown below.





These graphs show clearly that as the year went on, fewer people wore earmuffs and more people wore bikinis...

Pictograms

— these use pictures instead of numbers.

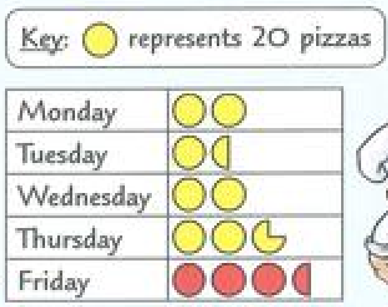
Every pictogram has a key telling you what one symbol represents.

With pictograms, you MUST use the KEY.

EXAMPLE: This pictogram shows how many pizzas were sold by a pizzeria on different days.

a) How many pizzas were sold on Tuesday?
There's 1 whole circle (= 20 pizzas)...
...plus half a circle (= 10 pizzas). **30 pizzas**

b) 70 pizzas were sold on Friday.
Use this information to complete the diagram.
You need 3 whole circles (= 60 pizzas), plus another half a circle (= 10 pizzas).



Picture this — a world with no maths...

= 500 talking cats

Have a go at drawing your own line graphs and pictograms to get the hang of them. You can come up with your own symbols for pictograms — it's time to unleash your inner artist. Mine's called Jon.

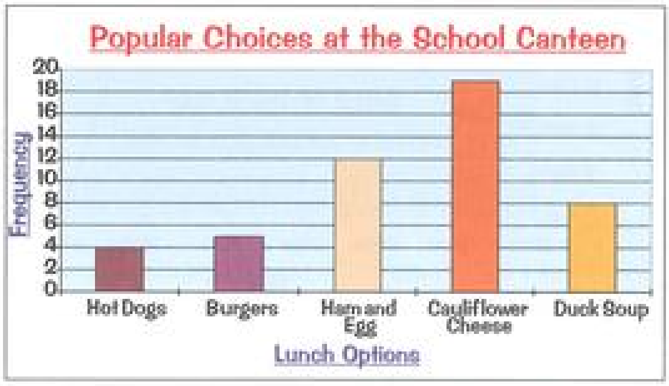
1) Using the line graph above, estimate how many more people wore earmuffs than bikinis in April.
2) Use the pictogram above to work out how many pizzas were sold on Thursday.

Bar Charts

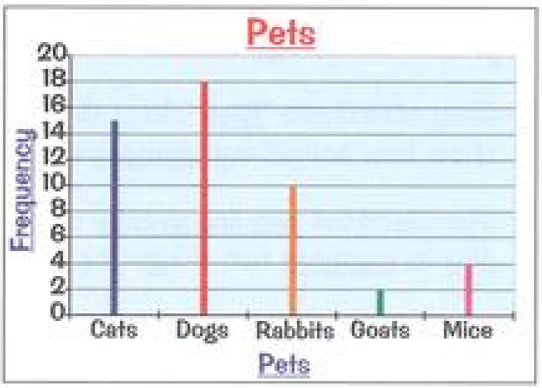
Here's the next exciting way of displaying data — bar charts.

Bar Charts Show Data as Bars

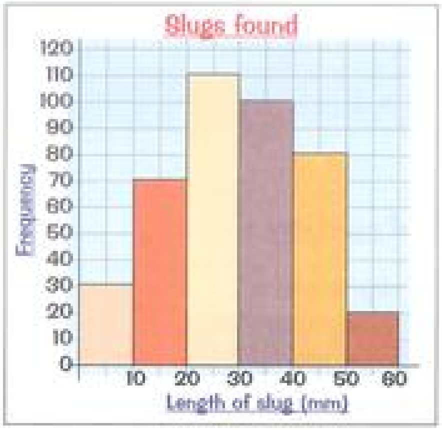
The height of each bar is the frequency (i.e. how many) for that group.



This bar chart compares separate items (e.g. hot dogs, burgers) so the bars don't touch. You can see from the graph that cauliflower cheese was the most popular choice, and only 4 people chose hot dogs.



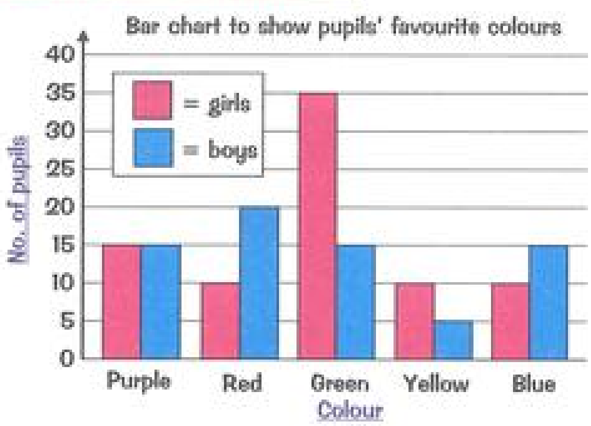
This is a bar-line graph. It's just like a bar chart, except it has thin lines instead of bars. Here, dogs were the most popular pets and 2 people have goats.



Sometimes the bars need to touch. This is when you need to cover a range of numbers on the x-axis, rather than separate categories — so there should be no spaces between the bars (e.g. for measurements like height or length).

If your data is continuous (see p.86), the bars should touch.

Dual Bar Charts



1) Dual bar charts show two sets of data together so you can compare them.

2) Each category has two bars — one for each set of data.

I'd like to see bar charts having a duel...

1) a) Using the bar-line graph above, work out how many people have rabbits.
   b) Using the dual bar chart, find the most popular colour for boys and for girls.

Pie Charts

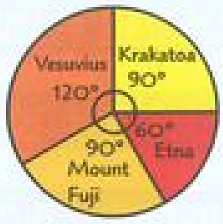
You must learn the Golden Rule for pie charts (which sadly don't have anything to do with pie):

The TOTAL of Everything = 360°

1) Fraction of the Total = Angle ÷ 360°

EXAMPLE:

Some geography students were asked to name their favourite volcano. The results are displayed in the pie chart.
What fraction of the students chose Etna?



Just remember that 'everything = 360°'.

Fraction that chose Etna = $\frac{\text{angle of Etna}}{\text{angle of everything}} = \frac{60°}{360°} = \frac{1}{6}$

2) Find a Multiplier to Calculate Your Angles

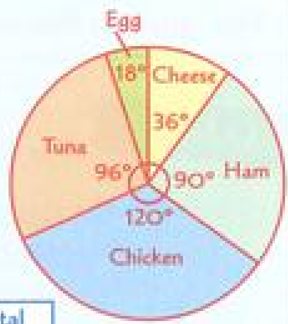
EXAMPLE: Draw a pie chart to show this information:

Sandwich	Cheese	Ham	Chicken	Tuna	Egg
Number	6	15	20	16	3

1. Find the total by adding. $6 + 15 + 20 + 16 + 3 = 60$
2. 'Everything = 360°' — so find the multiplier (or divider) that turns your total into 360°. Multiplier = 360 ÷ 60 = 6
3. Now multiply every number by 6 to get the angle for each sector.

Angle	6 × 6 = 36°	15 × 6 = 90°	20 × 6 = 120°	16 × 6 = 96°	3 × 6 = 18°	Total = 360°

4. Draw your pie chart accurately using a protractor.



3) Find How Many by Using the Angle for 1 Thing

EXAMPLE: In the example at the top of the page, 36 students were asked in total. How many chose Krakatoa as their favourite volcano?

1. 'Everything = 360°', so... ⟶ 36 students = 360°
2. Divide by 36 to find... ⟶ 1 student = 10°
3. The angle for Krakatoa is 90°, so you need to multiply both sides by 9: 9 students = 90°
   9 students chose Krakatoa

My favourite volcano is Popocatepetl...

Make sure you know the golden rule for pie charts — if you know it, you can answer this question:

1) How many students chose Vesuvius in the volcano example above?

Mean, Median, Mode and Range

Mean, median, mode and range pop up all the time — make sure you know what they are and how to find them. Learn the Golden Rule too — it's really important.

The Four Definitions

Not as smart as the median bear.

MODE = MOST common

MEDIAN = MIDDLE value (when values are in order of size)

MEAN = TOTAL of items ÷ NUMBER of items

RANGE = Difference between highest and lowest

REMEMBER:
Mode = most (emphasise the 'mo' in each when you say them)
Median = mid (emphasise the m*d in each when you say them)
Mean is just the average, but it's mean 'cos you have to work it out.

The Golden Rule

There's one vital step for finding the median that lots of people forget:

Always REARRANGE the data in ASCENDING ORDER (and check you have the same number of entries!)

You must do this when finding the median, but it's also really useful for working out the mode.

EXAMPLE: For the numbers 6, 4, 7, 1, 2, 6, 3, 5, find:

a) the median

The MEDIAN is the middle value (when they're arranged in order of size) — so first, rearrange the numbers.

1, 2, 3, 4, 5, 6, 6, 7

Check you have the same number of values after you've rearranged them.

When there are two middle numbers, the median is halfway between the two.

← 4 numbers this side ↑ 4 numbers this side →

Median = 4.5

b) the mode

MODE (or modal value) is the most common value. Mode = 6

Some data sets have more than one mode, or no mode at all.

c) the mean

$$\text{MEAN} = \frac{\text{total of items}}{\text{number of items}} \qquad \frac{1+2+3+4+5+6+6+7}{8} = \frac{34}{8} = 4.25$$

d) the range

RANGE = difference between highest and lowest values, i.e. between 7 and 1. $7 - 1 = 6$

Mode to Joy — Beethoven's most common symphony...

People often forget that the mode, median and mean are all averages (most people mean 'mean' when they say 'average'). Learn all four definitions and the golden rule, then try this question:

1) Find the median, mode, mean and range of these numbers: 10, 12, 8, 15, 9, 12, 11.

Frequency Tables

Frequency tables are like tally charts. The numbers can be arranged in either rows or columns. They're not too bad if you learn these key points:

1) The word FREQUENCY means HOW MANY.
So a frequency table is just a 'How many in each group' table.

2) The FIRST ROW (or column) gives the CATEGORY.

3) The FREQUENCY ROW (or column) tells you HOW MANY THERE ARE in that category.

Frequency tables show how many things there are in each category.

Category — How many

Vehicle	Frequency
Car	5
Bus	20
Lorry	31

Grouped frequency tables group the data into classes.

Height (h cm)	Frequency
5-10	12
11-15	15
16-20	11

Filling in Frequency Tables

46 pupils in a school were asked how many sisters they had. The results were put into a frequency table as shown:

In Rows:

Category

No. of Sisters	0	1	2	3	4	5	6	Total:
Tally	𝍸 ‖	𝍸 𝍸 𝍸	𝍸 𝍸 ‖	𝍸 |||	|||				
Frequency	7	15	12	8	3	1	0	46

The tally column is often left out.

The frequency is just a total of the tally for that group.

In Columns:

No. of Sisters	Frequency
0	7
1	15
2	12
3	8
4	3
5	1
6	0
Total:	46

You can use frequency tables to find averages — there's more on this on the next page.

I frequently wish I didn't have to do maths...

The best way to get to grips with frequency tables is to have a go at doing them. So here's a question for you to try (don't say I never give you anything).

1) 20 people were asked to name their favourite picnic food. Put the results into a frequency table.
sausage rolls, crisps, pork pies, quiche, sandwiches, sausage rolls, pork pies, pork pies, crisps, quiche, crisps, sausage rolls, pork pies, sandwiches, sausage rolls, sandwiches, sandwiches, sausage rolls, quiche, pork pies.

Averages from Frequency Tables

I hope you're happy with frequency tables, because there's one more thing to learn...

Find the Mean from a Frequency Table

To find the mean from a frequency table, you need to add an extra column to your table.

EXAMPLE: Some people were asked how many posters they have on their bedroom walls. The table shows the results. Find the mean of the data.

Number of posters	Frequency
0	1
1	10
2	12
3	9
4	6
5	2

1) If we had a list of the number of posters everyone had, it would look like this:

$$\underbrace{0}_{\text{One 0}},\ \underbrace{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}_{\text{10 lots of 1}},\ \underbrace{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2}_{\text{12 lots of 2}}, \ldots$$

2) There are 40 numbers in the list because 40 people were asked.
3) To find the mean you'd add all these numbers and divide by 40.
4) It's exactly the same for the table — except we cheat by adding an extra column to the table:

Number of posters	Frequency	No. of posters × Frequency
0	1	0
1	10	10
2	12	24
3	9	27
4	6	24
5	2	10
Total	40	95

This is the same as adding 10 lots of 1.

This is the same as adding 6 lots of 4.

This is the total number of posters.

To find the mode from a frequency table, just pick the category with the highest frequency.

total number of posters

$$\text{MEAN} = \frac{\text{3rd column total}}{\text{2nd column total}} = \frac{95}{40} = 2.375$$

total number of people asked

Always follow these steps:

1) Add up the **TOTAL** of the **SECOND COLUMN**.
2) Make a **THIRD COLUMN** by **MULTIPLYING** the **FIRST COLUMN** and **SECOND COLUMN** together.
3) Add up the **TOTAL** of the new **THIRD COLUMN**.
4) **MEAN = 3rd Column total ÷ 2nd Column Total**

If they're not Hugh Jackman posters, I'm not interested...

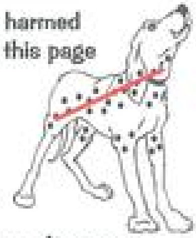
1) Find the mean number of sisters from the frequency table on the previous page.

Scatter Graphs

Scatter graphs are really useful — they show you if there's a link between two things.

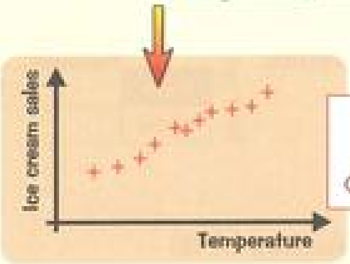
Scatter Graphs Show Correlation

No animals were harmed in the making of this page



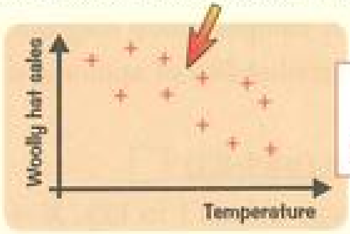
1) A scatter graph shows how closely two things are related. The fancy word for this is CORRELATION.
2) If the two things are related, then you should be able to draw a straight line (called a line of best fit) passing pretty close to most of the points on the scatter diagram.

STRONG correlation is when your points make a fairly straight line.



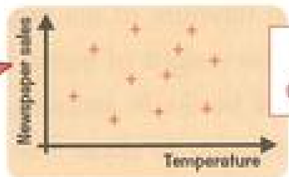
If the points form a line sloping uphill from left to right, then there is POSITIVE correlation — both things increase or decrease together.

WEAK correlation means your points don't line up quite so nicely (but you can still draw a line of best fit through them).



If the points form a line sloping downhill from left to right, then there is NEGATIVE correlation — as one quantity increases, the other decreases.

3) If the two things are not related, you get a load of messy points. This scatter graph is a messy scatter — so there's no correlation between the two things.



4) You can use a line of best fit to predict other values.

EXAMPLE: This graph shows the value of a car (in £'000s) plotted against its age in years.

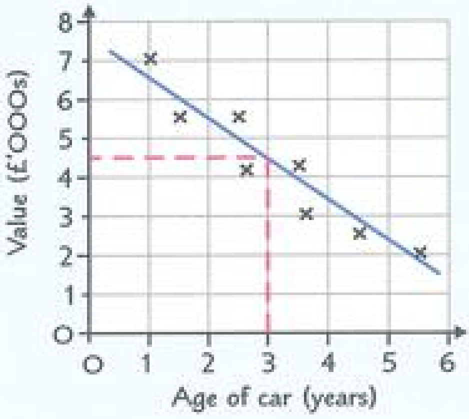
a) Describe the strength and type of correlation shown by the graph.

Strong negative correlation

b) Estimate the value of a 3 year old car.

1. Draw a line of best fit (shown in blue).
2. Draw a line up from 3 years to your line, and then across to the other axis.

3 years is roughly £4500



Make like a graph and scatter...

Disclaimer: CGP does not endorse drawing lines of best fit on Dalmatians or other spotted animals.

1) Use the graph in the example above to predict the age of a car worth £3500.

Revision Summary for Section 5

You know what's coming by now — here are some questions to check it's all sunk in.

- Try these questions and tick off each one when you get it right.
- When you've done all the questions for a topic and are completely happy with it, tick off the topic.

Probability (p81-83)

1) What does a probability of 1 mean?
2) In a game, you can either win or lose. If P(win) = 0.1, what is P(lose)?
3) In a bag of sweets, there are 5 cola bottles, 2 jelly snakes, 3 chocolate buttons and 2 chocolate mice. Find the probability of picking a cola bottle.
4) I have a spinner that is half black and half white. I spin it twice.
   a) Fill in this sample space diagram to show all the possible results.
   b) Find the probability of spinning a black and a white.

First spin \ Second spin	Black	White
Black		BW
White		

Venn Diagrams (p84-85)

5) Let ξ = {integers from 1 to 12}, X = {prime numbers}, Y = {factors of 8}
   a) Find X and Y.
   b) Draw a Venn diagram showing sets X and Y.

Types of Data, Graphs and Charts (p86-89)

6) What is primary data? What is secondary data?
7) Is 'favourite flavours of ice cream' quantitative or qualitative data?
8) I measure the weight of some baby dragons. Is this data discrete or continuous?
9) Luke reads 4 books in January, 6 books in February, 5 books in March and 3 books in April. Draw a pictogram to show this information.
10) The table on the right shows how many camels a dealer sells each week. Draw a bar chart to show this information.

Week	1	2	3	4
Number of camels	4	8	18	22

11) On a pie chart, the angle representing 'not if you paid me £1 000 000' is 30°. If 120 people took part in the survey, how many gave this answer?
12) Draw a pie chart to represent the data in the table on the right.

Colour of car	red	blue	black	silver
Number	15	5	10	30

Averages and Frequency Tables (p90-92)

13) Find the mode, median, mean and range of this data: 2, 8, 7, 5, 11, 5, 4.
14) The frequency table below shows the number of TVs per house. How many houses had 3 TVs?
15) Using the same frequency table, find the mode, mean and range for the number of TVs.

Number of TVs	0	1	2	3	4
Frequency	2	10	15	12	1

Scatter Graphs (p93)

16) Give an example of data that would show positive correlation on a scatter graph.

Answers

Section One

Page 1 — Calculating Tips
1) £40

Page 2 — Calculating Tips
1) a) 1 b) 21 c) 2
2) 10

Page 3 — Calculating Tips
1) 72

Page 4 — Ordering Numbers and Place Value
1) a) Nine million, nine hundred and five thousand, two hundred and eighty-five.
   b) Six million, fifty-four thousand, two hundred and three.
2) 2, 45, 54, 59, 98, 304, 442

Page 5 — Ordering Numbers and Place Value
1) 0.004, 0.032, 0.054, 0.54, 0.55, 1.23, 3.42, 5.63, 7.11, 9.54

Page 6 — Add, Subtract, Multiply and Divide
1) a) 44 b) 29
   c) 85 d) 31
2) a) 63 b) 84

Page 7 — Addition and Subtraction
1) a) 797 b) 852
2) 308 km

Page 8 — Adding and Subtracting Decimals
1) £19.62
2) 12.35 seconds

Page 9 — Multiplying by 10, 100, etc.
1) a) 490 b) 17290 c) 22.2
2) a) 22 b) 6600
   c) 350000

Page 10 — Dividing by 10, 100, etc.
1) a) 0.333 b) 561.12
   c) 0.08521
2) a) 52 b) 13 c) 70

Page 11 — Multiplying Without a Calculator
1) a) 990 b) 3300
   c) 17570

Page 12 — Dividing Without a Calculator
1) a) 16 b) 72 c) 45 r 10
2) 2 cm left over

Page 13 — Negative Numbers
1) a) 5 b) –15 c) 18 d) –3
2) 14 °C

Page 14 — Special Types of Number
1) 53, 61, 123, 7305
2) 1, 64

Page 15 — Prime Numbers
1) 73, 83

Page 16 — Multiples, Factors and Prime Factors
1) a) 9, 18, 27, 36, 45, 54, 63, 72
   b) 1, 2, 3, 4, 6, 9, 12, 18, 36
2) $2 \times 2 \times 3 \times 5$ or $2^2 \times 3 \times 5$

Page 17 — LCM and HCF
1) 21
2) 16

Page 18 — Fractions, Decimals and Percentages
1) a) $\frac{4}{5}$ b) $\frac{3}{10}$
   c) $\frac{1}{25}$ d) $\frac{3}{20}$
2) a) 30% b) 50% c) $\frac{17}{25}$

Page 19 — Fractions
1) a) $\frac{5}{7}$ b) $\frac{1}{4}$
2) $\frac{4}{5}$

Page 20 — Fractions
1) a) $\frac{2}{30} = \frac{1}{15}$ b) $\frac{24}{35}$ c) $\frac{10}{8} = \frac{5}{4}$
   d) $\frac{4}{5}$ e) $\frac{7}{8}$ f) $\frac{7}{12}$

Page 21 — Percentages
1) a) 196.95 b) 51
2) a) 30% b) 250%

Page 22 — Rounding Numbers
1) a) 16.8 b) 6.648
2) a) 7.70 b) 11.800

Page 23 — Rounding Numbers
1) a) 3000 b) 37 c) 0.0558
2) a) 18 b) 64600

Page 24 — Accuracy and Estimating
1) a) 0.048 b) –230
2) E.g. £800

Page 25 — Powers
1) a) 72 b) 56
2) a) 4^{16} b) $7^2 = 49$
3) 7^{12}

Page 26 — Square Roots and Cube Roots
1) a) 3 and –3 b) 11 and –11
   c) 13 and –13
2) a) 4.36 b) 8.63 c) 4.44

Revision Questions — Section 1
1) a) 9 b) 8 c) 10
2) a) One million, six hundred and forty-five thousand, one hundred
   b) Eight million, seven thousand, one hundred and eighty-two
3) 12, 19, 81, 87, 98, 564, 874, 911
4) 0.001, 0.02, 0.09, 0.51, 0.9, 1.8, 2.91
5) a) 121 b) 180
6) a) 611 b) 596 c) 2.673
7) a) 122.3 b) 15120
   c) 0.675 d) 0.0062
8) a) 414 b) 34
   c) 2489 d) 96
9) a) –2 b) –14
   c) 56 d) –9
10) a) Even numbers are whole numbers that divide by 2.
    b) Odd numbers are whole numbers that don't divide by 2.
    c) Square numbers are whole numbers multiplied by themselves.
    d) Cube numbers are whole numbers multiplied by themselves and then by themselves again.
11) a) 41, 43, 47 b) 83, 89
12) a) 11, 22, 33, 44, 55
    b) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
13) a) $2^3 \times 3$ b) 2×5^2
    c) $2 \times 3^2 \times 5$
14) a) 24 b) 16
15) a) $\frac{6}{10} = \frac{3}{5}$ and 60%
    b) $\frac{35}{100} = \frac{7}{20}$ and 0.35
16) a) E.g. $\frac{6}{10}$ and $\frac{9}{15}$
    b) $\frac{4}{30} = \frac{2}{15}$
    c) $\frac{1}{3}$
17) a) $\frac{8}{9}$ b) $\frac{1}{12}$
    c) $\frac{20}{33}$ d) $\frac{35}{60} = \frac{7}{12}$
18) a) 120 b) 210
19) a) 11.7 b) £132.44
    c) £98.01
20) 4.42%
21) a) 164.4 b) 76 000
    c) 765440
22) –45
23) a) 1200 b) 700 c) 50
24) a) 1 000 000 b) 555 c) 49
25) a) 6^{13} b) $3^4 = 81$ c) 2^{18}
26) a) 16 b) 1.77 c) 4

Answers

Section Two

Page 28 — Algebra — Simplifying

1) a) $3a$ b) $8d$ c) $3f$
2) a) $6x + 2y$ b) $-8x - 5y$

Page 29 — Algebra — Multiplying

1) a) d^5 b) $16ef$
2) a) $3x + 15$ b) $2x^2 + 3x$
   c) $x^2 + x - 15$

Page 30 — Formulas

1) a) 15 b) 13 c) 1
2) 8

Page 31 — Making Formulas from Words

1) a) $y = 5x - 3$ b) $y = \sqrt{x} + 1$
2) $c = 50s + 40$

Page 32 — Solving Equations

1) a) $x = 3$ b) $x = 11$
   c) $x = 3$ d) $x = 21$

Page 33 — Solving Equations

1) a) $x = 6$ b) $x = 4$
2) a) $x = 10$ b) $x = -5$ c) $y = 4$

Page 34 — Number Patterns and Sequences

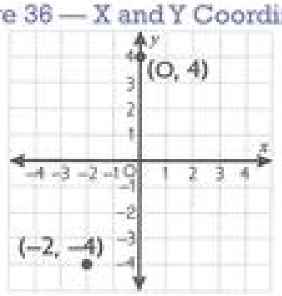
1) 14 — Subtract 6 from the previous term.

Page 35 — Number Patterns and Sequences

1) a) $3n + 2$ b) $-5n + 8$

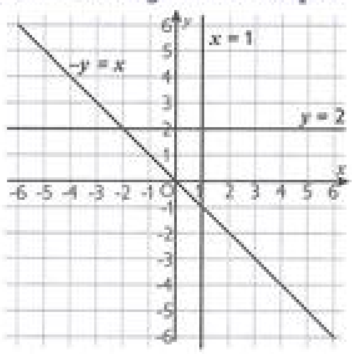
Page 36 — X and Y Coordinates

1)



Page 37 — Straight Line Graphs

1)

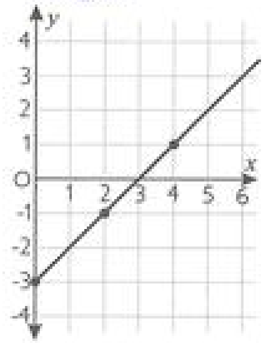


Page 38 — Straight Line Graphs

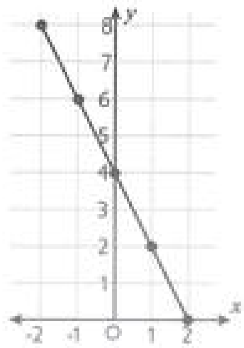
1) a) yes b) yes c) no
   d) no e) yes

Page 39 — Plotting Straight Line Graphs

1)



2)



Page 40 — Reading Off Graphs

1) 3.5 metres
2) About 0.1 seconds and 1.5 seconds

Page 41 — Travel Graphs

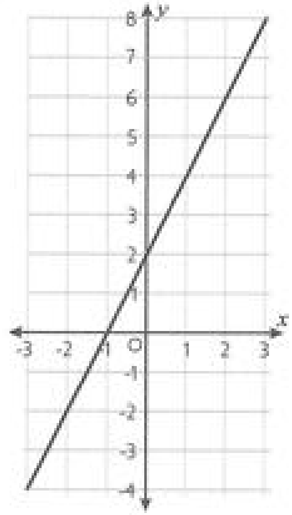
1) a) 3
   b) 3 hours and 30 minutes

Page 42 — Conversion Graphs

1) a) About 480 euros
   b) About 32 km

Revision Questions — Section 2

1) a) $5a$ b) $9b$
2) a) $6d + e$ b) $-2f + 9$
3) a) g^4 b) $9mn$
4) a) $3v + 24$ b) $-14w - 35$
5) a) $3x^2 + 10x$ b) $7y^2 + 8y - 15$
6) a) P = 29 b) P = −1
7) −4 °F
8) $P = 8s + 15t$
9) a) $x = 7$ b) $x = 22$
   c) $x = 12$ d) $x = 80$
10) a) $x = 3$ b) $x = 4$ c) $x = 2$
11) a) 26 — Add 6 to the previous term.
    b) 243 — Multiply the previous term by 3.
    c) 21 — Add the two previous terms together.
12) $2n + 3$
13) A (2, 2) B (3, −2)
    C (−2, −3) D (−2, 1)
14) a) No b) Yes
    c) Yes d) No
15)



16) a) On his way home.
    b) 15 minutes
17) Draw a straight line from the value you know on one axis to the graph. Then, change direction and draw another line straight to the other axis. Then read off the value.

Section Three

Page 44 — Ratios

1) a) 1 : 3 b) 6 : 5
   c) 2 : 7
2) a) 15 litres b) 25 litres

Page 45 — Proportion Problems

1) 39
2) Bread flour = 1600 g = 1.6 kg
   Soft Butter = 200 g
   Yeast = 80 g
   Water = 1000 ml = 1 litre

Page 46 — Proportion Problems

1) 1 litre of the juice for £3

Page 47 — Percentage Increase and Decrease

1) £3400
2) £16

Page 48 — Metric and Imperial Units

1) a) 1000 mm b) 36 inches
   c) 224 ounces

Page 49 — Conversion Factors

1) a) 5.6 litres b) 98 pounds
2) 5 pounds

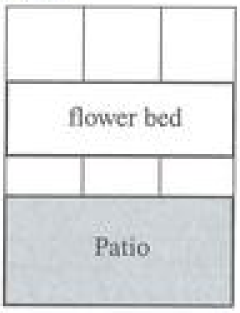
Answers

Page 50 — Conversion Factors
1) 300 kg
2) 180 cm, 2 metres, 7 feet

Page 51 — Reading Timetables
1) 1.32 pm

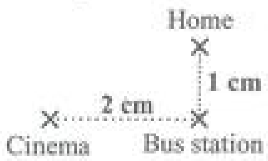
Page 52 — Maps and Scale Drawings
1) 20 km
2) 7 cm

Page 53 — Maps and Scale Drawings
1) E.g.



Page 54 — Speed
1) 200 metres
2) 12 km/h

Revision Questions — Section 3
1) **a)** 1 : 5 **b)** 7 : 8 **c)** 3 : 2
2) 21
3) 250 g
4) 30 cars
5) 84p
6) 90 olives
7) 750 g for £3
8) £6.90
9) £780
10) £156
11) See p.48
12) 600 cm
13) 10 yards
14) 22 pounds
15) **a)** 50 miles **b)** 75 cm
16) 100 minutes
17) 4 cm
18) 50 m by 25 m
19)



20) 6 m/s
21) 75 km

Section Four

Page 56 — Symmetry
1) 1
2) 2

Page 57 — Quadrilaterals
1) Kite

Page 58 — Triangles and Regular Polygons
1) 20 lines of symmetry, order of rotational symmetry = 20

Page 59 — Congruence and Similarity
1) Any regular hexagon is similar to shape E.

Page 60 — Perimeter and Area
1) 20 cm

Page 61 — Areas
1) 40 cm^2
2) 28 m^2

Page 62 — Area of Compound Shapes
1) 45 cm^2

Page 63 — Circles
1) Circumference = 62.8 mm (1 d.p.)
Area = 314.2 mm^2 (1 d.p.)

Page 64 — Circle Questions
1) 1253.50 m^2 (2 d.p.)

Page 65 — 3D Shapes
1) **a)** Regular tetrahedron
b) Faces = 4, edges = 6, vertices = 4

Page 66 — Nets and Surface Area
1) 96 cm^2

Page 67 — Nets and Surface Area
1) 24 cm^2

Page 68 — Volume
1) 168 cm^3

Page 69 — Lines and Angles
1) **a)** obtuse **b)** reflex **c)** acute

Page 70 — Measuring and Drawing Angles
1), 2) See page for method

Page 71 — Five Angle Rules
1) $x = 25°$

Page 72 — Five Angle Rules
1) 54°

Page 73 — Parallel Lines
1) $y = 102°$

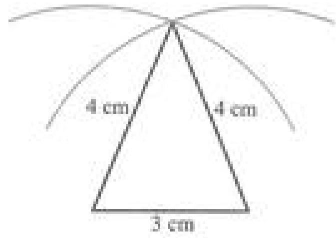
Page 74 — Interior and Exterior Angles
1) Exterior angle = 45°
Interior angle = 135°

Page 75 — Transformations
1) Translation by the vector $\begin{pmatrix}2\\-7\end{pmatrix}$

Page 76 — Transformations
1) Rotation 90° anticlockwise about point (–2, –1)

Page 77 — Enlargements
1) Scale factor = 3

Page 78 — Triangle Construction
1)



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Page 79 — Constructions
1) See constructions on page

Revision Questions — Section 4
1) Lines of symmetry = 2
Order of rotational symmetry = 2
2) Rhombus & parallelogram
3) Equilateral, right-angled, isosceles and scalene
4) Heptagon
5) Congruent shapes are exactly the same shape and size.
Similar shapes are the same shape but different sizes.
6) Perimeter = 32 cm,
Area = 55 cm^2
7) 48 cm^2
8) 30 cm^2
9) Circumference = 50.3 cm (1 d.p.)
Area = 201.1 cm^2 (1 d.p.)
10) 4 times
11) Faces = 5, edges = 9, vertices = 6
12) Surface area = 62 cm^2
Volume = 30 cm^3
13) See p.67
14) 400 cm^3

Answers

15) a) E.g. 72° (any value from 0°-89°)
 b) E.g. 111° (any value from 91°-179°)
 c) E.g. 260° (any value from 181°-359°)

16) See method on page 70

17) See p.71

18) Alternate angles

19) $x = 70°$, $y = 70°$, $z = 40°$

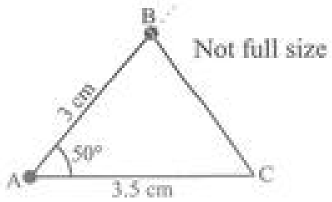
20) Exterior angle = 36°
 Interior angle = 144°

21) 720°

22) The angle of rotation, the direction of rotation and the centre of rotation.

23) (5, 3), (14, 3) and (11, 12)

24)



25) E.g.

Section Five

Page 81 — Probability

1) 0.4

Page 82 — Equal and Unequal Probabilities

1) $\frac{8}{15}$

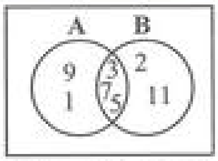
Page 83 — Listing Outcomes

1)

Dice		Coin: Heads	Tails
	1	1H	1T
	2	2H	2T
	3	3H	3T
	4	4H	4T
	5	5H	5T
	6	6H	6T

Page 84 — Venn Diagrams

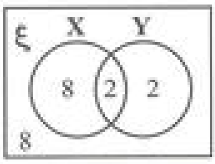
1)



2) n(P) = 5, n(Q) = 4

Page 85 — Venn Diagrams

1) X = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
 Y = {1, 4, 9, 16}



Page 86 — Types of Data

1) a) Discrete b) Qualitative
 c) Discrete d) Continuous

Page 87 — Line Graphs and Pictograms

1) 50

2) 55

Page 88 — Bar Charts

1) a) 10
 b) Boys = red, girls = green

Page 89 — Pie Charts

1) 12

Page 90 — Mean, Median, Mode and Range

1) Median = 11, mode = 12, mean = 11, range = 7

Page 91 — Frequency Tables

1)

Food	Sausage rolls	Crisps	Pork pies	Quiche	Sand-wiches
Frequency	5	3	5	3	4

Page 92 — Averages from Frequency Tables

1) Mean = $[(0 \times 7) + (1 \times 15) + (2 \times 12) + (3 \times 8) + (4 \times 3) + (5 \times 1) + (6 \times 0)] \div 46 = 80 \div 46 = 1.74$ (2 d.p.)

Page 93 — Scatter Graphs

1) About 4 years

Revision Questions — Section 5

1) A probability of 1 means that something is certain to happen.

2) P(lose) = 0.9

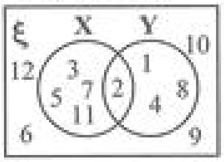
3) $\frac{5}{12}$

4) a)

First spin		Second spin: Black	White
	Black	BB	BW
	White	WB	WW

 b) $\frac{1}{2}$ or 0.5

5) a) X = {2, 3, 5, 7, 11}
 Y = {1, 2, 4, 8}
 b)

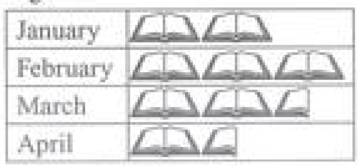


6) Primary data is data you've collected yourself. Secondary data is data someone else has collected.

7) Qualitative

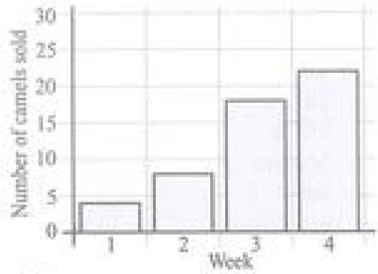
8) Continuous

9) E.g.



Key: [book] = 2 books

10)



11) $\frac{30}{360} \times 120 = 10$

12)

13) Mode = 5, Median = 5, Mean = 6, Range = 9

14) 12

15) Mode = 2, Mean = 2, Range = 4

16) E.g. heights and weights of people.

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