



# Multiple View Geometry: Exercise Sheet 6

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<http://vision.in.tum.de/teaching/ss2017/mvg2017>

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## Part I: Theory

The following exercises should be **solved at home**. You do not have to hand in your solutions, however, writing it down will help you present your answer during the tutorials.

1. The essential matrix  $E = \hat{T}R$  has the singular value decomposition  $E = U\Sigma V^T$ . Let  $R_Z(\pm\frac{\pi}{2})$  be the rotation by  $\pm\frac{\pi}{2}$  around the  $z$ -axis.

Show the following properties:

- (a)  $\hat{T} \in so(3)$  (i.e.  $\hat{T}$  is a skew-symmetric matrix)
- (b)  $R \in SO(3)$  (i.e.  $R$  is a rotation matrix)

Hint: Use the equalities:  $\hat{T} = UR_Z(\pm\frac{\pi}{2})\Sigma U^T$  and  $R = UR_Z(\pm\frac{\pi}{2})^T V^T$ .  
(see Chapter 5, Slide 9)

2. Consider the matrices  $E = \hat{T}R$  and  $H = R + Tu^T$  with  $R \in \mathbb{R}^{3 \times 3}$  and  $T, u \in \mathbb{R}^3$ . Show that the following holds:

- (a)  $E = \hat{T}H$
- (b)  $H^T E + E^T H = 0$

3. Let  $F \in \mathbb{R}^{3 \times 3}$  be the fundamental matrix for the cameras  $C_1$  and  $C_2$ . Show that the following holds for the epipoles  $e_1$  and  $e_2$ :

$$Fe_1 = 0 \quad \text{and} \quad e_2^T F = 0$$

## Part II: Practical Exercises

This exercise is to be solved **during the tutorial**.

1. Download the package `ex6.zip` from the website and extract the images `batinria0.tif` and `batinria1.tif`.
2. **Get the 2D coordinates of corresponding point pairs:** Show the first image and mark at least 8 points. You can retrieve the pixel coordinates of mouse clicks with the command `[x,y] = ginput(gcf)`. Then show the second image and click at the corresponding points in the same order. Again you can get the pixel coordinates with `ginput`. Now you should have the 2D coordinates of corresponding point pairs.
3. **Implement the 8-point algorithm** from the lecture and run it with these point pairs. To this end, you have to transform the coordinates. The intrinsic camera matrices are:

$$K1 = \begin{pmatrix} 844.310547 & 0 & 243.413315 \\ 0 & 1202.508301 & 281.529236 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K2 = \begin{pmatrix} 852.721008 & 0 & 252.021805 \\ 0 & 1215.657349 & 288.587189 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Reconstruct the depths of the points as described in Chapter 5 on Slides 17 and 18.

Hints:

- The file `additional_information.txt` provides  $K1$  and  $K2$
- Use `kron` and `reshape`
- It might occur that one of the matrices  $U$  or  $V$  of the SVD of  $E$  has a determinant less than zero. In this case, determine the SVD of  $-E$ .