Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2010

Mathematics

MPC3

Unit Pure Core 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

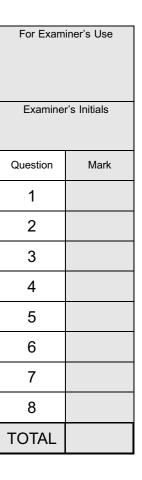
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.



Answer all questions in the spaces provided.

- The curve $y = 3^x$ intersects the curve $y = 10 x^3$ at the point where $x = \alpha$.
 - (a) Show that α lies between 1 and 2. (2 marks)
 - **(b) (i)** Show that the equation $3^x = 10 x^3$ can be rearranged into the form $x = \sqrt[3]{10 3^x}$. (1 mark)
 - (ii) Use the iteration $x_{n+1} = \sqrt[3]{10 3^{x_n}}$ with $x_1 = 1$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

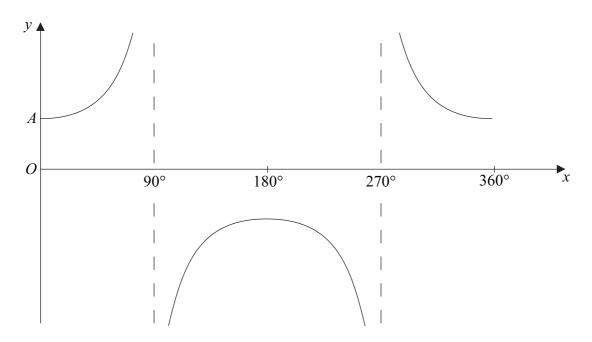
QUESTION PART REFERENCE	



QUESTION PART REFERENCE	
•••••	
•••••	
•••••	
•••••	
•••••	
•••••	

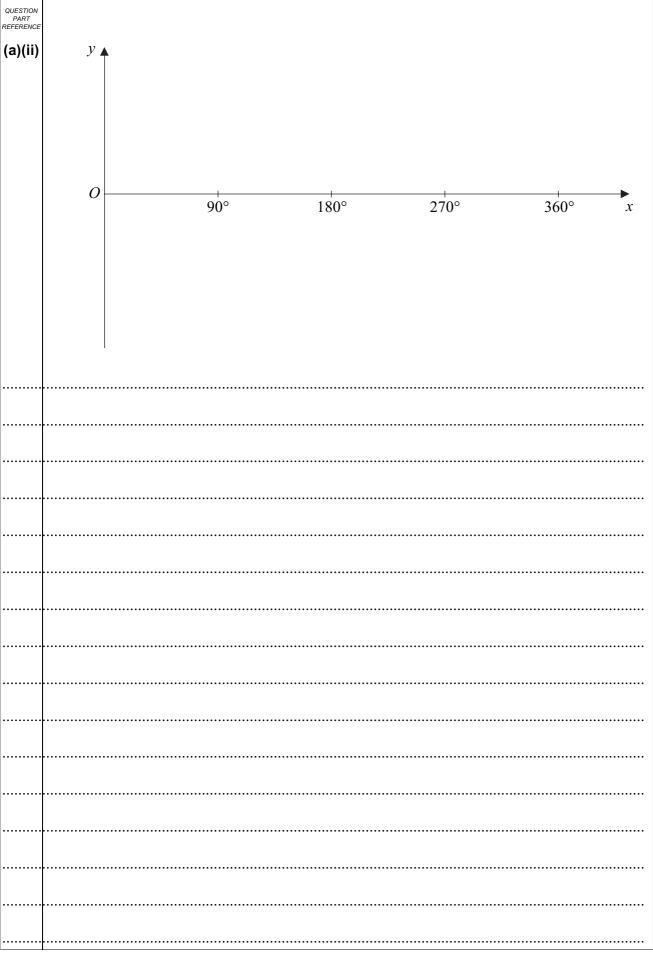


2 (a) The diagram shows the graph of $y = \sec x$ for $0^{\circ} \le x \le 360^{\circ}$.



- (i) The point A on the curve is where x = 0. State the y-coordinate of A. (1 mark)
- (ii) Sketch, on the axes given on page 5, the graph of $y = |\sec 2x|$ for $0^{\circ} \le x \le 360^{\circ}$.
- Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0^{\circ} \le x \le 360^{\circ}$. (2 marks)
- Solve the equation $|\sec(2x 10^\circ)| = 2$, giving all values of x in degrees in the interval $0^\circ \le x \le 180^\circ$.

QUESTION PART	
REFERENCE	





3 (a) Find $\frac{dy}{dx}$ when:

(i)
$$y = \ln(5x - 2)$$
;

(2 marks)

(ii)
$$y = \sin 2x$$
.

(2 marks)

(b) The functions f and g are defined with their respective domains by

$$f(x) = \ln(5x - 2)$$
, for real values of x such that $x \ge \frac{1}{2}$

$$g(x) = \sin 2x$$
, for real values of x in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

(i) Find the range of f.

(2 marks)

(ii) Find an expression for gf(x).

(1 mark)

(iii) Solve the equation gf(x) = 0.

(3 marks)

(iv) The inverse of g is g^{-1} . Find $g^{-1}(x)$.

(2 marks)

QUESTION PART REFERENCE	
KEFEKENCE	
 .	



QUESTION PART REFERENCE	
•••••	
•••••	
•••••	
•••••	
•••••	



4 (a)	Use Simpson's rule with 7 ordinates (6 strips) to find an approximation	
	to $\int_{0.5}^{2} \frac{x}{1+x^3} dx$, giving your answer to three significant figures.	(4 marks)

(b)	Find the exact value of	$\int_0^1 \frac{x^2}{1+x^3} \mathrm{d}x .$	(4 marks)
-----	-------------------------	--	-----------

QUESTION PART REFERENCE	
PART	
INLI LINLINGL	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	



QUESTION PART REFERENCE	
•••••	
•••••	
••••••	
••••••	
••••••	
•••••	
••••••	
•••••	
•••••	



5	(a)	Show	that	the	equation
---	-----	------	------	-----	----------

$$10\csc^2 x = 16 - 11\cot x$$

can be written in the form

$$10\cot^2 x + 11\cot x - 6 = 0 (1 mark)$$

(b) Hence, given that $10 \csc^2 x = 16 - 11 \cot x$, find the possible values of $\tan x$.

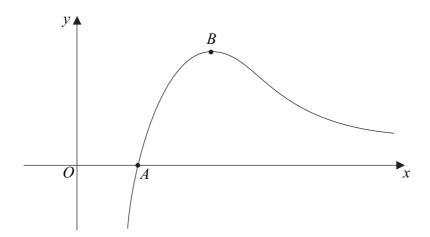
QUESTION	
PART	
PART REFERENCE	
• • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	



QUESTION PART REFERENCE	
•••••	
•••••	
•••••	
•••••	
•••••	



6 The diagram shows the curve $y = \frac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(a) State the coordinates of A. (1 mark)

(b) Find the coordinates of the stationary point, B, of the curve, giving your answer in an exact form. (5 marks)

Find the exact value of the gradient of the normal to the curve at the point where $x = e^3$. (3 marks)

QUESTION PART	
REFERENCE	
l	

QUESTION PART REFERENCE	
•••••	
•••••	
•••••	
•••••	
•••••	
•••••	



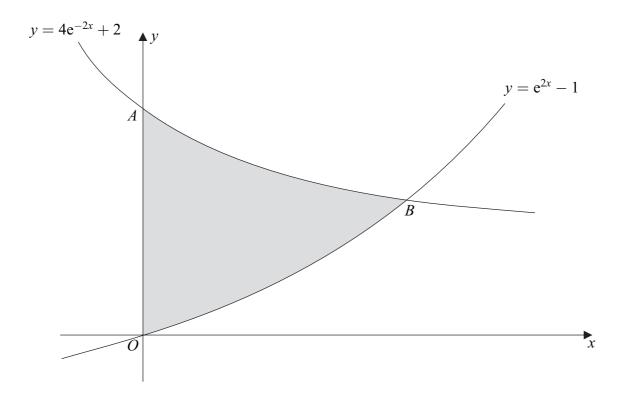
7 (a)	Use integration by parts to find:	
	(i)	$\int x \cos 4x \mathrm{d}x;$	(4 marks)
	(ii)	$\int x^2 \sin 4x \mathrm{d}x .$	(4 marks)
(b)	The region bounded by the curve $y = 8x\sqrt{(\sin 4x)}$ and the lines $x = 0$ and is rotated through 2π radians about the x-axis. Find the value of the volume solid generated, giving your answer to three significant figures.	
QUESTION PART REFERENCE			
••••••	• • • • • • • • •		•••••••
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
••••••	• • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
•••••	• • • • • • • • •		••••••
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
•••••	• • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • • • • • • • • • • • • • •		
	• • • • • • • •		
. ,			
•••••			



QUESTION PART REFERENCE	
•••••	
•••••	
•••••	
•••••	
•••••	
•••••	



8 The diagram shows the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$.



The curve $y = 4e^{-2x} + 2$ crosses the y-axis at the point A and the curves intersect at the point B.

- Describe a sequence of two geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x} 1$. (4 marks)
- (b) Write down the coordinates of the point A. (1 mark)
- (c) (i) Show that the x-coordinate of the point B satisfies the equation

$$(e^{2x})^2 - 3e^{2x} - 4 = 0 (2 marks)$$

- (ii) Hence find the exact value of the x-coordinate of the point B. (3 marks)
- (d) Find the exact value of the area of the shaded region bounded by the curves $y = e^{2x} 1$ and $y = 4e^{-2x} + 2$ and the y-axis. (5 marks)

QUESTION PART REFERENCE	
KEFEKENCE	

QUESTION PART REFERENCE	
REFERENCE	
•••••	



QUESTION PART REFERENCE	
•••••	
•••••	
••••••	
••••••	
••••••	
•••••	
••••••	
•••••	
•••••	



QUESTION PART REFERENCE	
REFERENCE	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
• • • • • • • • • • • • • • • • • • • •	
•••••	
• • • • • • • • • • • • • • • • • • • •	
••••••	
	END OF QUESTIONS



