Multi-Model Slip Ratio Prediction for Energy Management Optimization in Electric Vehicles

I. Trace of Slip Ratio on Energy Consumption

A. Slip Ratio Definition

The slip ratio for longitudinal motion of a road vehicle is defined as follows

$$\kappa = \frac{R_\omega \cdot \omega - V_{veh}}{max(R_\omega, V)} \tag{1}$$

Where:

κ: Slip Ratio

V_{veh}: Vehicle Velocity

R_ω: Wheel Radius

B. Tyre Traction

The tyre-road friction interaction is modeled using Pacejka's Magic Formula:

$$F_x = \mu\left(\kappa\right) \cdot F_{vertical} \tag{2}$$

$$\mu\left(\kappa\right) = D \cdot \sin\left(C \cdot \arctan\left(B\kappa - E\left[B\kappa - \arctan(B\kappa)\right]\right)\right) \tag{3}$$

Where:

F_x: Longitudinal Traction Force

D: Peak Factor $\mu_{max} \cdot F_z$

C: Shape Coefficient

B: Stiffness Coefficient

κ: Wheel Slip

E: Curvature Coefficient

The ammount of Force/Torque produced by the Tractive force is capped by the ammount of Torque that is generated by the shaft to the wheel. This interaction is modeled as in the figure above.

C. Wheel Torque

The torque acting at each wheel is modeled through the Wheel Dynamics equation:

$$T_w = I \cdot \frac{d\omega}{dt} + R_\omega \cdot F_x \tag{4}$$

Where:

T_w: Wheel Torque

I: Moment of Inertia of the Wheel

ω: Angular velocity of the wheel

R_ω: Wheel Radius

F_x: Traction Force

D. Total Motor Torque

The transmission is responsible for the flow of mechanical energy from the motor to each wheel. The torque needed from the motor to power each wheel is modeled as being proportional to the summation of the wheel torques. In the literature, the product of gear ratio τ and the gear efficiency η are used a proportionality constants

$$T_m = \sum_{i \in \{fR, fL, rR, rL\}} \eta \cdot \tau \cdot T_i \tag{5}$$

Where:

T_m: Motor Torque

τ Gear ratio

η: Gear Efficiency

T_w: Wheel Torque

E. Brushed DC Motor

In EVs, the electric motor converts electric power into mechanical power. The required electrical power is:

$$P_e = \frac{\omega \cdot T_m}{\eta_{(\omega, T_m)}} \tag{6}$$

ω: Angular velocity of the motor axis

T_m: Motor Torque

$\eta_{(\omega,T_m)}$: Motor Efficiency

The Brushed DC Motor is modelled as an inductive and resistive load with the EMF from the coil fields considered. The Dynamic Equation of current is therefore:

$$L \cdot \frac{dI_{arm}}{dt} = V_{supply} - K_e \cdot \omega - \frac{R_{arm} \cdot I_{arm}}{L_{arm}} \tag{7}$$

L: Armature inductance

I_{arm}: Armature current

R_{arm}: Armature resistance

V_{supply}: External supply voltage

K_e : Back electromotive force (EMF) constant

ω: Angular velocity of the motor shaft

F. Linear Vehicle Dynamics Derivation

From Newton's second Law of motion:

$$M_{veh} \cdot \dot{V}_{veh} = \Sigma F_x - F_r - F_a - F_g \tag{8}$$

F_x: Wheel Tractive Force

F_r: Rolling Resistance

F_a: Aerodynamic Drag Force

F_g: Grade Force

Since our vehicle has 4 wheels, it will also have 4 tractive forces applied on each of its wheels.

$$M_{veh} \cdot \dot{V}_{veh} = 4 \cdot \mu(\kappa) \cdot F_{vertical} - F_{r+a} - F_{grade} \tag{9}$$

$$=> \dot{V}_{veh} = \frac{4 \cdot \mu(\kappa) \cdot F_{vertical} - F_{r+a} - F_{grade}}{M_{veh}} \tag{10}$$

where $F_{roll+aero}$ is a second order approximation of the the rolling force and grade force acting on the vehicle as a function of velocity.

If we take a linear approximation of the road friction as a function of slip ratio ($\mu(\kappa) \approx C_\kappa \cdot \kappa$) and only consider the first-order approximation of rolling and aerodynamic forces using the coast-down coefficients, the dynamic equation of velocity becomes:

$$\dot{V}_{veh} = \frac{4 \cdot C_\kappa \cdot \kappa \cdot F_{vertical} - C_0 - C_1 \cdot V_{veh} - F_{grade}}{M_{veh}} \tag{11}$$

Substituting the definition of wheel slip, normal force, and grade force, and assuming that the vehicle is moving on a surface with slope α:

$$\dot{V}_{veh} = \frac{4 \cdot C_\kappa \left(\frac{R_\omega \cdot \omega - V_{veh}}{R_\omega}\right) F_{vertical} - C_0 - C_1 \cdot V_{veh} - M_{veh} \cdot g \cdot sin\left(\alpha\right)}{M_{veh}} \tag{12}$$

Simplifying, we obtain the following:

$$\dot{V}_{veh} = \left(\frac{-4 \cdot C_\kappa \cdot g \cdot cos\left(\alpha\right)}{R_\omega} - \frac{C_1}{M_{veh}}\right) \cdot V_{veh} + 4 \cdot C_\kappa \cdot g \cdot cos\left(\alpha\right) \cdot \omega - \frac{C_0}{M_{veh}} - g \cdot sin\left(\alpha\right) \tag{13}$$

For the dynamic equation of angular velocity, we start with the wheel dynamics equations (4) Rearranging, we get:

$$\frac{d\omega}{dt} = \frac{T_w - R_\omega \cdot F_x}{I_m + I_w} \tag{14}$$

I_m: Motor inertia

I_w: Wheel inertia

Substituting the definition of traction force, we get

$$\frac{d\omega}{dt} = \frac{T_w - R_\omega \cdot C_\kappa \left(\frac{R_\omega \cdot \omega - V_{veh}}{R_\omega}\right) F_{vertical}}{I_m + I_w} \tag{15}$$

With the equation (7) of the DC Motor above, the state space model of this system is as follow:

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{4C_\kappa g\cos(\alpha)}{R_\omega} - \frac{C_1}{M_{veh}} & C_\kappa g\cos(\alpha) & 0 \\ \frac{C_\kappa g\cos(\alpha)}{I_m + I_w} & -\frac{C_\kappa R_\omega M_{veh} g\cos(\alpha)}{I_m + I_w} & \frac{K_T}{I_m + I_w} \\ 0 & \frac{-K_e}{L_{arm}} & \frac{-R_{arm}}{L_{arm}} \end{bmatrix} \begin{bmatrix} v \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_{arm}} \end{bmatrix} V_{in} \tag{16}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_{in} \tag{17}$$

All the corresponding parameters for the state space model are redefined for clarity:

- v: Vehicle velocity (m/s).
- ω: Angular velocity of the wheel (rad/s).
- i: Armature current (A).
- V_{in}: Input voltage to the motor (V).

- C_κ: Linear approximation coefficient of road friction as a function of slip ratio.
- g: Acceleration due to gravity ($9.81\,\text{m/s}^2$).
- α: Slope of the road surface (radians).
- R_ω: Wheel radius (m).
- C_1: 1rst Order Coast-down coefficient for aerodynamic drag (kg/s).
- M_{veh}: Vehicle mass (kg).
- I_m: Motor inertia (kg·m^2).
- I_w: Wheel inertia (kg·m^2).
- K_T: Motor torque constant (N·m/A).
- K_e: Back electromotive force (EMF) constant (V·s/rad).
- L_{arm}: Armature inductance (H).
- R_{arm}: Armature resistance (Ω).

G. State Space Analysis

The controllability matrix $\mathcal{C}$ is given by:

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B \end{bmatrix}, \tag{18}$$

where

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_{arm}} \end{bmatrix}, \tag{19}$$

$$AB = \begin{bmatrix} 0 \\ \frac{K_T}{(I_m+I_w)L_{arm}} \\ -\frac{R_{arm}}{L_{arm}^2} \end{bmatrix}, \tag{20}$$

$$A^2B = \begin{bmatrix} \frac{K_T C_\kappa g\cos(\alpha)}{(I_m+I_w)L_{arm}} \\ -\frac{C_\kappa R_\omega M_{veh} g\cos(\alpha) K_T}{(I_m+I_w)^2 L_{arm}} - \frac{K_T R_{arm}}{(I_m+I_w)L_{arm}^2} \\ -\frac{K_e K_T}{(I_m+I_w)L_{arm}^2} + \frac{R_{arm}^2}{L_{arm}^3} \end{bmatrix}. \tag{21}$$

Thus, the full controllability matrix is:

$$\mathcal{C} = \begin{bmatrix} 0 & 0 & \frac{K_T C_\kappa g \cos(\alpha)}{(I_m+I_w)L_{arm}} \\ 0 & \frac{K_T}{(I_m+I_w)L_{arm}} & -\frac{C_\kappa R_\omega M_{veh} g \cos(\alpha) K_T}{(I_m+I_w)^2 L_{arm}} - \frac{K_T R_{arm}}{(I_m+I_w)L_{arm}^2} \\ \frac{1}{L_{arm}} & -\frac{R_{arm}}{L_{arm}^2} & -\frac{K_e K_T}{(I_m+I_w)L_{arm}^2} + \frac{R_{arm}^2}{L_{arm}^3} \end{bmatrix}. \tag{22}$$

Since every column in the controllability matrix is linearly independent, the system is therefore controlable.

The system is also mostly time invariant except for the road friction coefficient C_κ and for the road slope α, which are dynamic parameters which can change depending on the external conditions.