Mandatory Assignment 1

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1)

2)

For both the LES and RANS models, we start from the Navier-Stokes equations for incompressible flow:

$$\nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \mathbf{u}) = -\frac{\partial p}{\partial x} + \mu \nabla^2 u$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v \mathbf{u}) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v$$

Lengthy derivations of the LES and RANS equations will not be given, but a short explanation of each will give the general process.

For Large Eddy Simulation (LES), we use spatial filtering to separate varying sizes of eddies. A cutoff width Δ is introduced, for which information about eddies smaller than the given width will be ignored/destroyed. A spatial filtering using a filter finction $G(\mathbf{x}, \mathbf{x}', \Delta)$ is introduced, giving in the following form (3.84 in the book):

$$\overline{\phi}(\mathbf{x}, t) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(\mathbf{x}, \mathbf{x}', \Delta) \phi(\mathbf{x}', t) \mathrm{d}x_1' \mathrm{d}x_2' \mathrm{d}x_3'$$

where $\overline{\phi}(\mathbf{x}, t)$ and $\phi(\mathbf{x}, t)$ are the filtered and unfiltered functions respectively. The filter function $G(\mathbf{x}, \mathbf{x}', \Delta)$ can be given in several ways, but the one used in finite volume implementations is the top-hat/box filter function

$$G(\mathbf{x}, \mathbf{x}', \Delta) = \begin{cases} \frac{1}{\Delta^3} & |\mathbf{x} - \mathbf{x}'| \leq \Delta/2 \\ 0 & |\mathbf{x} - \mathbf{x}'| > \Delta/2 \end{cases}$$

Using this filtering on the Navier-Stokes equations, we get the LES momentum equations (the intermediate step from 3.88a-c to 3.89a-c in the book for rewriting the term $\nabla \cdot (\rho \overline{\phi \mathbf{u}})$ is not shown):

$$\frac{\partial(\rho \overline{u})}{\partial t} + \nabla \cdot (\rho \overline{u}\, \overline{\mathbf{u}}) = -\frac{\partial \overline{p}}{\partial x} + \mu \nabla^2 \overline{u} - (\nabla \cdot (\rho \overline{u \mathbf{u}}) - \nabla \cdot (\rho \overline{u}\, \overline{\mathbf{u}}))$$

$$\frac{\partial(\rho \overline{v})}{\partial t} + \nabla \cdot (\rho \overline{v}\, \overline{\mathbf{u}}) = -\frac{\partial \overline{p}}{\partial y} + \mu \nabla^2 \overline{v} - (\nabla \cdot (\rho \overline{v \mathbf{u}}) - \nabla \cdot (\rho \overline{v}\, \overline{\mathbf{u}}))$$

3)

4)

5)

Since turbulence is a three-dimensional phenomenon, I would assume that some critical information could be lost using LES as a 2D model. One example could be an eddy with primarily extension in the z-direction, that might have a width below the cutoff value in the x- or y-directions. Thus the impact of this eddy on the mean flow, which might be significant, could potentially be lost by using only a 2D approach.