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Cyclic skip-spawning in half-sibling close-kin mark-recapture models

When breeding on-cycle, the pool of potential parents includes the proportion of adults breeding annually (1-ψ) plus the proportion of skip breeders on that cycle ψ/a. However, it is important to remember that the true number of available breeders integrates multiple year classes. This means that survival (ϕ) will also affect the breeder pool and will disproportionately affect the number of skip breeders available because, by definition, to be present they will have had to survive more years than annual breeders.

E.g., when there is no skip-spawning (ψ = 0) the number of potential parents $\widetilde{N}$ for a given total adult N and age-at-maturity M:

$$\widetilde{N} = \sum_{i=M}^{n} N_M + \phi N_{M+1} + \phi^2 N_{M+2} + \phi^3 N_{M+3} + \cdots + \phi^n N_{M+n}$$

Whereas if all adults skips-spawn on a two-year cycle (ψ =1, a=2) number of potential parents is:

$$\widetilde{N} = \sum_{i=M}^{n} N_M + \phi^2 N_{M+2} + \phi^4 N_{M+4} + \phi^6 N_{M+6} \ldots + \phi^n N_{M+n}$$

And, therefore, the effect of mortality means that when the two are mixed the pool of potential parents is more heavily weighted towards annual spawners than would be expected at first glance. For example, if half of parents skip-spawn of a two-year cycle (ψ = 0.5, a=2), the pool of potential parents will be:

$$\widetilde{N} = \sum_{i=M}^{n} N_M + (1-\psi)\phi N_{M+1} + \phi^2 N_{M+2} + (1-\psi)\phi^3 N_{M+3} + \phi^4 N_{M+4} + \cdots + \phi^n N_{M+n}$$

Or, broken down into the annual and skip spawning components as a function of a:

$$\widetilde{N}_{annual} = \sum_{i=M}^{n} (1-\psi) N_M + (1-\psi)\phi N_{M+1} + (1-\psi)\phi^2 N_{M+2} + \cdots + (1-\psi)\phi^n N_{M+n}$$

Which, given a stable survival rate, is equivalent to:

$$\widetilde{N}_{annual} = N(1-\psi)$$

And:

$$\widetilde{N}_{skip} = \sum_{i=M}^{n} \psi N_M + \psi\phi^a N_{M+a} + \psi\phi^{2a} N_{M+2a} + \psi\phi^{3a} N_{M+3a} + \cdots + \psi\phi^n N_{M+n}$$

Which is equivalent to:

$$\widetilde{N}_{skip} = N\frac{(1-\phi)}{(1-\phi^a)}$$

With both naturally combining to equal $\widetilde{N}$:

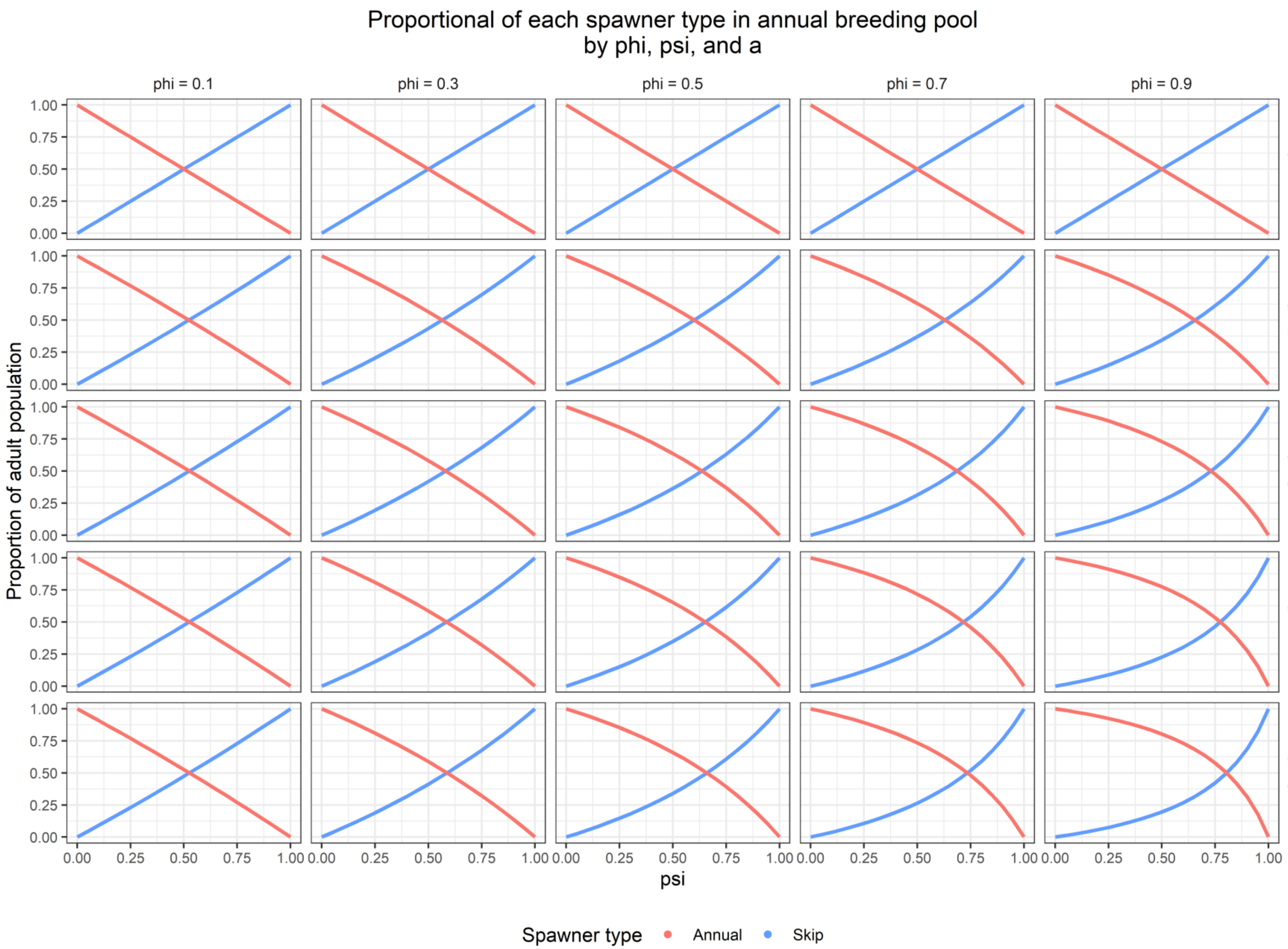
$$\widetilde{N} = \widetilde{N}_{annual} + \widetilde{N}_{skip}$$

Thus, the probability that any two individuals are half-siblings differs depending on whether or not their age difference is on-cycle (making skip-spawning parents available). If they are in on-cycle, the probability is a relatively simple function of $\widetilde{N}$ and the probability that the older individual's parent died before the younger individual was spawned

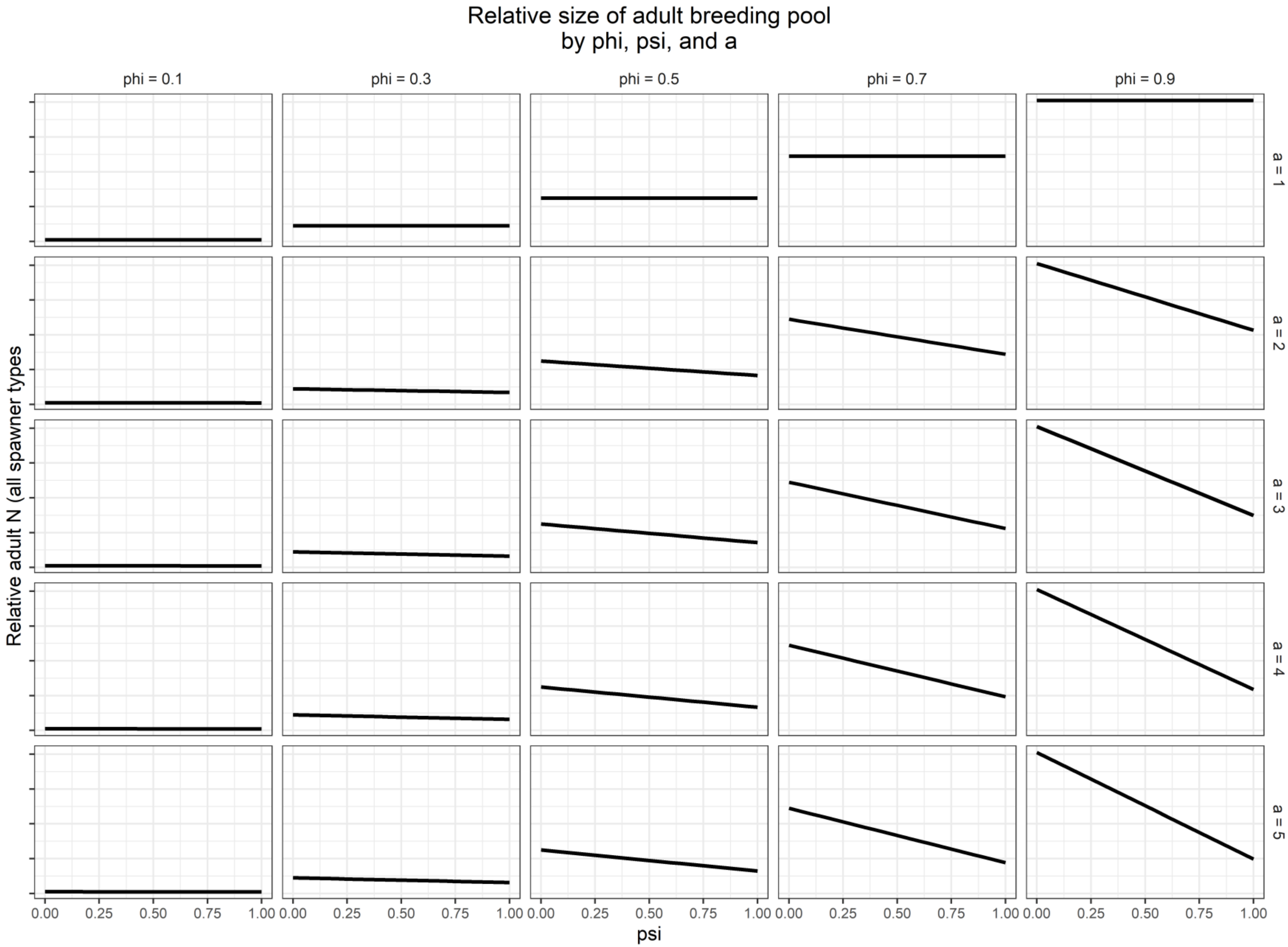
$$P(MHSP \mid c_1, c_2) = \frac{\phi^{|c_1 - c_2|}}{\widetilde{N}_{annual} + \widetilde{N}_{skip}}$$

I.e., the number of adults that spawn annually plus the number of adults that skip spawn on that breeding cycle. However, if the two individuals are off-cycle (meaning that a parental match must necessarily involve an annual spawner) the probability is a function of the proportion of the yearly spawning pool that is annual spawners, the probability that the older individual's parent died before the younger individual was spawned, and the overall number of annual spawners present:

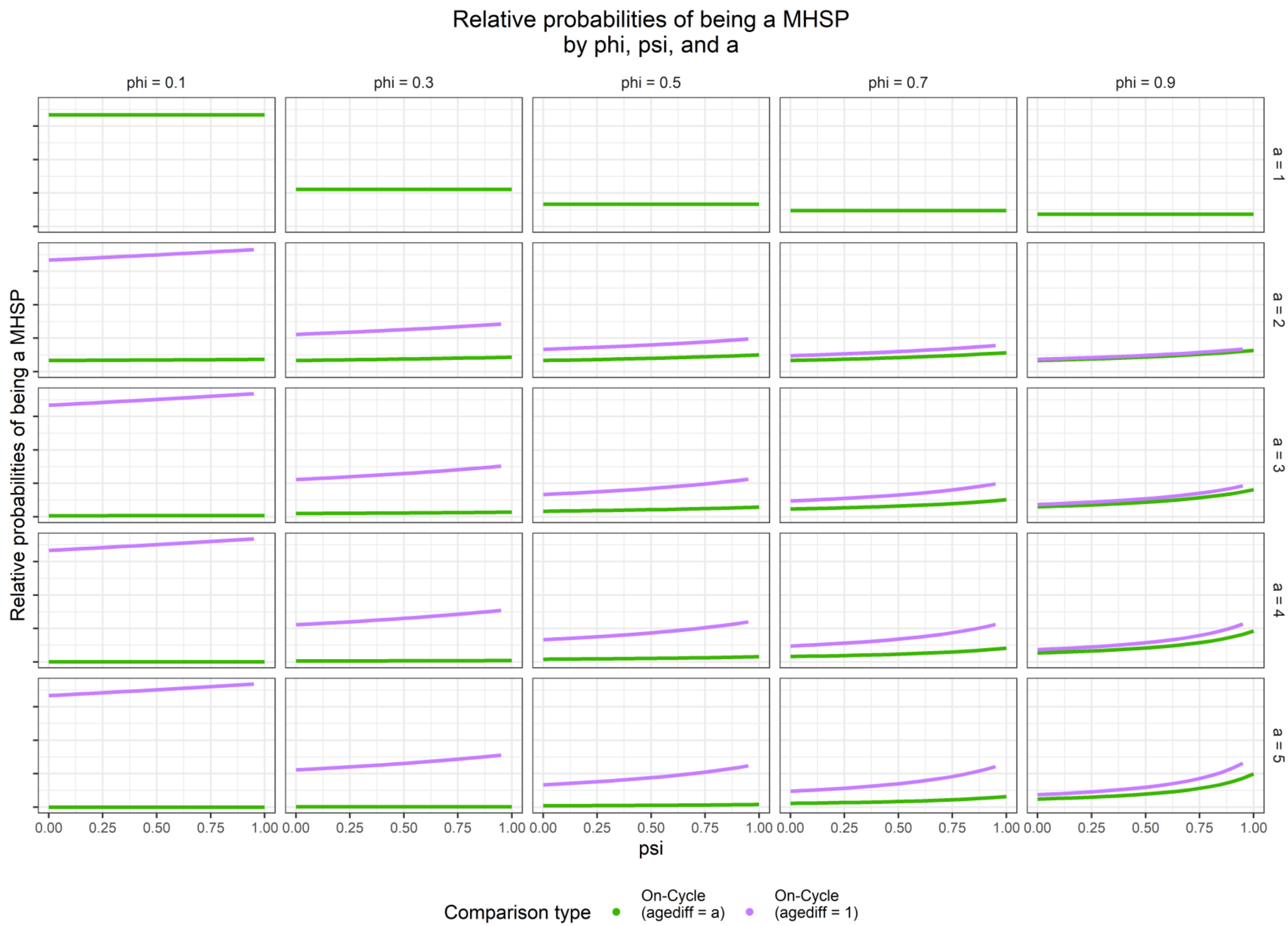
$$P(MHSP \mid c_1, c_2) = \frac{\phi^{|c_1 - c_2|} \frac{\widetilde{N}_{annual}}{\widetilde{N}}}{\widetilde{N}_{annual}}$$



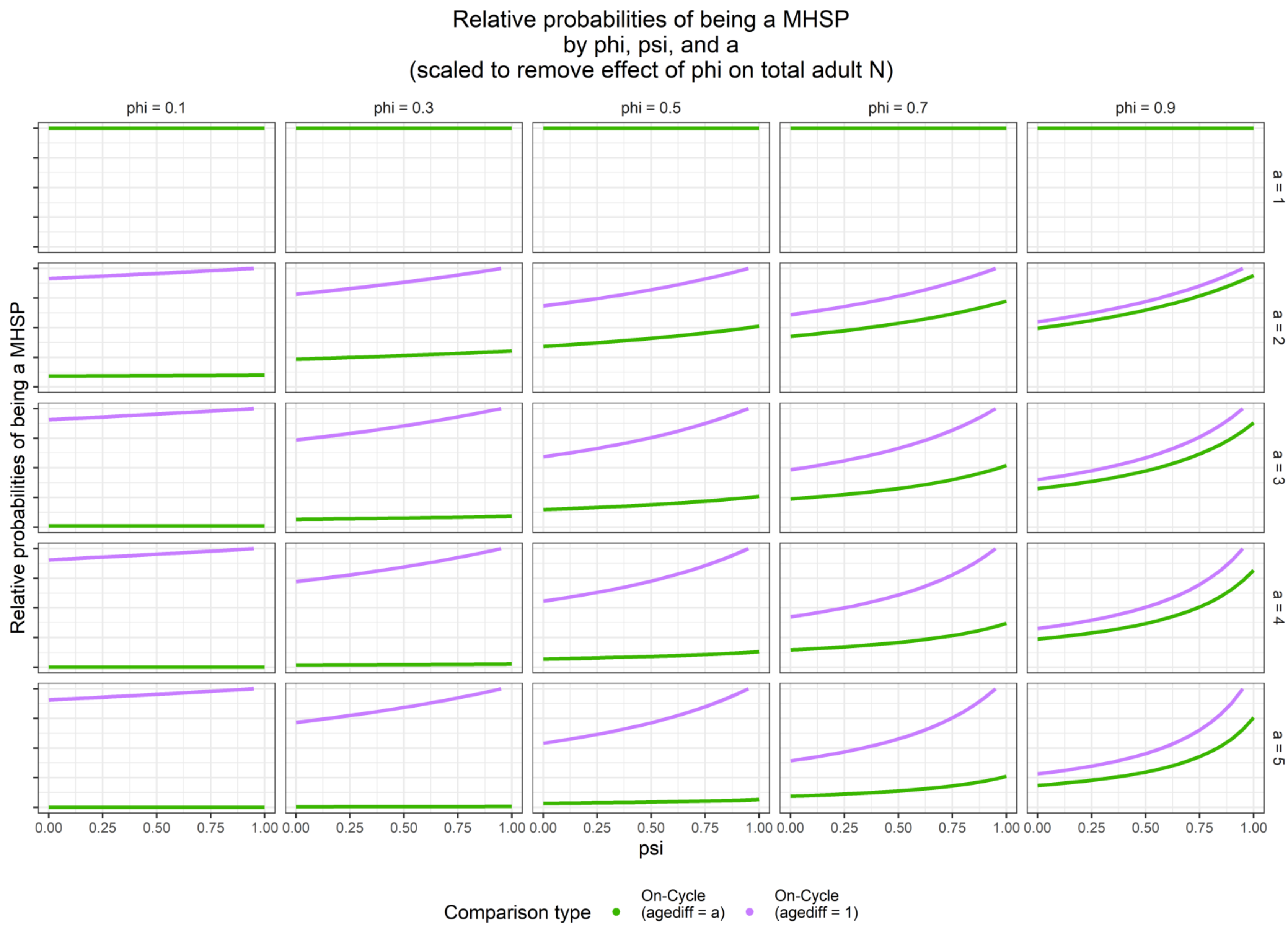
Proportional of each spawner type in annual breeding pool
by phi, psi, and a
phi = 0.1
phi = 0.3
phi = 0.5
phi = 0.7
phi = 0.9
a = 1
a = 2
a = 3
a = 4
a = 5
1.00
0.75
0.50
0.25
0.00
Proportion of adult population
0.00 0.25 0.50 0.75 1.00
psi
Spawner type
Annual
Skip



Relative size of adult breeding pool
by phi, psi, and a
phi = 0.1
phi = 0.3
phi = 0.5
phi = 0.7
phi = 0.9
a = 1
a = 2
a = 3
a = 4
a = 5
Relative adult N (all spawner types)
0.00
0.25
0.50
0.75
1.00
psi



Relative probabilities of being a MHSP
by phi, psi, and a
phi = 0.1
phi = 0.3
phi = 0.5
phi = 0.7
phi = 0.9
a = 1
a = 2
a = 3
a = 4
a = 5
Relative probabilities of being a MHSP
0.00
0.25
0.50
0.75
1.00
psi
Comparison type
On-Cycle (agediff = a)
On-Cycle (agediff = 1)



Relative probabilities of being a MHSP
by phi, psi, and a
(scaled to remove effect of phi on total adult N)
phi = 0.1
phi = 0.3
phi = 0.5
phi = 0.7
phi = 0.9
a = 1
a = 2
a = 3
a = 4
a = 5
Relative probabilities of being a MHSP
0.00
0.25
0.50
0.75
1.00
psi
Comparison type
On-Cycle (agediff = a)
On-Cycle (agediff = 1)