# **Bayesian framework (updated 11/08/2021)**

In October, I revamped the CKMR code in two major ways:

1. Changed FROM setting kinship probabilities for each comparison and maximizing the joint likelihood of all observations TO estimating abundance from the number of matches relative to the number of comparisons.
2. Changed FROM a frequentist based approach and optimization via optimx TO a Bayesian approach and optimization via JAGS

In addition, I added in survival as an estimable parameter and the model is good at jointly estimating abundance and survival.

When I tried pushing my luck to estimate lambda as well, the model didn’t do so well ☹

# **Frequentist framework (updated 06/07/2021)**

After meeting with Charlotte and Dovi on 5/20, Charlotte revamped the simulation code to make it jive with the model code (details below).

For now, the main script I’m using is:

CKMR\_DoviIBS\_Lemon\_sharks\_AvgN\_6yrs\_06.07.2021\_Lemon\_CB

1. First goal is to run without the ‘try’ function and make sure the model isn’t failing during any specific iteration.
2. Second goal is to add skipped-breeding into the data generation model.
3. Third goal is to adapt the model to account for skipped-breeding.

I also need to reorganize and set up my infrastructure better to work with this new code …

**Details about what Charlotte did:**

Charlotte started by using the binomial distribution during mating - so a maximum of 1 pup per female per year - then went back and added multiple mates and changed the binomial to poisson. Now, the model is giving unbiased estimates *as long as skipped breeding is not introduced.* Once skipped-breeding is introduced, the model fails (see below):

Here are the results after Charlotte’s edits to the code *without* skipped breeding:

Chart, box and whisker chart

Description automatically generated

And here are the results after Charlotte’s edits when the data generation model includes skipped-breeding that the CKMR model doesn’t account for:

Chart, box and whisker chart

Description automatically generated

Clearly, we need to account for skipped-breeding in the model.

The main issues that Charlotte addressed were:

1. the stable age distribution
   1. even though there's a burn-in, the cohort effect likely has a legacy effect when we start with everyone at age 12
2. we need to count the truth after mortality
   1. the population that produces the pups is the population after mortality from the previous year
   2. the N that's producing the pups is not the N that we were counting
   3. this *should* have resulted in negatively biased estimates, however, so probably not a huge contributor.
3. fecundity calculations
   1. used the Euler-Lotka equation

05/20/2021

I combed back through old code and results and found a model that gave abundance estimates that were less biased. This model does not incorporate a parameter for population growth, but rather estimates the average abundance over the years of sampling. As such, I've taken a step back and am working with scripts that estimate average abundance over the (approximate) sample period.

The main scripts I'm testing are the scripts with shortcuts here: 02\_IBS/currently\_testing

Specifically, **the primary scripts I’m working with right now are:**

**CKMR\_DoviIBS\_Lemon\_sharks\_AvgN\_6yrs\_05.20.2021\_Lemon.R:** This script uses Dovi's IBS simulation to simulate a population of Lemon Sharks, and samples the population over six years.

**CKMR\_DoviIBS\_AvgN\_6yrs\_05.25.2021\_CNR.R:** This scriptsimulates a population of cownose rays (1 pup per year per female) and samples the population over six years.

**fishSim\_CKMR\_sex-specific\_and\_aggregated\_loop\_AvgN\_6yrs\_05.13.2021\_Lemon.R**: this script uses fishSim to simulate a population of Lemon Sharks and samples this population over six years. The script returns relatively unbiased abundance estimates, with a median relative bias for males, females, and all adults around 2-4%.

**fishSim\_CKMR\_sex-specific\_and\_aggregated\_loop\_AvgN\_6yrs\_05.13.2021\_CNR.R:** this script uses fishSim to simulate a population of Cownose Rays and samples the population over six years. The script returns abundance estimates that are quite biased, with a median relative bias closer to 20%

\*All of these scripts include the kinship probability and likelihood functions as part of the script i.e. they're not sourced from the 00\_functions folder\*

**Agenda for meeting on 5/26/2021**

1. John share results from simulations
2. Discuss what degree of bias is acceptable?
   1. How/when will we know if the model is robust enough to test our research questions?
3. Peruse code and model to find potential source(s) of bias, focusing on one of the purple scripts listed above. Four parts to check:
   1. Are the simulation parameters correctly defined? (Dovi)
   2. Am I sampling the population appropriately? (Dovi)
   3. Any obvious errors in data formatting and constructing the pairwise comparison matrix? (All)
   4. Is the model mathematically sound? (Charlotte)
4. If no issue is identified following the above, what makes sense to try next?

# **Simulation Parameters**

These are the parameters that are setting up the individual-based simulation with Dovi’s IBS code.

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Value for Lemon Sharks** | **Value for Cownose Ray** |
| **Initial population size** | 3000 | 3000 |
| **Operating sex ratio** | 0.5 | 0.5 |
| **Age of reproductive maturity** | 12 | 7 |
| **Maximum age** | 30 | 19 |
| **Mating periodicity** | 2 (years) | 1 (year) |
| **Potential number of mates** | 1:3 (per year) | 1 (per year) |
| **Average number of offspring per mate** | 3 | 1 |
| **Birth sex ratio** | 0.5, 0.5 | 0.5, 0.5 |
| **Adult survival** | 0.9 | 0.9 |
| **Juvenile survival** | 0.9 | 0.86 |
| **Young-of-year survival** | 0.8 | 0.75 |
| **Years of simulation** | 90 | 90 |
| **Sample years** | 85:90 | 58:60 |
| **Sample size per year** | varies | Focusing on 150 for now |

# **CKMR parameters (prior to 05/20/2021)**

These are the parameters that are being used or estimated in the CKMR model.

year\_est: This is the year on which the abundance estimate is focused.

Lambda: I’ve been fixing lambda to the mean population growth rate from the birth year of the oldest sampled individual (min\_cohort) to the last year of the simulation. In the model, I make abundance in a given year a function of abundance in year\_est x lambda raised to the number of years between year\_est and the younger sibling birth year (ys\_birth).

Adult survival: I’ve been fixing this value to the mean observed survival rate of adults through the simulation.

Female abundance: Trying to estimate.

Male abundance: Trying to estimate.

# **Kinship probability**

Where

k = individual adult female

F = all adult females in population

ys\_ birth = the birth year of the younger individual in the comparison

os\_birth = the birth year of the older individual in the comparison

yr\_est = the birth year of the oldest sampled individual i.e. the year for which we’re estimating abundance

= survival of mother k from the year of the older sibling birth to the year of the younger sibling birth.

* Note that the younger individual birth year will be a larger number than the older individual birth year (e.g. 2021 vs 2010)
* I have been fixing this value to the observed survival of adults through the simulation

= the number of total females alive in the year of estimation

* This is the value I’m estimating

= The mean population growth rate raised to the number of years between the birth year of the younger individual and the year of estimation (i.e. birth year of oldest sampled individual).

* I have been fixing this value to the mean population growth between yr\_est and the last year of the simulation.
* This seems to have a major effect on the estimate of NF
* I’m presently trying to include this as an estimated parameter, but constraining it to +/-1%. I wonder if giving this parameter a little wiggle room will help.

The same equation above is constructed for males. The male and female equation are saved together as an R function called get\_P\_lemon. The negative log likelihood of this function is minimized with respect to the parameters of interest (male and female abundance).

# **Process**

1. I simulate a population setting the simulation parameters above to emulate either lemon shark or cownose ray life history.
2. After the simulation, I randomly sample the population over multiple years.
   1. I filter the sample set for:
      1. Recaptures of self (keep just one instance of capture for each individual)
      2. Full siblings (if found, keep just one)
3. Then, I set up a pairwise comparison matrix, where each individual is compared against every other.
   1. I remove within-cohort comparisons from the pairwise comparison matrix before proceeding. This is supposed to help ensure that comparisons are independent, since the variance in reproductive output should not affect the probability of recapture (i.e. sampling a half-sib on the second occasion)
4. Half siblings are identified
5. The pairwise comparison matrix is split into four separate matrices:
   1. Maternal half-sibs
   2. Paternal half-sibs
   3. Maternal negative comparisons
      1. Includes all comparisons that were not maternal half-sibs
   4. Paternal negative comparisons
      1. Includes all comparisons that were not paternal half-sibs
6. I fit the CKMR model to the data and maximize the likelihood with respect to abundance using a Hessian matrix and the BFGS method
   1. Initial parameter values for male and female abundance are set to the values at the beginning of the simulation.
7. I calculate standard error with the delta method (though I don’t really understand this yet)
8. I determine the true value of adult abundance by extracting the number of adult males and females from the year of estimation
9. I examine the relative bias via

# **Kinship probability code**

Note:

Pars1 = vector of parameter values (in this case female abundance and male abundance)

maxAge = maximum age (obviously)

min\_est\_cohort = the birth year of the oldest sampled individual

f\_age\_at\_mat = the year at maturity for females (m\_age\_at\_mat is for males)

The code below is for cownose rays. The functions for lemon sharks are identical.

#Set up empty array that will be filled with function below

P\_Mother = P\_Father = array(0,dim=c(n\_yrs,n\_yrs)) #Dimensions are older sib birth year and younger sib birth year (all of which are specified by n\_yrs)

#CKMR model: populate array with kinship probabilities based on birth years

get\_P\_cownose <- function(Pars1,P\_Mother,P\_Father,t\_start,t\_end){

N\_F=exp(Pars1[1]) #number of mature females

for(os\_birth in min\_est\_cohort:(n\_yrs-1)){

for(ys\_birth in (os\_birth+1):n\_yrs){

if((ys\_birth - os\_birth) <= ((maxAge+1) - f\_age\_at\_mat)){

#Fill in array with kinship probability from CKMR equation

P\_Mother[os\_birth, ys\_birth] <- (surv^(ys\_birth - os\_birth))/(N\_F\*lam^(ys\_birth-min\_est\_cohort))

} else P\_Mother[os\_birth, ys\_birth] <- 0

}

}

##Repeat the above with males

N\_M=exp(Pars1[2]) #number of mature males

for(os\_birth in min\_est\_cohort:(n\_yrs-1)){

for(ys\_birth in (os\_birth+1):n\_yrs){

if((ys\_birth - os\_birth) <= ((maxAge+1) - m\_age\_at\_mat)){

#Fill array with kinship probability from half-sib CKMR equation

P\_Father[os\_birth,ys\_birth] <- (surv^(ys\_birth - os\_birth))/(N\_M\*lam^(ys\_birth-min\_est\_cohort))

} else P\_Father[os\_birth,ys\_birth] <- 0

}

}

return(list(P\_Mother=P\_Mother, P\_Father=P\_Father)) #return makes sure this is moved out of the loop into the environment

}

# **Likelihood function**

Note: The Negatives and Pairs dataframes refer to the separate pairwise comparison matrices for positive and negative kinship comparisons.

cownose\_neg\_log\_lik <- function(Pars1, Negatives\_Mother, Negatives\_Father, Pairs\_Mother, Pairs\_Father, P\_Mother, P\_Father, t\_start, t\_end) {

P=get\_P\_cownose(Pars1 = Pars1, P\_Mother = P\_Mother, P\_Father = P\_Father, t\_start = t\_start, t\_end = t\_end)

loglik=0

#likelihood contributions for all negative comparisons

for(irow in 1:nrow(Negatives\_Mother)){

loglik = loglik + Negatives\_Mother[irow, 3] \* log(1 - P$P\_Mother[Negatives\_Mother[irow, 1], Negatives\_Mother[irow, 2]])

}

for(irow in 1:nrow(Negatives\_Father)){

loglik = loglik + Negatives\_Father[irow, 3] \* log(1 - P$P\_Father[Negatives\_Father[irow, 1], Negatives\_Father[irow, 2]])

}

#likelihood contributions for positive comparisons

for(irow in 1:nrow(Pairs\_Mother)){

loglik = loglik + Pairs\_Mother[irow, 3] \* log(P$P\_Mother[Pairs\_Mother[irow, 1], Pairs\_Mother[irow, 2]])

}

for(irow in 1:nrow(Pairs\_Father)){ loglik = loglik + Pairs\_Father[irow, 3] \* log(P$P\_Father[Pairs\_Father[irow, 1], Pairs\_Father[irow, 2]])

}

-loglik

}

# **Results**

The models that I’ve been using have returned unbiased results when fit to data from a population simulated from a Leslie Matrix, where kinship was assigned based on the probabilities specified in the CKMR model (Figure 1).

Chart, box and whisker chart

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**Figure 1:** Relative bias by sample size under three scenarios of population growth. The populations were simulated from a Leslie Matrix, and kinship was assigned based on the probabilities specified in the model. As sample size increases, so does precision. These results suggest that at the population level, half-sibling CKMR performs as expected.

However, when I use individual-based simulation and fit a model to those data, the model consistently returns estimates that are positively biased. I have tried this with both Lemon Shark (6 pups per female) and Cownose Ray (1 pup per female) life histories and they all give positively biased estimates (Figure 2A).

Timeline

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**Figure 2:** Relative bias by sample size. This simulation was run to test whether there's a difference in bias when ages are misassigned. The key reason to include this here is that the estimates in A are positively biased, even when ages are correct.

# **Troubleshooting**

I’ve run various tests to examine where the bias in Figure 2 might originate. None have proven successful so far.

**Test 1: Include only one individual from each litter**

**Why?** CKMR assumes that the odds of sampling an individual are independent of that individual’s reproductive output. This forms the basis of the Expected Relative Reproductive Output (ERRO) framework outlined in Bravington et. al. (2016). If multiple litter mates are included in the comparison, then the kinship probabilities are dependent not just on the expected reproductive output, but also the variance.

**Test 2: Use cownose ray life history (1 pup per female per year)**

**Why?** In case the bias is caused by persistent variation in litter size among individuals, using a cownose ray life history – where individuals have just one pup per year – should clear up that issue.

**Test 3: Allow lambda to vary in the model**

**Why?** Using a CKMR model that relies on population growth (lambda) to set a single year for abundance estimation results in a model that is sensitive to the value of lambda. But with a non-deterministic population, lambda varies among years. So, I copied code from Paul Conn to include lambda as an estimable parameter, constraining it to +/-1%.

# **Presently …**

04/22/20121

**Current test 1:** I am looping over different values of lambda and survival and seeing where the discrepancy lies.

* values of lambda from obs\_lambda-0.01 : obs\_lambda+0.01, by = .002
* values of survival from obs\_survival-.02 : obs\_survival+0.02, by = 0.01

**Current test 2:** I am removing lambda and setting the true value to the average abundance from all the cohort years.

Taking post-breeding census i.e. age 0 individuals are included.

Does mortality occur before birth/sampling?

Sampling occurs from each dataframe after the entire simulation is run.

Are adults reproductively mature AT age 7?

YES

**Order:**

Birthdays

Mate

Mortality

Sampling occurs after birthday and mate, but doesn’t include mortality.