Simulating Fixation Times with Interacting Acoustic Factors

Simulation Strategy

This simulation models **total fixation time** (in milliseconds) as a function of three binary within-subject factors:

- f_0 (SD): Standard deviation of f_0 (Low vs High)
- f_0 (mean): Mean f_0 (Low vs High)
- D_f: Distance from a perceptual category boundary (Low vs High)

Each of **1000 participants** views stimuli representing **all 8 combinations** of these factors. Fixation time is sampled from a normal distribution, with the following logic:

- **Main effects**:
  - f_0 (SD): Strongest effect. Adds 100 ms when High.
  - f_0 (mean): Adds 75 ms when High.
  - D_f: Adds 50 ms when High.
- **Interactions**:
  - f_0 (SD) $\times$ f_0 (mean): Adds 100 ms when both High.
  - f_0 (SD) $\times$ D_f: Adds 100 ms when both High.
  - f_0 (mean) $\times$ D_f: Adds 50 ms when both High.
  - 3-way interaction: Adds 120 ms when all three are High.

A normal distribution with SD = 1000 ms is used to generate noisy trial-level data. Values are clamped between 0 and 5000 ms.

Simulate the Data

```r
set.seed(42)

# Create all 8 condition combinations
stimulus_conditions <- expand_grid(
  f0_sd    = c("Low", "High"),
  f0_mean  = c("Low", "High"),
  Df       = c("Low", "High")
)

# Generate 1000 participant IDs
participant_ids <- str_c("P", str_pad(1:1000, width = 4, pad = "0"))

# Full design: each participant sees all conditions
design <- expand_grid(
  ID = participant_ids,
  stimulus_conditions
)

# Simulate fixation times
simulated_data <- design |>
  mutate(
    f0_sd_val   = if_else(f0_sd == "Low", 0, 1),
    f0_mean_val = if_else(f0_mean == "Low", 0, 1),
    Df_val      = if_else(Df == "Low", 0, 1),

    base_mean = 2500 + 100 * f0_sd_val,
    effect_mean = 75 * f0_mean_val,
    effect_df   = 50 * Df_val,

    interaction_effect = 100 * f0_sd_val * f0_mean_val +
      100 * f0_sd_val * Df_val +
      50  * f0_mean_val * Df_val +
      120 * f0_sd_val * f0_mean_val * Df_val,

    mu = base_mean + effect_mean + effect_df + interaction_effect,

    fixation_time = round(rnorm(n(), mean = mu, sd = 1000)),
    fixation_time = pmin(pmax(fixation_time, 0), 5000)
  ) |>
  select(ID, f0_sd, f0_mean, Df, fixation_time)
```

Fit the Linear Mixed-Effects Model

```
mod <- lmer(fixation_time ~ f0_sd * f0_mean * Df + (1 | ID), data = simulated_data)
anova(mod)
```

```
Analysis of Variance Table
                 npar    Sum Sq   Mean Sq  F value
f0_sd               1 123939509 123939509 126.3168
f0_mean             1  55029861  55029861  56.0854
Df                  1  48917697  48917697  49.8560
f0_sd:f0_mean       1   6209270   6209270   6.3284
f0_sd:Df            1   3393438   3393438   3.4585
f0_mean:Df          1   9645563   9645563   9.8306
f0_sd:f0_mean:Df    1     60836     60836   0.0620
```

Estimate Partial Eta-Squared for Fixed Effects

```
eta_squared(mod, partial = TRUE)
```

```
# Effect Size for ANOVA (Type III)

Parameter        | Eta2 (partial) |       95% CI
-------------------------------------------------
f0_sd            |           0.02 | [0.01, 1.00]
f0_mean          |       7.96e-03 | [0.00, 1.00]
Df               |       7.08e-03 | [0.00, 1.00]
f0_sd:f0_mean    |       9.04e-04 | [0.00, 1.00]
f0_sd:Df         |       4.94e-04 | [0.00, 1.00]
f0_mean:Df       |       1.40e-03 | [0.00, 1.00]
f0_sd:f0_mean:Df |       8.87e-06 | [0.00, 1.00]

- One-sided CIs: upper bound fixed at [1.00].
```

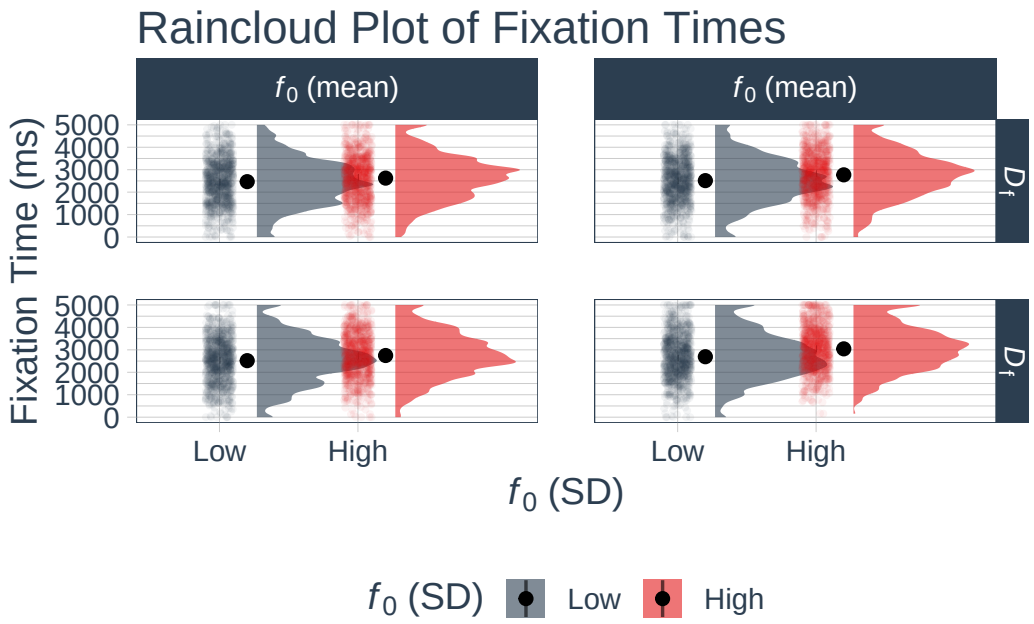
Raincloud Plot

```
# Prep factor labels and plotmath expressions
plot_data <- simulated_data |>
  mutate(
    f0_mean = factor(f0_mean, levels = c("Low", "High")),
    Df = factor(Df, levels = c("Low", "High")),
    f0_sd = factor(f0_sd, levels = c("Low", "High"))
  )

label_f0_mean <- c("Low" = "italic(f)[0]*' (mean)'",
                   "High" = "italic(f)[0]*' (mean)'")
label_Df <- c("Low" = "italic(D)[f]",
              "High" = "italic(D)[f]")

# Create raincloud plot
ggplot(plot_data, aes(x = f0_sd, y = fixation_time, fill = f0_sd)) +
  stat_halfeye(
    adjust = 0.5,
    justification = -0.3,
    .width = 0,
    point_colour = NA,
    alpha = 0.6
  ) +
  geom_jitter(
    aes(color = f0_sd),
    width = 0.1,
    alpha = 0.05,
    size = 0.7
  ) +
  stat_summary(
    fun = mean,
    geom = "point",
    size = 2,
    color = "black",
    position = position_nudge(x = 0.2)
  ) +
  stat_summary(
    fun.data = mean_se,
    geom = "errorbar",
    width = 0.1,
    color = "black",
```

```
  position = position_nudge(x = 0.2)
) +
facet_grid(
  Df ~ f0_mean,
  labeller = labeller(
    f0_mean = as_labeller(label_f0_mean, default = label_parsed),
    Df = as_labeller(label_Df, default = label_parsed)
  )
) +
labs(
  title = "Raincloud Plot of Fixation Times",
  x = expression(italic(f)[0]*" (SD)"),
  y = "Fixation Time (ms)",
  fill = expression(italic(f)[0]*" (SD)"),
  color = expression(italic(f)[0]*" (SD)")
) +
scale_colour_tq() +
scale_fill_tq() +
theme_tq(base_size = 14)
```



Summary

This simulation demonstrates how within-subject manipulations of acoustic features (f_0 SD, f_0 mean, and D_f) can be modeled and visualized effectively using mixed models and raincloud plots. The structure supports testing complex interactions while remaining grounded in a realistic psycholinguistic context.