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Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Subsidiary Examination June 2010

# **Mathematics**

MPC1

**Unit Pure Core 1** 

Monday 24 May 2010 1.30 pm to 3.00 pm

### For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.



# Examiner's Initials Question Mark 1 2 3 4 5 6 7 TOTAL

### Time allowed

• 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

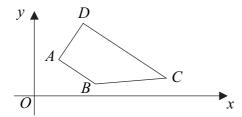
### **Advice**

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.



## Answer all questions in the spaces provided.

1 The trapezium *ABCD* is shown below.



The line AB has equation 2x + 3y = 14 and DC is parallel to AB.

(a) Find the gradient of AB.

(2 marks)

- (b) The point D has coordinates (3, 7).
  - (i) Find an equation of the line DC.

(2 marks)

- (ii) The angle BAD is a right angle. Find an equation of the line AD, giving your answer in the form mx + ny + p = 0, where m, n and p are integers. (4 marks)
- (c) The line BC has equation 5y x = 6. Find the coordinates of B. (3 marks)

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- **2 (a)** Express  $(3-\sqrt{5})^2$  in the form  $m+n\sqrt{5}$ , where m and n are integers. (2 marks)
  - (b) Hence express  $\frac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}}$  in the form  $p+q\sqrt{5}$ , where p and q are integers.

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<b>3</b> The polynomial $p(x)$ is given	by
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$$p(x) = x^3 + 7x^2 + 7x - 15$$

- (a) (i) Use the Factor Theorem to show that x + 3 is a factor of p(x). (2 marks)
  - (ii) Express p(x) as the product of three linear factors. (3 marks)
- (b) Use the Remainder Theorem to find the remainder when p(x) is divided by x 2.

  (2 marks)
- (c) (i) Verify that p(-1) < p(0). (1 mark)
  - (ii) Sketch the curve with equation  $y = x^3 + 7x^2 + 7x 15$ , indicating the values where the curve crosses the coordinate axes. (4 marks)

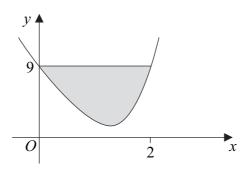
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4 The curve with equation  $y = x^4 - 8x + 9$  is sketched below.



The point (2, 9) lies on the curve.

(a) (i) Find 
$$\int_0^2 (x^4 - 8x + 9) dx$$
.

(5 marks)

- (ii) Hence find the area of the shaded region bounded by the curve and the line y = 9.

  (2 marks)
- **(b)** The point A(1, 2) lies on the curve with equation  $y = x^4 8x + 9$ .
  - (i) Find the gradient of the curve at the point A.

(4 marks)

(ii) Hence find an equation of the tangent to the curve at the point A.

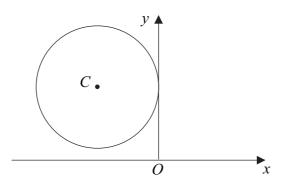
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A circle with centre C(-5, 6) touches the y-axis, as shown in the diagram.



(a) Find the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = r^2$$
 (3 marks)

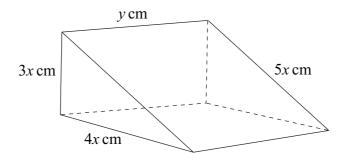
- (b) (i) Verify that the point P(-2, 2) lies on the circle. (1 mark)
  - (ii) Find an equation of the normal to the circle at the point P. (3 marks)
  - (iii) The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O. (4 marks)

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The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths 3x cm, 4x cm and 5x cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is 144 cm<sup>2</sup>.

- (a) (i) Show that  $xy + x^2 = 12$ . (3 marks)
  - (ii) Hence show that the volume of the block,  $V \text{ cm}^3$ , is given by

$$V = 72x - 6x^3 (2 marks)$$

- **(b) (i)** Find  $\frac{dV}{dx}$ .
  - (ii) Show that V has a stationary value when x = 2. (2 marks)
- (c) Find  $\frac{d^2V}{dx^2}$  and hence determine whether V has a maximum value or a minimum value when x=2. (2 marks)

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7 (a) (i)	Express $2x^2 - 20x + 53$	in the form $2(x-p)^2 + q$	, where $p$ and $q$ are integers.
			(2 marks)

- (ii) Use your result from part (a)(i) to explain why the equation  $2x^2 20x + 53 = 0$  has no real roots. (2 marks)
- **(b)** The quadratic equation  $(2k-1)x^2 + (k+1)x + k = 0$  has real roots.
  - (i) Show that  $7k^2 6k 1 \le 0$ .

(4 marks)

(ii) Hence find the possible values of k.

(4 marks)

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