Multiple View Geometry: Exercise Sheet 4

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Part I: Theory

1. **Image Formation**

(a) Compute λ and show that (2) is equivalent to

$$n = \frac{fX}{Z} + o_x\,, \quad m = \frac{fY}{Z} + o_y\,.$$

Performing the matrix multiplication in (2), one obtains

$$\begin{pmatrix} \lambda n \\ \lambda m \\ \lambda \end{pmatrix} = \begin{pmatrix} fX + o_x Z \\ fY + o_y Z \\ Z \end{pmatrix}$$

From the third row, it directly follows that $\lambda = Z$. Using $n = \frac{\lambda n}{\lambda}$ and $m = \frac{\lambda m}{\lambda}$ and inserting, one immediately obtains the result.

(b) A classic ambiguity of the perspective projection is that one cannot tell an object from another object that is exactly *twice as big but twice as far*. Explain why this is true.
Let $\tilde{\mathbf{X}} = (X\ Y\ Z)^\top$ be a point on the smaller object and $\tilde{\mathbf{X}}' = (X'\ Y'\ Z')^\top$ a point on the larger object. Since $\tilde{\mathbf{X}}'$ is twice as far away, we have $Z' = 2Z$, and since it is twice as big we have $X' = 2X$ and $Y' = 2Y$. Thus,

$$n' = \frac{fX'}{Z'} + o_x = \frac{2fX}{2Z} + o_x = \frac{fX}{Z} + o_x = n$$

and analogous for $m' = m$.

(c) For a camera with $f = 540$, $o_x = 320$ and $o_y = 240$, compute the pixel coordinates n and m of a point $\tilde{\mathbf{X}} = (60\ 100\ 180)^\top$.

$$n = \frac{fX}{Z} + o_x = \frac{540 \cdot 60}{180} + 320 = 500$$
$$m = \frac{fY}{Z} + o_y = \frac{540 \cdot 100}{180} + 240 = 540$$

Explain with the help of (b) why the units of $\tilde{\mathbf{X}}$ are not needed for this task.
Using different units (mm, cm, m, etc.) can be interpreted as scaling the point coordinates by a constant factor (10, 100, ...). The argument of (b) for a factor of 2 can easily be generalized to any factor α.
Will the projected point be in the image if it has dimensions 640×480?
No, the point $(n, m) = (500, 540)$ is not in $[0, 640] \times [0, 480]$.

(d) Using the generic projection π, show that (1), (2) and (3) are equivalent to

$$\begin{pmatrix} n \\ m \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} .$$

Insert in the RHS of the equation:

$$K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & o_x \\ 0 & f & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X/Z \\ Y/Z \\ 1 \end{pmatrix} = \begin{pmatrix} fX/Z + o_x \\ fY/Z + o_y \\ 1 \end{pmatrix}$$

2. **Radial Distortion**

(a) Can this model be used for lenses with a field of view of more than $180°$?
No, it can only model points for which the viewing ray intersects the image plane.

(b) Derive a closed form solution for f in the undistortion formula

$$\pi(\tilde{\mathbf{X}}) = f\left(\|\pi_d(\tilde{\mathbf{X}})\|\right) \cdot \pi_d(\tilde{\mathbf{X}})$$

using (4) and $g(r) = g_{\text{ATAN}}(r)$.
Define $r := \|\pi(\tilde{\mathbf{X}})\|$ and $r_d := \|\pi_d(\tilde{\mathbf{X}})\|$. The norms of (4) and (6) are:

$$r = f(r_d)r_d \quad \text{and} \quad r_d = g(r)r$$

Inserting $g = g_{\text{ATAN}}$ yields

$$r_d = \frac{1}{\omega r} \arctan\left(2r\tan\left(\frac{\omega}{2}\right)\right) r = \frac{1}{\omega} \arctan\left(2r\tan\left(\frac{\omega}{2}\right)\right)$$

$$\Rightarrow \tan(r_d\omega) = 2r\tan\left(\frac{\omega}{2}\right)$$

$$\Rightarrow r = \frac{\tan(r_d\omega)}{2\tan\left(\frac{\omega}{2}\right)} = f(r_d)r_d \quad \Rightarrow f(r_d) = \frac{\tan(r_d\omega)}{2r_d\tan\left(\frac{\omega}{2}\right)}$$

3. **Image Rectification**

Let I_d be a distorted image taken by a camera with intrinsic parameter matrix K_d.

Write down expressions for the pixel coordinates n_d, m_d of a point $\tilde{\mathbf{X}} \in \mathbb{R}^3$ in the distorted image and for the pixel coordinates n, m of the same point projected by a perfect pinhole camera with intrinsic camera matrix K.

$$\begin{pmatrix} n_d \\ m_d \\ 1 \end{pmatrix} = K_d \begin{pmatrix} \pi_d(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix} = K_d \begin{pmatrix} g\left(\|\pi(\tilde{\mathbf{X}})\|\right)\pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix}, \quad \begin{pmatrix} n \\ m \\ 1 \end{pmatrix} = K \begin{pmatrix} \pi(\tilde{\mathbf{X}}) \\ 1 \end{pmatrix}$$