UKRIDA
Universitas Kristen Krida Wacana

Pusat MEDIA Pembelajaran UKRIDA

Variabel Random dan Distribusi Probabilitas

Dr. Budi Marpaung, ST., MT.

Being Transformed To Win The Future

Variabel Random (1)

Populasi : sekumpulan unsur atau objek yang memiliki ciri atau karakteristik yang sama

Dalam kenyataannya sulit mengukur semua unit yang ada dalam populasi (biaya, waktu, tenaga, teknis)

Dalam pengambilan sampel secara random, setiap objek memiliki peluang yang sama untuk terpilih menjadi sampel

Variabel yang nilainya ditentukan oleh apa yang terjadi pada suatu percobaan/ eksperimen

A **random variable** is a function that associates a real number with each element in the sample space.

Variabel Random (2)

Dua madu mata dadu dilempar sekaligus, hasilnya sbb.

Dadu 1	Dadu 2					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Variabel Random (3)

Dadu 1	Dadu 2					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

A = nilai dadu 1 ganjil, nilai dadu 2 genap

A = (1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)

$$P(A) = \frac{9}{36} = \frac{1}{4} = 0.25$$

Variabel Random (4)

Dadu 1	Dadu 2					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

B = jumlah nilai kedua dadu tidak lebih dari 5

B = $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

$$P(B) = \frac{10}{36} = \frac{5}{18} = 0.28$$

Variabel Random (5)

Dadu 1	Dadu 2					
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

C = jumlah nilai kedua dadu tepat 6

C = (1,5), (2,4), (3,3), (4,2), (5,1)

$$P(C) = \frac{5}{36} = 0.14$$

Soal Latihan 1

Sebuah dadu dilempar dua kali. Anda diminta untuk menyatakan distribusi probabilitas jumlah angka mata yang muncul.

x	2	3	4	5	6	7	8	9	10	11	12
P(x)											

2 Jenis Variabel Random

Diskrit

Terbatas dan biasanya dinyatakan pada bilangan bulat

- Banyak peserta
- Jumlah kecelakaan
- Jumlah yang antri

Kontinu

Tidak terbatas dan biasanya dinyatakan dalam interval

- ✓ Tinggi badan manusia (55 cm sd 210 cm)
- ✓ Berat produk (20 sd 45 gram)

Prob. Fungsi – Diskrit (1)

The set of ordered pairs $(x, f(x))$ is a **probability function, probability mass function**, or **probability distribution** of the discrete random variable X if, for each possible outcome x,

1. $f(x) \geq 0$,
2. $\sum_x f(x) = 1$,
3. $P(X = x) = f(x)$.

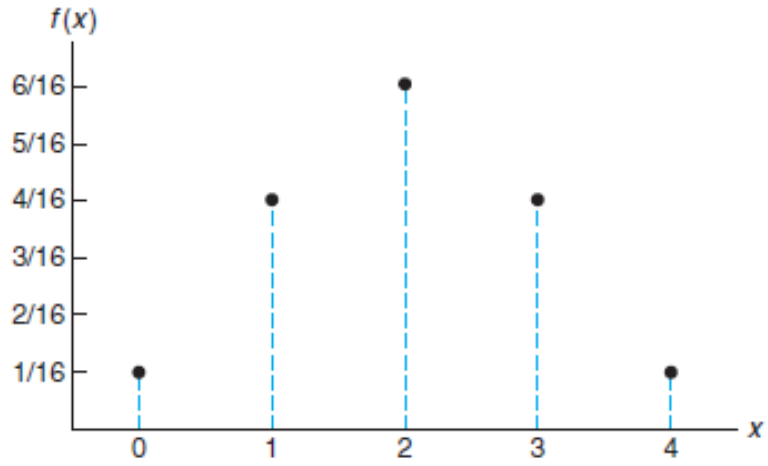

Figure 3.1: Probability mass function plot.

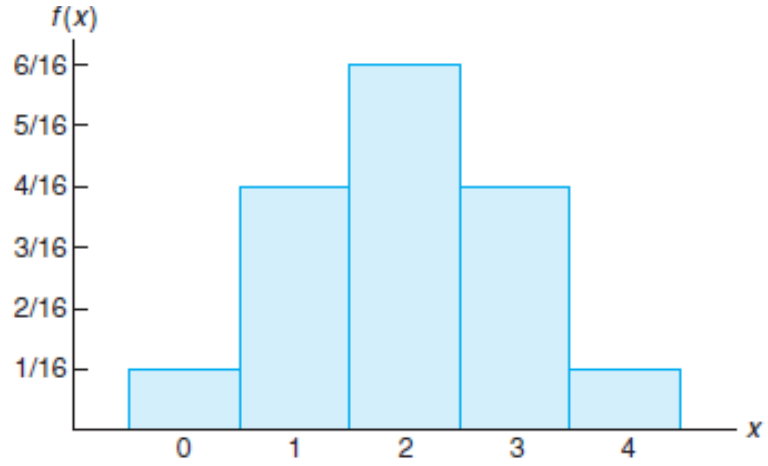

Figure 3.2: Probability histogram.

Prob. Fungsi – Diskrit (2)

The **cumulative distribution function** $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -\infty < x < \infty.$$

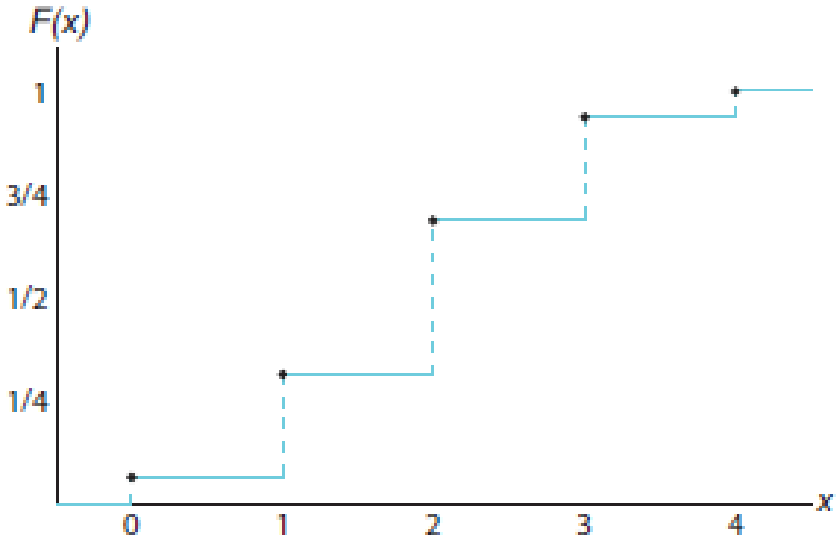

Figure 3.3: Discrete cumulative distribution function.

Contoh

Dari 20 laptop yang dikirim ke retailer, 2 di antaranya cacat. Bila diambil 3 laptop secara random, tentukan distribusi probabilitas barang yang cacat.

$$f(0) = P(X = 0) = \frac{\binom{3}{0}\binom{17}{2}}{\binom{20}{2}} = \frac{68}{95}, \quad f(1) = P(X = 1) = \frac{\binom{3}{1}\binom{17}{1}}{\binom{20}{2}} = \frac{51}{190}$$

$$f(2) = P(X = 2) = \frac{\binom{3}{2}\binom{17}{0}}{\binom{20}{2}} = \frac{3}{190}.$$

Thus, the probability distribution of X is

x	0	1	2
$f(x)$	$\frac{68}{95}$	$\frac{51}{190}$	$\frac{3}{190}$

Soal Latihan 2

3.67 An industrial process manufactures items that can be classified as either defective or not defective. The probability that an item is defective is 0.1. An experiment is conducted in which 5 items are drawn randomly from the process. Let the random variable X be the number of defectives in this sample of 5. What is the probability mass function of X?

Walpole, Halaman 107

Soal Latihan 3

3.11 A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

Walpole, Halaman 92

Prob. Fungsi – Kontinu (1)

The function $f(x)$ is a **probability density function** (pdf) for the continuous random variable X, defined over the set of real numbers, if

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$.
3. $P(a < X < b) = \int_a^b f(x)\, dx$.

The **cumulative distribution function** $F(x)$ of a continuous random variable X with density function $f(x)$ is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt, \quad \text{for } -\infty < x < \infty.$$

Prob. Fungsi – Kontinu (2)

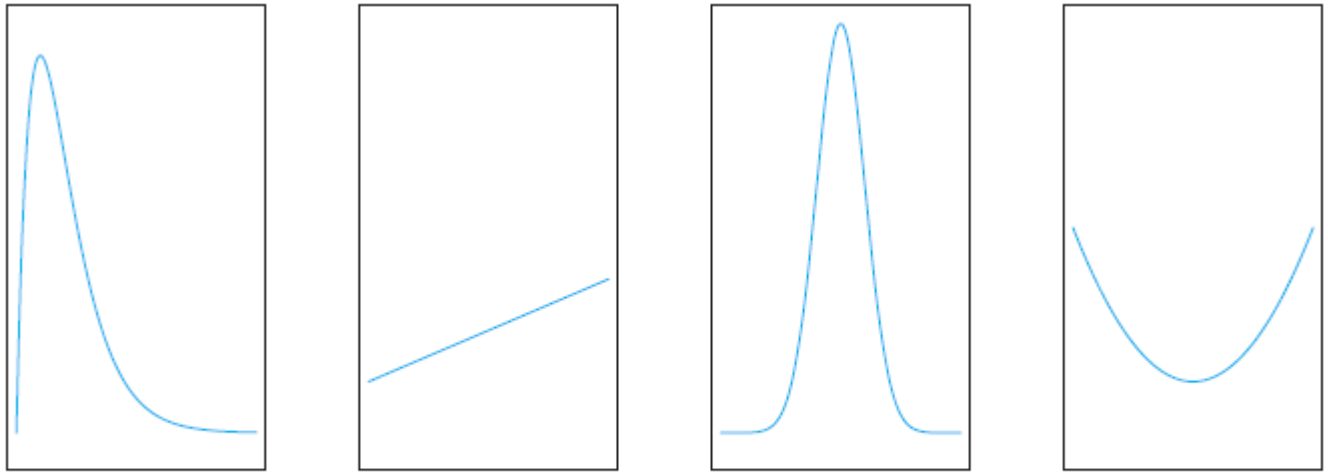

Figure 3.4: Typical density functions.

Prob. Fungsi – Kontinu (3)

$$P(a < X < b) = \int_a^b f(x)\, dx.$$

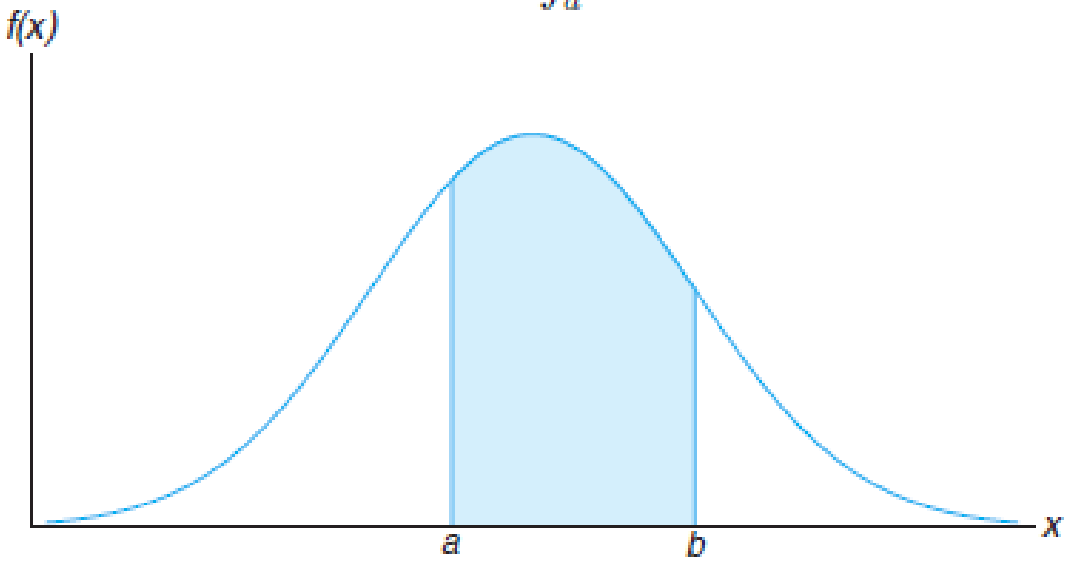

Figure 3.5: $P(a < X < b)$.

Contoh

Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

.

(a) Verify that $f(x)$ is a density function.

(b) Find $P(0 < X \leq 1)$.

Walpole, Halaman 49

Soal Latihan 4

3.21 Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}\,, 0 < x < 1 \\ 0, \text{elsewhere} \end{cases}$$

(a) Evaluate k.
(b) Find F(x) and use it to evaluate P(0.3 <X< 0.6).

Walpole, Halaman 93

Soal Latihan 5

3.73 Impurities in a batch of final product of a chemical process often reflect a serious problem. From considerable plant data gathered, it is known that the proportion Y of impurities in a batch has a density function given by

$$f(y) = \begin{cases} 10(1-y)^9, 0 \le y \le 1 \\ 0, \text{elsewhere} \end{cases}$$

(a) Verify that the above is a valid density function.

(b) A batch is considered not sellable and then not acceptable if the percentage of impurities exceeds 60%. With the current quality of the process, what is the percentage of batches that are not acceptable?

Walpole, Halaman 108

SEKIAN