Problem Sheet 4
1. Xn he du position aster noteps de a particle that personn an unbiased random walk on I statingato.
Show that $(X_n^2 - n)_{n \neq 0}$ is a martingale w.r.+ $(X_n)_{n \neq 0}$
desine $(Y_n)=(x_n^2-n)_{n>0}.(Y_n)_{n>0}$ is a martingar is the sollowing three conditions hold.
i) E(14,1) < 00 This is great!
i) $E( Y_n ) < \infty$ $E( X_n^2 ) + E( n ) : Ericugle inequality$ $= E( X_n^2 ) + E( n ) :  X_n^2  =  x_n  +  x_n  =  x_n  +  x_n  =  x_n  +  x_n  +  x_n  =  x_n  +  x_n$
$= E(x_n^2) + n : expectatein Os constant is a$
$\max(X_n) = n^2$ $= E((\Xi_{i=1}^n, 3_i)^2) + n : X_n = \Xi_{i=1}^n 3_i$ where $3_i = \pm 1$ w.p. $\frac{1}{2}$
$=E\left(\sum_{i=1}^{n}3_{i}^{*}+\sum_{i\neq j}^{n}3_{i}^{*}\right)+n$
1) represents all times where both 3's are equal to   or -1  (2) represents all times where one 3 = 1 and the other 3 =-1
$= \sum_{i=1}^{n} E(3_{i}^{2}) + \sum_{i\neq j} E(3_{i} \cdot 3_{j}^{2}) + n$
$=\sum_{i=1}^{n}\cdot 1+\sum_{i\neq j}\cdot 0+n$
where $E(3^2) = (1)^2 (1/2) + (-1)^2 (1/2) = 1$ and $E(3^2) = E(3^2) = (-1)(1/2) + (-1)(1/2) + (-1)(1/2) + (-1)(1/2) = 0$ independence $(-1/2) + (-1)(1/2) + (-1)(1/2) + (-1)(1/2) = 0$
I note protect

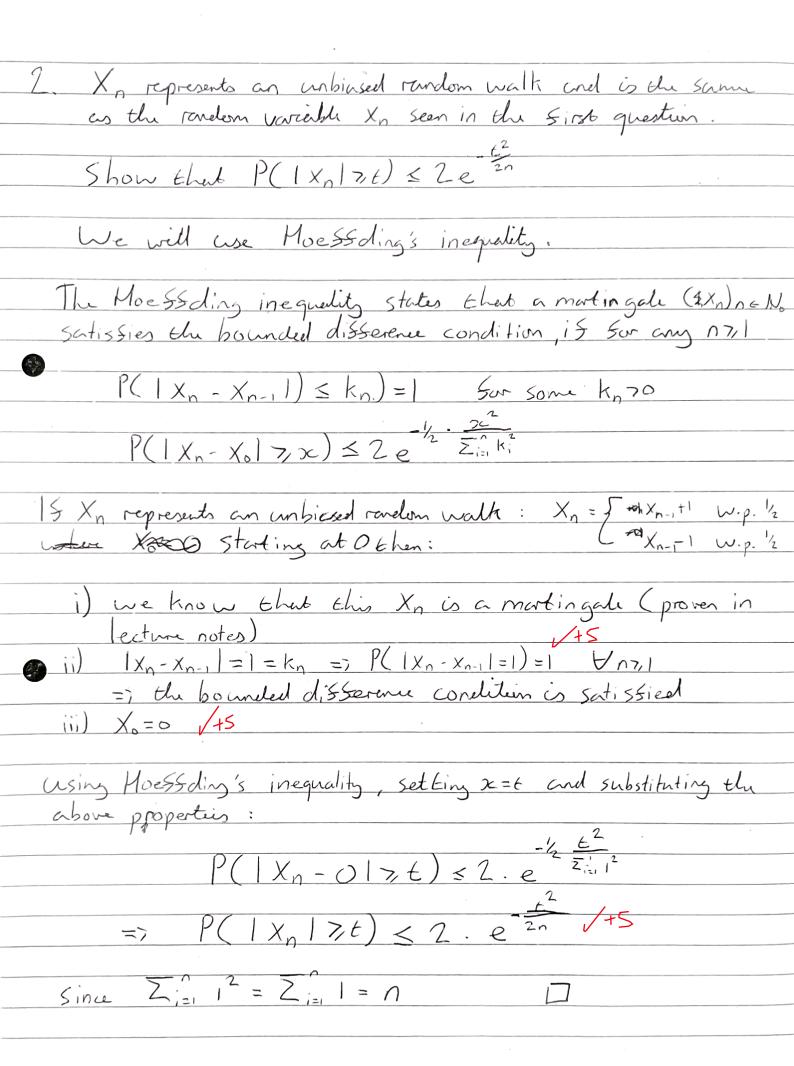
= n + n < 12n < 00 0

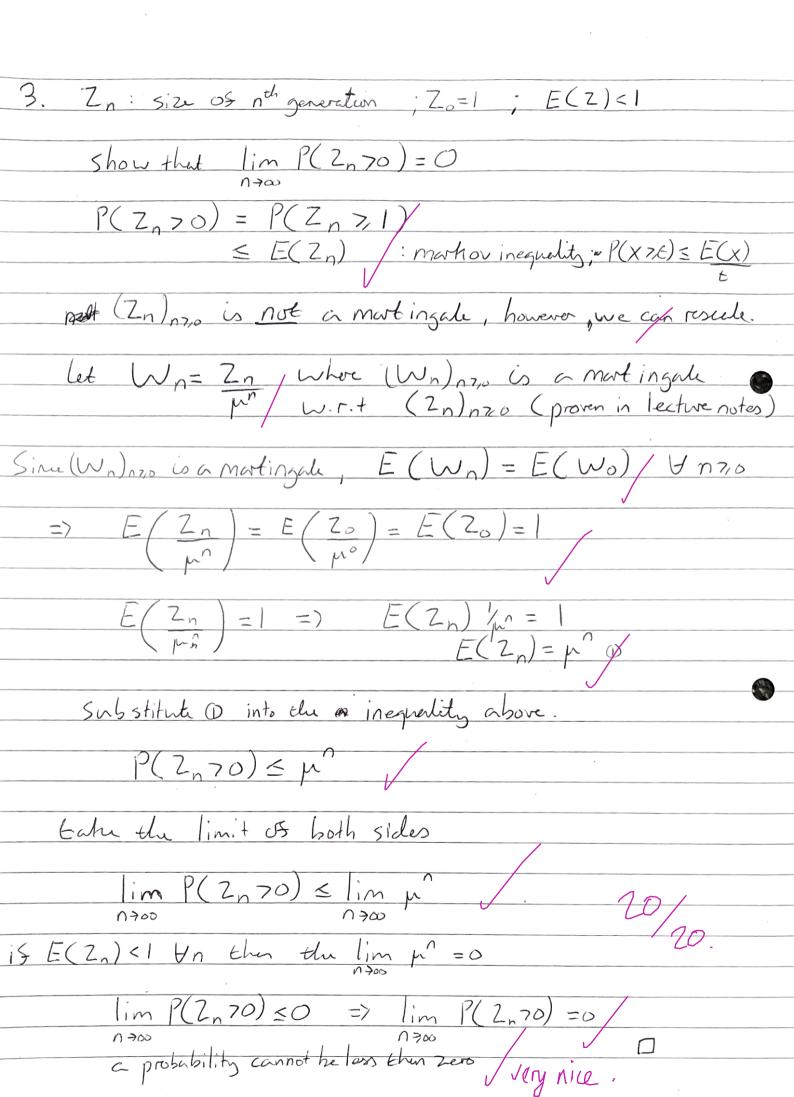
ii) 
$$E(Y_0 \mid X_{n-1}, \dots, X_0) = Y_{n-1}$$

$$E(X_0^2 - n \mid X_{n-1}, \dots, X_0) = E(X_0^2 \mid X_{n-1}, \dots, X_0) - E(n \mid X_{n-1}, \dots, X_0)$$

$$= E(X_0^2 - n \mid X_{n-1}, \dots, X_0) = E(X_0^2 \mid X_{n-1}, \dots, X_0) - n$$

$$= E(X_0^2 \mid X_{n-1}, \dots, X_0) + 2E(X_{n-1}, \dots, X_0) \mid X_{n-1}, \dots, X_0) + E(X_0^2 \mid X_0^2 \mid X_0$$





4.)  $X_n$ : biased random walk  $X_n = \sum_{n-1}^{\infty} x_{n-1} + 1$  w.p. p = 1-pwhere oxpx1. Show that the  $(Y_n)_{n \neq 0}$ , where  $Y_n = ({}^{q_1}p)^{\chi_n}$  is a most ingale with respect to  $(\chi_n)_{n \neq 0}$ . (At the desire  $X_n = \overline{2}$  3: where 3 := 5 + 1 w.p.p. q = 1 - pthe Sollowing three conditions must hold Sur (Yn) 17,0 to be a martingale W. r. + (Xn) 17,6 i)  $E(|Y_n|) < 00$   $E(|Y_n|) = E(|(q_{ip})^{x_n}|)$   $= E((q_{ip})^{x_n})$  is  $0 thin <math>q_{ip} > 0$  and  $(q_{ip})^{x_n} > 0$ ?  $= E((q_{ip})^{x_n} > 0$ ?  $= E((q_{ip})^{x_n} > 0$ ?  $= E((a_{/p})^{3}, (a_{/p})^{3}, \dots (a_{/p})^{3})$  $= E((2/p)^{3}) \cdot E((2/p)^{3}) \cdot ... E((2/p)^{3})$  $E((a_{1p})^{3}) = (a_{1p})(p) + (a_{1p})(q)$   $= a_{1} \cdot p + p_{1} \cdot q$  = q + p = q + p $= \sum_{i=1}^{n} E(|Y_{n}|) = \prod_{i=1}^{n} E((^{q}p)^{3}i) = \prod_{i=1}^{n} | = | < \infty | D$ 

