Thermodynamic Cycle Optimization of a Turbofan Engine: Variable parameter analysis

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Abstract: This study employs numeric simulations to optimize the thermodynamic cycle of a turbofan engine by varying compressor pressure ratios (CPR), bypass ratios (BPR), turbine inlet temperatures (TiT), and fan pressure ratios (FPR) for two case studies: Take-off and Cruise at 10 km altitude. Parameters are explored within specified ranges. Through iterative optimization, the research identifies optimal parameter combinations to enhance engine performance by analyzing specific thrust output and specific fuel consumption. Additionally, a comparison of results is carried out with data from the Pratt & Whitney PW1100G-JM. By selecting key design factors, the net thrust and CO_2 footprint are determined.

Index Terms: Turbofan, optimization, thermodynamic cycle

Nomenclature

ΔP_b	burner pressure loss $[kPa]$
$\dot{m}$	mass flow $[kg/s]$
γ	adiabatic coefficient
τ_{Core}	core thrust $[kN]$
τ_{Fan}	fan thrust $[kN]$
τ_{Net}	engine net thrust $[kN]$
A	area $[m^2]$
a	speed of sound $[m/s]$
BPR	bypass ratio
C	velocity $[m/s]$
C_p	specific heat $[kJ/kgK]$
CO_2fp	carbon footprint $[CO_2/pax]$
CPR	compressor pressure ratio
$engines$	amount of aircraft engines
er_{A1}	CO_2 Jet-A1 emission rate $[kgCO_2/KgA1]$
f	fuel to air ratio
FPR	fan pressure ratio
LHV	lower heating value$[kJ/kg]$
M	mach number
P	pressure $[kPa]$
pax	amount of aircraft passenger
R	universal gas constant $[kJ/kgK]$
$s\tau$	specific thrust$[m/s]$
SFC	specific fuel consumption$[lbm/h/lbf]$
T	temperature $[K]$
t	flight duration $[s]$
TiT	turbine inlet temperature$[K]$

Isentropic efficiencies

η_F	fan efficiency
η_I	inlet efficiency
η_T	turbine efficiency
η_B	combustion efficiency
η_{CN}	core nozzle efficiency
η_C	compressor efficiency
η_M	mechanical efficiency
η_{NF}	nozzle fan efficiency

Subscripts

$*$	critical condition
00	atmosphere conditions
013	fan intake stage
019	fan nozzle exit
02	inlet stage
03	compressor intake stage
04	burner stage
05	turbine stage
07	diffuser stage
09	nozzle stage
19	fan nozzle exit total condition
9	core nozzle exit total condition
a	air
f	fuel
g	gas

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1. Introduction

This research aims to determine the optimum performance of a turbofan engine by utilizing a parametric iteration process. The primary objective is to identify key parameters that optimize the turbofan engine's specific thrust while minimizing specific fuel consumption, crucial factors for enhancing aircraft efficiency and performance.

The iterative process delves into the intricacies of the turbo-machinery, focusing on the thermodynamic cycle of the aircraft engine. This detailed analysis encompasses various stages of the turbofan, including the inlet, fan, fan nozzle, compressor, burner, turbine, diffuser, and core nozzle. By examining thermodynamic variables derived from theoretical analyses, we gain insights into the interplay between each stage, facilitating a holistic understanding of the engine's performance.

Following the exploration of thermodynamics, an algorithm is employed to vary ambient conditions (take-off and cruise), compressor pressure ratio (CPR), bypass ratio (BPR), turbine inlet temperature (TiT), and fan pressure ratio (FPR). Through iterative processes involving these parameters, the thermodynamic cycle is solved for each combination of values. Upon completion of simulations, the optimization process becomes evident as we compute the minimum specific fuel consumption and maximum specific thrust. Subsequently, an optimized curve is generated, capturing the key values derived from the optimization process.

The significance of this research extends to real-world applications, where the findings are validated using a Pratt & Whitney PW1100G-JM turbofan engine. By comparing numerical solutions with real-world data, we assess the accuracy and applicability of our optimization approach, thus providing valuable insights into the practical implications of our findings.

2. Objectives

1) Identify optimal settings for a turbofan engine to maximize thrust while minimizing fuel consumption.
2) Explore how variations in environmental conditions and engine parameters affect turbofan performance.
3) Investigate the interplay between different stages of the turbofan engine's thermodynamic cycle.
4) Develop a graphical representation illustrating the relationship between thrust and fuel consumption for different parameter combinations.
5) Validate optimization results using real-life data from a Pratt & Whitney PW1100G-JM turbofan engine.

3. Theoretical analysis and methodology

3.1. Atmosphere

To derive the values of T_0 and P_0, we utilized the International Standard Atmosphere (ISA).

3.2. Ambiance conditions at Take-Off (0 m)

$$T_0 = 288.15K \tag{1}$$

$$P_0 = 10.13kPA \tag{2}$$

3.3. Ambiance conditions at Cruise (10 km)

$$T_0 = 223.25K \tag{3}$$

$$P_0 = 26.5kPa \tag{4}$$

To initiate the optimization process, it is imperative to traverse through the turbo-engine machinery, considering the energy balances across each stage of the turbofan. By calculating the thermodynamic variables at each stage, we establish relationships among the inlet, fan, nozzle fan, compressor, burner, turbine, diffuser, and core nozzle [1] [2]. Through preliminary analysis, the equations can be formulated as follows:

3.4. Inlet Stage[0 → 2]

$$T_{02} = T_0 \left(1 + \frac{(\gamma_a - 1)}{2} \cdot \left(\frac{C_0}{a_0}\right)^2\right) \tag{5}$$

$$P_{02} = P_0 \cdot \left(1 + \eta_I \cdot \frac{(\gamma_a - 1)}{2} \cdot \left(\frac{C_0}{a_0}\right)^2\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{6}$$

3.5. Fan Stage [2 → 13]

$$T_{013} = T_{02} \cdot \left(1 + \frac{1}{\eta_F} \cdot \left(\mathrm{FPR}^{\frac{\gamma_a - 1}{\gamma_a}} - 1\right)\right) \tag{7}$$

$$P_{013} = P_{02} \cdot FPR \tag{8}$$

3.6. Nozzle Fan Stage [13 → 19]

$$\frac{P_{013}}{P*} = \frac{1}{\left(1 - \frac{1}{\eta_{NF}} \cdot \frac{\gamma_a - 1}{\gamma_a + 1}\right)^{\frac{\gamma_a}{\gamma_a - 1}}} \tag{9}$$

3.6.1. Choked Condition

- $\frac{P_{013}}{P_0} \geq \frac{P_{013}}{P*}$

$$P_{19} = P* \tag{10}$$

$$T_{19} = T* = T_{019} \cdot \frac{2}{\gamma_a + 1} \tag{11}$$

$$C_{19} = \sqrt{\gamma_a \cdot R \cdot T_{19}} \tag{12}$$

3.6.2. Unchoked Condition

- $\frac{P_{013}}{P_0} < \frac{P_{013}}{P*}$

$$P_{19} = P_0 \tag{13}$$

$$T_{19} = T_{013} \cdot \left(1 - \eta_{Nf} \cdot \left(1 - \left(\frac{P_{19}}{P_{013}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}\right)\right) \tag{14}$$

$$C_{19} = \sqrt{2 \cdot C_{p_a} \cdot (T_{013} - T_{19})} \tag{15}$$

3.7. Compressor Stage [2 → 3]

$$T_{03} = T_{013} \cdot \left(1 + \frac{1}{\eta_c} \cdot \left(\mathrm{CPR}^{\frac{\gamma_a - 1}{\gamma_a}} - 1\right)\right) \tag{16}$$

$$P_{03} = \mathrm{CPR} \cdot P_{013} \tag{17}$$

3.8. Burner Stage [3 → 4]

$$T_{04} = TiT \tag{18}$$

$$f = \frac{C_{p_g} \cdot T_{04} - C_{p_a} \cdot T_{03}}{\eta_b \cdot \mathrm{LHV} - C_{p_g} \cdot T_{04}} = \frac{\dot{m}_f}{\dot{m}_a} \tag{19}$$

$$P_{04} = P_{03} - \Delta P_b \tag{20}$$

3.9. Turbine Stage [5 $\rightarrow$ 7]

$$T_{05} = T_{04} - \frac{(T_{03} - T_{02}) + \text{BPR} \cdot (T_{013} - T_{02})}{\eta_m \cdot (1+f) \cdot \left(\frac{CP_g}{CP_a}\right)} \tag{21}$$

$$P_{05} = P_{04} \cdot \left(1 - \left(\frac{T_{04} - T_{05}}{\eta_T \cdot T_{04}}\right)\right)^{\frac{\gamma_g}{\gamma_g - 1}} \tag{22}$$

$$T_{07} = T_{05} \tag{23}$$

$$P_{07} = P_{05} \tag{24}$$

3.10. Core Nozzle Stage [7 $\rightarrow$ 9]

$$\frac{P07}{P*} = \frac{1}{\left(1 - \frac{1}{\eta_n} \cdot \frac{\gamma_g - 1}{\gamma_g + 1}\right)^{\frac{\gamma_g}{\gamma_g - 1}}} \tag{25}$$

3.10.1. Choked

- $\frac{P_{07}}{P_0} \geq \frac{P_{07}}{P*}$

$$P_9 = P* \tag{26}$$

$$T_9 = T* = T_{09} \cdot \frac{2}{\gamma_g + 1} \tag{27}$$

$$C_9 = \sqrt{\gamma_g \cdot R \cdot T_9} \tag{28}$$

3.10.2. Unchoked

- $\frac{P_{07}}{P_0} < \frac{P_{07}}{P*}$

$$P_9 = P_0 \tag{29}$$

$$T_9 = T_{07} \cdot \left(1 - \eta_n \cdot \left(1 - \left(\frac{P_9}{P_{07}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right)\right) \tag{30}$$

$$C_9 = \sqrt{2 \cdot C_{p_g} \cdot (T_{07} - T_9)} \tag{31}$$

3.11. Engine outputs

3.11.1. Specific Thrust ($s\tau$)

$$s\tau = (1+f)\frac{C_9}{1+\text{BPR}}\left(1 + \frac{1}{\gamma_g M_9^2}\left(1 - \frac{P_0}{P_9}\right)\right) + \frac{\text{BPR} \cdot C_{19}}{1+\text{BPR}}\left(1 + \frac{1}{\gamma_a M_{19}^2}\left(1 - \frac{P_0}{P_{19}}\right)\right) - C_0 \tag{32}$$

3.11.2. Specific Fuel Consumption (SFC)

$$\text{SFC} = \frac{1}{1+\text{BPR}} \cdot \frac{f}{s\tau} \tag{33}$$

3.11.3. Net Thrust (τ)

$$\tau_{net} = \tau_{Core} + \tau_{Fan} \tag{34}$$

$$\tau_{Core} = \dot{m}_9 C_9 - \dot{m}_9 C_0 + A_9(p_9 - p_0) \tag{35}$$

$$\tau_{Fan} = \dot{m}_{19} C_{19} - \dot{m}_{19} C_0 + A_{19}(p_{19} - p_0) \tag{36}$$

$$\dot{m}_a = \dot{m}_{19} + \dot{m}_9 = \dot{m}_9(BPR + 1) \tag{37}$$

$$\dot{m}_a = \rho \cdot A \cdot C \tag{38}$$

3.11.4. Carbon Footprint (CO_2/pax)

$$er_{A1} = \frac{3kgC0_2}{kgA1} \tag{39}$$

$$CO_2fp = \frac{\dot{m}_f \cdot er_{A1} \cdot t \cdot engines}{pax} \tag{40}$$

4. Optimization Process

The optimization process was initiated by establishing the constants relevant to the problem, including atmospheric values, gas and air constants, and the low heating value (LHV). In this ideal turbofan engine scenario, each stage is assumed to have an isentropic efficiency of 1, and there is no pressure loss in the burner.

For the parametric iterative process, the performance of the turbo-engine was analyzed by varying the following parameters: compressor pressure ratio (CPR) from 5 to 40 with a step size of 5, bypass ratio (BPR) from 5 to 15 with a step size of 2.5, turbine inlet temperature (TiT) from 1200 K to 1700 K with a step size of 100 K, and fan pressure ratio (FPR) from 1.5 to 3.0 with a step size of 0.25.

Subsequently, the thermodynamic expressions and parameters were derived by solving the aforementioned equations using these values. Once the thermodynamic computations were completed, specific fuel consumption (SFC) and thrust-specific fuel consumption ($s\tau$) were computed for each parametric condition.

Upon solving the system of equations, the maximum values of $s\tau$ and the minimum values of SFC for each iteration were determined. This allowed the development of optimal curves for each simulation state.

A pseudo-code is provided to enhance understanding of the process.Algorithm19

5. Results

The case study was dictated by two primary flight conditions: take-off and cruise. By tailoring the optimization process to accommodate these key factors, we can delve into the following analysis.

The optimization process computes $20,160$ values, aiming to determine the optimum ones.

Following the algorithm, the optimization process for take-off conditions was conducted.

Finally, by extracting the minimum values from the previous figures, we can derive the optimized curve of the thermodynamic cycle of the Turbofan.

To validate the optimization model obtained, the Pratt and Whitney PW1100G-JM turbofan model has been selected ??.

According to the Product Card [3] the parameters are:

- $\phi_{Fan} = 2.057m$

TABLE I: ST Data-frame: [3360 rows x 6 columns]

Height	CPR	BPR	TiT	FPR	ST
0	5.0	5.0	1200	1.50	244.895089
0	5.0	5.0	1200	1.75	265.332965
0	5.0	5.0	1200	2.00	265.494053
0	5.0	5.0	1200	2.25	NaN
0	5.0	5.0	1200	2.50	NaN
...	...	...	...	...	...
10000	40.0	15.0	1700	2.00	NaN
10000	40.0	15.0	1700	2.25	NaN
10000	40.0	15.0	1700	2.50	NaN
10000	40.0	15.0	1700	2.75	NaN
10000	40.0	15.0	1700	3.00	NaN

TABLE II: SFC Data-frame: [3360 rows x 6 columns]

Height	CPR	BPR	TiT	FPR	SFC
0	5.0	5.0	1200	1.50	0.489827
0	5.0	5.0	1200	1.75	0.439727
0	5.0	5.0	1200	2.00	0.428303
0	5.0	5.0	1200	2.25	NaN
0	5.0	5.0	1200	2.50	NaN
...	...	...	...	...	...
10000	40.0	15.0	1700	2.00	NaN
10000	40.0	15.0	1700	2.25	NaN
10000	40.0	15.0	1700	2.50	NaN
10000	40.0	15.0	1700	2.75	NaN
10000	40.0	15.0	1700	3.00	NaN

- $\tau_{Net} = 146.791kN$
- $BPR = 12.5$

This turbofan engine is used in Airbus A320 NEO [3], which is twin-engine. and it's possible configured to 146 passenger[4], so:

$$engines = 2, pax = 146 \tag{41}$$

For the analysis, we chose to focus on a flight duration of one hour. Following this

$$t = 3600S \tag{42}$$

5.1. Take-off

5.1.1. Net Thrust (τ)

To obtain τ_{net} the equation 3.11.3 was used and the following value were obtained:

- $123.805kN$

5.2. Cruise

5.2.1. Net Thrust (τ)

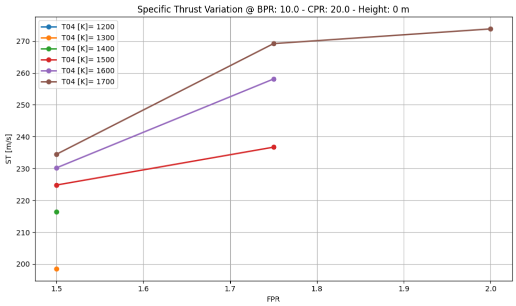
To obtain τ_{net} The algorithm was used and the following value was obtained:

- $79.184kN$

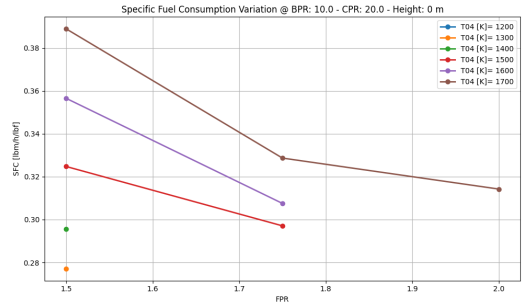
5.2.2. carbon footprint

To obtain CO_2fp the equation 3.11.4 was used and the following value was obtained:

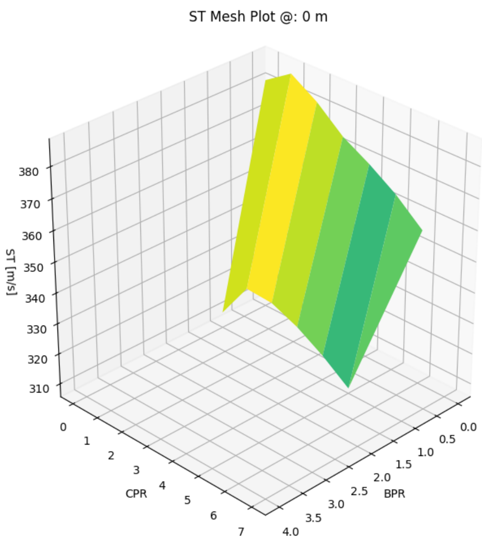
- $1854.8\frac{kgCO_2}{pax}$



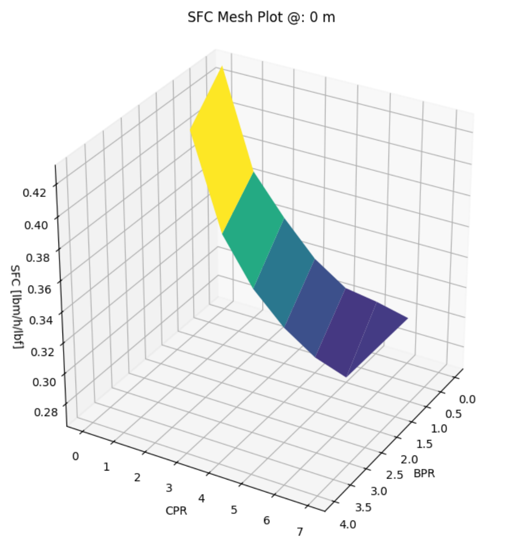
(a) Specific Thrust



(b) Specific Fuel Consumption



(c) Optimum Curve for Specific Thrust



(d) Optimum Curve for Specific Fuel Consumption

Fig. 1: Take-off parametric iteration

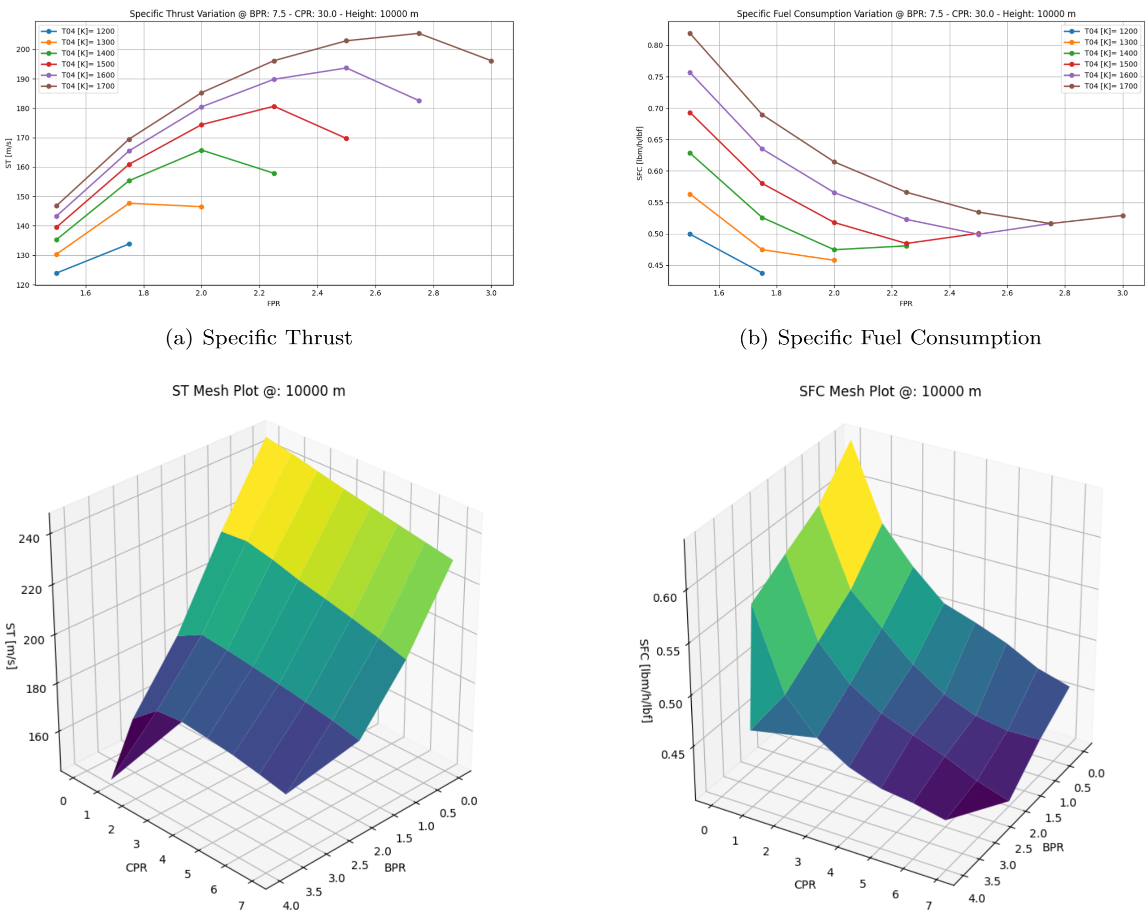
6. Conclusions

- Based on the theoretical thrust value of 146.79 kN and the obtained value of 79.184 kN, it is evident that there exists a significant deviation. This discrepancy can be attributed to the difference in engine configurations. The theoretical calculation assumed a single spool engine, while in reality, the engine is of the two-spool type. Such oversight highlights the importance of accurately aligning theoretical models with practical configurations to ensure precise predictions. This underscores the necessity for meticulous consideration of all relevant parameters to enhance the accuracy of future calculations and simulations.
- The increase in bypass ratio (BPR) enhances specific fuel consumption (SFC) but results in a notable decrease in specific thrust.
- The optimum fan pressure ratio (FPR) increases with turbine inlet temperature (TIT)
- The optimum fan pressure ratio (FPR) decreases with the increase of bypass ratio (BPR).

References

[1] A. F. El-Sayed, Aircraft Propulsion and Gas Turbine Engines, 2nd. Boca Raton, FL: CRC Press, 2014.

[2] S. Farokhi, Aircraft Propulsion, 2nd. Hoboken, NJ: Wiley, 2014.



(a) Specific Thrust (b) Specific Fuel Consumption

(c) Optimum Curve for Specific Thrust (d) Optimum Curve for Specific Fuel Consumption

Fig. 2: Cruise parametric iteration

[3] MTU Aero Engines, GTF Engine Family, https://www.mtu.de/engines/commercial-aircraft-engines/narrowbody-and-regional-jets/gtf-engine-family/, Accessed: March 21, 2024.

[4] All Nippon Airways. “ANA - Seat Map - A320.” Accessed: March 21, 2024. (), [Online]. Available: https://www.ana.co.jp/en/it/travel-information/seat-map/a320/#:~:text=There%20are%208%20Business%20Class,in%20rows%201%20and%202..

Algorithm 1 Turbofan Optimization Process: Thermodynamic Calculations

```
Define constants: atmospheric values, isentropic efficiencies, chemical properties, etc.
function Turbofan(FPR,T04,T0,C0,a0,P0,BPR,CPR)
    Calculate Inlet variables
    Calculate Fan variables
    Calculate Fan Nozzle variables
    Calculate Compressor variables
    Calculate Burner variables
    Calculate Turbine variables
    Calculate Core Nozzle variables
    Calculate specific thrust (ST) and specific fuel consumption (SFC)
    return ST and SFC
end function
Initialize arrays ST_results and SFC_results with zeros of appropriate dimensions
for each combination of parameters in CPR, BPR, TiT, FPR do
    Call TurboFan function with current parameters
    Store results in ST_results and SFC_results arrays
end for
Calculate maximum and minimum values varying BPR
Calculate maximum and minimum values varying CPR (Optimum Curves)
```