Hoofdstuk 1

Bézier-curven

1.1 Definities

i Bernstein-veelterm van graad n:

$$B_i^n = \binom{n}{i}(1-t)^{n-i}t^i$$

$B_i^n = 0$ als $i < 0$ of $i > n$.

Bézier curve met $n+1$ controlepunten:

$$\vec{x}(g) = \sum_{i=0}^{n} \vec{b_i} B_i^n(t)$$

1.2 Eigenschappen

1. $$\sum_{i=0}^{n} B_i^n(t) = 1$$

   Bewijs.

   $$\sum_{i=0}^{n} B_i^n(t) = \sum_{i=0}^{n} \binom{n}{i}(1-t)^{n-i}t^i = ((1-t)+t)^n = 1$$

   □

2. Voor $0 \leq t \leq 1$:

   $$B_i^n(t) \geq 0$$

   Bewijs.

   $$\binom{n}{i}(1-t)^{n-i}t^i$$

   $\binom{n}{i}$ is positief, $(1-t)$ is positief want $0 \leq t \leq 1$ en t is ook positief. □

3. Zij $n > 0$.
   - $B_i^n(0) = B_i^n(1) = 0$ als $i \neq$, $i \neq n$.
   - $B_0^n(0) = 1$
   - $B_0^n(1) = 0$
   - $B_n^n(0) = 0$
   - $B_n^n(1) = 1$

   Bewijs. Vul eenvoudigweg alles in.
   - $B_i^n(0) = B_i^n\binom{n}{i}1^{n-i}0^i = 0$ en $B_i^n(1) = B_i^n\binom{n}{i}0^{n-i}1^i = 0$
   - $B_0^n(0) = 1^n = 1$
   - $B_0^n(1) = 0^n = 0$
   - $B_n^n(0) = 0^n = 0$
   - $B_n^n(1) = 1^n = 1$

   $\square$

4. Symmetrie
$$B_i^n(t) = B_{n-i}^n(1-t)$$

   Bewijs.
$$\binom{n}{n-i}(1-1+t)^{n-n+i}(1-t)^{n-i} = \binom{n}{i}t^i(1-t)^{n-i} = \binom{n}{i}(1-t)^{n-i}t^i$$

   $\square$

5. Afgeleide
$$\frac{d}{dt}B_i^n(t) = n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t))$$

   Bewijs.
$$\frac{d}{dt}B_i^n(t) = \frac{d}{dt}\binom{n}{i}(1-t)^{n-i}t^i$$
$$\binom{n}{i}\frac{d}{dt}\left((1-t)^{n-i}t^i\right) = \binom{n}{i}\left(t^i\frac{d}{dt}(1-t)^{n-i} + (1-t)^{n-i}\frac{d}{dt}t^i\right)$$
$$\binom{n}{i}\frac{d}{dt}\left((1-t)^{n-i}t^i\right) = \binom{n}{i}\left((1-t)^{n-i}it^{i-1} - t^i(n-i)(1-t)^{n-i-1}\right)$$
$$= \left(\frac{n!}{i!(n-i)!}(1-t)^{n-i}it^{i-1} - \frac{n!}{i!(n-i)!}t^i(n-i)(1-t)^{n-i-1}\right)$$
$$= n\left(\frac{(n-1)!}{(i-1)!(n-i)!}(1-t)^{n-i}t^{i-1} - \frac{(n-1)!}{i!(n-i-1)!}t^i(1-t)^{n-i-1}\right)$$
$$= n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t))$$

   $\square$