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# Analysis and validation of a new hydraulic cylinder nominal dynamics

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***Abstract*— The paper provides analysis and validation of a new model of hydraulic cylinder dynamics. First, we analyze the conventional nominal model via a power preserving interconnection as a port-based approach. Second, we propose and analyze a new nominal model of hydraulic cylinder dynamics. Remarkably, the mechanical part of both models are the same in the representation and the fluid part of the proposed model is a generalized version of that of the convectional model. Furthermore, some properties as simplicity of the conventional nominal model are found in the proposed nominal model again. Finally, we show the effectiveness of the proposed nominal model by a comprehensive numerical validation as well as a non-comprehensive experimental validation.**

## I. Introduction

Hydraulic robots can achieve high power-weight ratio in comparison with electric robots and also realize a gravity compensation without any additional control. A new modeling of hydraulic cylinder dynamics will be a foundation to improve control performance against the nonlinearity, uncertainty, parameter perturbation, and so on (e.g., [14] [21] [8]).

The balance between accuracy and simplicity is always a main issue in modeling (e.g., [3] [11]). Indeed, even though hydraulic robots are a class of fluid-mechanical systems, instead of the infinite dimensional model, the finite dimensional models are familiar, especially as the nominal models, in many controller design procedures (e.g., [22], [1] [13] [9] [7] [10] ) as well as the parameter identification procedure [17] and the fast computation procedure [16].

A starting point of the study in this paper is an analysis of one of the most familiar conventional nominal models via a power preserving interconnection as a port-based approach ([2] [3] [5]) by which we decompose the conventional nominal model into the several components. Then, from the physical dimension (the dimension of physical quantities [4]) point of view, we can check links to fruitful results (e.g., robust stabilization, learning [5] [19] [6]) that the general nonlinear models (e.g., $\dot{x} = f(x) + g(x)u$ and $y = h(x)$) can not have. It is needless to say that if the conventional nominal model does not keep the links, it is of importance to propose a new nominal model in which such links hold at least.

The rest of this paper is organized as follows. In Section II, we review the conventional nominal model of hydraulic cylinder dynamics and analyze it via a power preserving interconnection. Unfortunately, one of the main components of the conventional nominal model has the different physical dimension from the standard component. This means that

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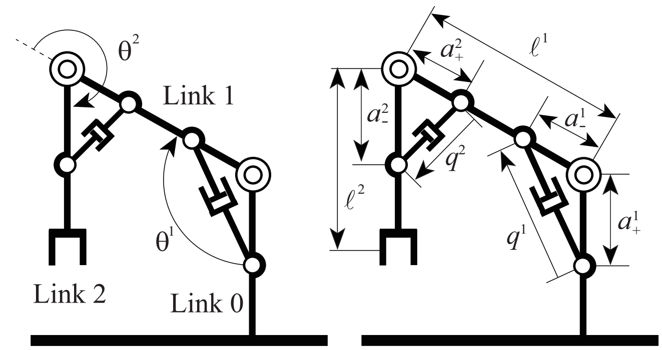


Fig. 1. Hydraulic robots.

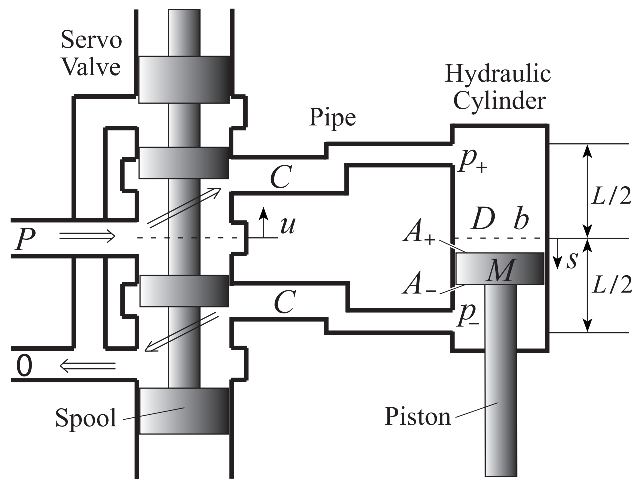


Fig. 2. Hydraulic actuators.

the links to fruitful results can be lost in the conventional nominal model. In Section III, we provide and analyze a new nominal model of hydraulic cylinder dynamics whose fluid part is a generalized version of that of the conventional one and the mechanical parts are the same in the representation. It is shown that some properties such as simplicity of the conventional nominal model are found in the proposed nominal model successfully. In Section IV, the effectiveness of the proposed nominal model is confirmed by non-comprehensive experimental validation as well as comprehensive numerical validation based on our previous result [18]. In Section V, this paper is concluded.

## II. Conventional nominal model

This section presents a review and an analysis of the conventional nominal model of hydraulic cylinder dynamics.

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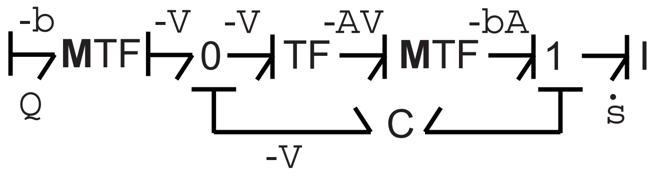


Fig. 3. The conventional nominal model in a bond graph representation.

### A. Review of the conventional nominal model

Figure 2 shows a hydraulic cylinder. Let us review the conventional nominal model in the original representation: [18]

**Definition 1 (conventional nominal model)** *Consider the conventional nominal model*

$$\begin{cases} M\dfrac{d^2 s}{dt^2} = -D\dfrac{ds}{dt} + A_+ p_+ - A_- p_- \\ \dfrac{dp_+}{dt} = bV_+(s)^{-1}\left[-A_+\dfrac{ds}{dt} + Q_+(p_+,u)\right] \\ \dfrac{dp_-}{dt} = bV_-(s)^{-1}\left[+A_-\dfrac{ds}{dt} - Q_-(p_-,u)\right] \end{cases} \tag{1}$$

*where the displacement $s(t)$ [m], the cap pressure $p_+(t)$ [Pa], the rod pressure $p_-(t)$ [Pa], and the spool displacement (the input) $u(t)$ [m] are the functions of time $t$ [s]. The subscript $+$ and $-$ denote the cap-side and the rod-side, respectively, and the subscript $\pm$ denotes both sides. The driving force is $f(t) = A_+ p_+(t) - A_- p_-(t)$ [N]. The mass $M$ [kg], the damping constant $D$ [Ns/m], the piston areas $A_+ \geq A_-$ [m$^2$], and the bulk modulus $b$ [Pa] are the positive constants. The cylinder volumes $V_+(s(t)) = A_+(L/2 + s(t))$, $V_-(s(t)) = A_-(L/2 - s(t))$ [m$^3$] with the constant stroke $L$ [m] are the functions of the displacement $s(t)$. The input $Q_+$ and $Q_-$ [m$^3$/s], are approximated by Bernoulli's principle:*

$$Q_+ = B(p_+, +u)u, \quad Q_- = B(p_-, -u)u \tag{2}$$

*with*

$$B(r,u) = \begin{cases} C\sqrt{-r+P} & (u > 0) \\ 0 & (u = 0) \\ C\sqrt{+r-0} & (u < 0) \end{cases}$$

*where the flow gain $C$ [$\sqrt{\mathrm{m}^5/\mathrm{kg}}$] and the source pressure $P$ [Pa] are the positive constants.*

The conventional nominal model introduces the restricted domain $s \in (-L/2, L/2)$ and $p_\pm \in [0, P]$ and the absolute notation within the square root functions (2) is dropped.

**Remark (Uncertainty)** The conventional nominal model (1) (2) is not our result but well-known. The equations (1) ignore the nonlinear friction effect and also the internal and external leakage effects at least. The equations (2) assume the steady flow and the negligible servo dynamics of the zero-lapped spool valve. On the other hand, the stroke $L$ can include the pipeline length effect and the bulk modulus $b$ includes the pipeline (or tube) flexibility effect. Of course, the difference (e.g. the nonlinear friction effect by the actual sealing) between the conventional nominal model and the experimental setup exists and depends on each experimental setup but would change continuously. In the context of robust control ([20] [23]), the difference is taken into account in the controller design procedure as uncertainty.

The conventional nominal model (1) (2) is converted into the input-state equation of the physical form ([9] [7] [12] [20] [5]):

$$\begin{cases} \dfrac{dx}{dt} = \underbrace{\begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & -D & J_{23} & J_{24} \\ 0 & -J_{23} & 0 & 0 \\ 0 & -J_{24} & 0 & 0 \end{bmatrix}}_{F} \nabla_x H + \underbrace{\begin{bmatrix} 0 \\ 0 \\ +bV_+^{-1}Q_+ \\ -bV_-^{-1}Q_- \end{bmatrix}}_{gu} \\ y = g^{\mathrm{T}} \nabla_x H \end{cases} \tag{3}$$

with the state $x = (s, p_m, p_+, p_-)^{\mathrm{T}}$,

$$J_{23}(s) = +bV_+(s)^{-1}A_+, \quad J_{24}(s) = -bV_-(s)^{-1}A_-,$$

and the conventional energy

$$H = p_m^2/(2M) - V_+(s)(b + p_+) - V_-(s)(b + p_-).$$

Here, the notation $\nabla_x$ denotes the gradient with respect to the variable $x$. The variable $p_m = Mv$ is the momentum with the velocity $v = \frac{ds}{dt}$.

**Remark (Casimirs)** The above conventional energy drops Casimir functions (the exponential terms) in the original energy [9]. Since the Casimir functions change the output of the physical form only (3) and do not change the input-state response, the following results in this paper hold even in the presence of the Casimir functions.

### B. Analysis of the conventional nominal model

Now let us introduce a famous concept of the power-preserving interconnection by which the conventional and the proposed nominal model in this paper are analyzed.

**Definition 2 (Dirac structure)** ([20] [5]) *A Dirac structure on $F \times F^*$ is a subspace $\mathscr{D} \subset F \times F^*$ such that*

$$\mathscr{D} = \mathscr{D}^\perp$$

*where $\perp$ denotes the orthogonal complement with respect to*

$$<< (f_1, e_1), (f_2, e_2) >>= e_1^{\mathrm{T}} f_2 + e_2^{\mathrm{T}} f_1.$$

*Here, the dual space $F^*$ is a set of the linear functional $e$ acting on $f \in F$ which is a finite linear space. The inner product is introduced for a simplicity.*

The physical form 3 makes the conventional nominal model (1) (2) decomposed into the several elements as shown in Figure 2 in the bond graph representation ([2] [3] [5]):

$$\mathrm{I}: \begin{cases} \dfrac{dp_m}{dt} = -f_2 \\ e_2 = \nabla_{p_m} K \ (= p_m/M = \dot{s}) \end{cases} \tag{4}$$

with the energy $K = p_m^2/(2M)$,

$$\mathrm{C}_+ : \begin{cases} \dfrac{ds}{dt} &= -f_1 \\ \dfrac{dp_+}{dt} &= -f_3 \\ e_1 &= \nabla_s U_+ \\ e_3 &= \nabla_{p_+} U_+ \ (= -V_+(s)) \end{cases} \tag{5}$$

with the energy $U_+(s, p_+) = -(p_+ + b)V_+(s)$, and

$$\mathrm{C}_- : \begin{cases} \dfrac{ds}{dt} &= -f_1 \\ \dfrac{dp_-}{dt} &= -f_4 \\ e_1 &= \nabla_s U_- \\ e_4 &= \nabla_{p_-} U_- \ (= -V_-(s)) \end{cases} \tag{6}$$

with the energy $U_-(s, p_-) = -(p_- + b)V_-(s)$, via the corresponding Dirac structure:

$$\mathscr{D} \ni \left( -\frac{dx}{dt}, f_c, \nabla H, e_c \right) \tag{7}$$

where $f_c$ and $e_c$ are the input $u$ and output $y$, respectively.

It is observed that the $\mathrm{C}_\pm$ element can not be the standard C element in the hydraulic systems ([2] [5]) in the sense that the physical dimension of the outputs $e_3$ and $e_4$ of the $\mathrm{C}_\pm$ element is not pressure while the output of the standard C element is pressure. This means that the links to fruitful results (e.g., robust stabilization, learning [5] [19] [6]) can be lost in the conventional nominal model. It is of importance to propose a new nominal model in which such links are recovered at least.

## III. A NEW NOMINAL MODEL

As the the main contribution of this paper, in this section, let us propose a new nominal model of hydraulic cylinder dynamics. Remarkably, the mechanical part of both models are the same in the representation and the fluid part of the proposed nominal model is a generalized version of that of the convectional model. Some properties as simplicity of the conventional nominal model are found in the proposed nominal model.

### A. Definition of a proposed nominal model

**Definition 4 (Proposed nominal model)** *Consider the following dynamics:*

$$\begin{cases} M\dfrac{d^2s}{dt^2} = -D\dfrac{ds}{dt} + A_+p_+ - A_-p_- \\ \dfrac{dp_+}{dt} = \tilde{b}(p_+)V_+(s)^{-1}\left[-A_+\dfrac{\tilde{b}(p_+)}{b}\dfrac{ds}{dt} + Q_+(p_+,u)\right] \\ \dfrac{dp_-}{dt} = \tilde{b}(p_-)V_-(s)^{-1}\left[+A_-\dfrac{\tilde{b}(p_-)}{b}\dfrac{ds}{dt} - Q_-(p_-,u)\right] \end{cases} \tag{8}$$

*with*

$$\tilde{b}(p_\pm) = b\sqrt{1 + (2p_\pm/b)}.$$

*The proposed nominal model is defined as a set of the dynamics (8) and the input flows (2).*

The following two properties are observed immediately.

**Remark (Property 1)** The mechanical part of the proposed nominal model (8) (2) (the right hand side of the first equation in (8)) and that of the conventional nominal model (1) (2) (the right hand side of the first equation in (1)) are the same in the representation.

**Remark (Property 2)** If we apply a replacement,

$$\tilde{b}(p_\pm) \to b$$

which implies $p_\pm/b \to 0$, then the fluid part of the proposed nominal model (8) (2) (the right hand side of the second and third equations in (8)) and that of the conventional nominal model (1) (2) (the right hand side of the second and third equations in (1)) are the same in the representation.

In a word, under the replacement, the proposed nominal model (8) (2) and the conventional nominal model (1) (2) are the same in the representation. Also, as the conventional nominal model, we can see the physical form of the proposed nominal model.

**Proposition 5 (Property 3)** *Consider the proposed nominal model (8) (2). Then the proposed nominal model is equivalent to the input-state equation of the following physical form:*

$$\begin{cases} \begin{bmatrix} \dot{s} \\ \dot{p_m} \\ \dot{p}_+ \\ \dot{p}_- \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & -D & \bar{J}_{23} & \bar{J}_{24} \\ 0 & -\bar{J}_{23} & 0 & 0 \\ 0 & -\bar{J}_{24} & 0 & 0 \end{bmatrix}}_{=F_c} \nabla H + \underbrace{\begin{bmatrix} 0 \\ 0 \\ g_3 Q_+ \\ g_4 Q_- \end{bmatrix}}_{g_c u} \\ y = g_c^{\mathrm{T}} \nabla H \end{cases} \tag{9}$$

*with*

$$\begin{cases} \bar{J}_{23}(s,p_+) &= -\dfrac{A_+(1+2p_+/b)}{V_+(s)}, \\ \bar{J}_{24}(s,p_-) &= +\dfrac{A_-(1+2p_-/b)}{V_-(s)}, \\ g_3(s,p_+) &= +\dfrac{b\sqrt{1+2p_+/b}}{V_+(s)}, \\ g_4(s,p_+) &= -\dfrac{b\sqrt{1+2p_-/b}}{V_-(s)} \end{cases}$$

*and the energy*

$$H = \frac{p_m^2}{2M} + \frac{V_+(s)}{2b}\left(-b + b\sqrt{1+\frac{2p_+}{b}}\right)^2 + \frac{V_-(s)}{2b}\left(-b + b\sqrt{1+\frac{2p_-}{b}}\right)^2.$$

**Proof of Proposition 5.** By the direct substitution of the gradient $\nabla H$:

$$\begin{bmatrix} \frac{A_+}{2b}\left(-b+b\sqrt{1+\frac{2p_+}{b}}\right)^2-\frac{A_-}{2b}\left(-b+b\sqrt{1+\frac{2p_-}{b}}\right)^2 \\ \frac{p_m}{M} \\ \frac{V_+(s)(-b+b\sqrt{1+(2p_+)/b}}{b\sqrt{1+(2p_+)/b}} \\ \frac{V_-(s)(-b+b\sqrt{1+(2p_-)/b}}{b\sqrt{1+(2p_-)/b}} \end{bmatrix}$$

into the input-state equation, the proof is completed. ■

*B. Analysis of the proposed nominal model*

Now let us analyze the proposed nominal model starting from the physical form (9).

**Proposition 6** *Consider the proposed nominal model (8) (2). Then there exists a coordinate transformation by which the physical form (9) is converted into a physical form:*

$$\left\{\begin{aligned} \frac{d}{dt}\begin{bmatrix} s \\ p_m \\ \tilde{V}_+ \\ \tilde{V}_- \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & -D & +A_+ & -A_- \\ 0 & -A_+ & 0 & 0 \\ 0 & +A_- & 0 & 0 \end{bmatrix}}_{F_A}\nabla H+\underbrace{\begin{bmatrix} 0 \\ 0 \\ Q_+ \\ Q_- \end{bmatrix}}_{g_A u} \\ y &= g_A^{\mathrm{T}}\nabla H \end{aligned}\right. \tag{10}$$

*with the state $\hat{x} := (s, p_m, \tilde{V}_+, \tilde{V}_-)^{\mathrm{T}}$, and the energy*

$$H = p_m^2/(2M)+U_+(s,\tilde{V}_+)+U_-(s,\tilde{V}_-).$$

*with the energy $U_+(s,\tilde{V}_+) = b/(2V_+(s))\tilde{V}_+(s)^2$ and $U_-(s,\tilde{V}_-) = b/(2V_-(s))\tilde{V}_-(s)^2$. Here $U_+$ and $U_-$ are the well-known potential energy due to the compressibility (elasticity).*

**Proof of Proposition 6.** For the form (10), let us take the first coordinate transformation:

$$\tilde{x} := \begin{bmatrix} s \\ p_m \\ \tilde{p}_+ \\ \tilde{p}_- \end{bmatrix} := \begin{bmatrix} s \\ p_m \\ \frac{b\tilde{V}_+}{V_+(s)} \\ \frac{b\tilde{V}_-}{V_-(s)} \end{bmatrix} =: \phi_1(\hat{x}).$$

Then, since we have

$$\nabla_{\hat{x}}\phi_1(\hat{x}) = \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \frac{-\tilde{V}_+A_+}{V_+(s)^2} & 0 & b/V_+(s) & 0 \\ \frac{-\tilde{V}_-A_-}{V_-(s)^2} & 0 & 0 & b/V_-(s) \end{bmatrix} = T_1,$$

the form (10) is transformed to the following:

$$\left\{\begin{aligned} \frac{d}{dt}\begin{bmatrix} s \\ p_m \\ \tilde{p}_+ \\ \tilde{p}_- \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & -D & \tilde{J}_{23} & \tilde{J}_{24} \\ 0 & -\tilde{J}_{23} & 0 & 0 \\ 0 & -\tilde{J}_{24} & 0 & 0 \end{bmatrix}}_{T_1F_AT_1^{\mathrm{T}}=F_B}\nabla H+\underbrace{\begin{bmatrix} 0 \\ 0 \\ +\tilde{J}_{23}(s,0)Q_+ \\ -\tilde{J}_{24}(s,0)Q_- \end{bmatrix}}_{T_1g_Au=:g_Bu} \\ y &= g_B^{\mathrm{T}}\nabla H \end{aligned}\right. \tag{11}$$

with

$$\begin{aligned} \tilde{J}_{23}(s,\tilde{p}_+) &= +\frac{\tilde{p}_++b}{V_+(s)}A_+, \\ \tilde{J}_{24}(s,\tilde{p}_-) &= -\frac{\tilde{p}_-+b}{V_-(s)}A_-, \end{aligned}$$

and the energy

$$H = \frac{p_m^2}{2M}+V_+(s)\frac{\tilde{p}_+^2}{2b}+V_-(s)\frac{\tilde{p}_-^2}{2b}.$$

Note that the above $F_B$ can be $F$ of the conventional nominal model in case of $\tilde{p}_\pm = 0$.

Finally, let us take the second coordinate transformation:

$$\bar{x} := \begin{bmatrix} s \\ p_m \\ p_+ \\ p_- \end{bmatrix} = \begin{bmatrix} s \\ p_m \\ \frac{\tilde{p}_+^2}{2b}+\tilde{p}_+ \\ \frac{\tilde{p}_-^2}{2b}+\tilde{p}_- \end{bmatrix} = \phi_2(\tilde{x})$$

where

$$p_\pm = \left(\frac{\tilde{p}_\pm}{2b}+1\right)\tilde{p}_\pm \Leftrightarrow \tilde{p}_\pm = b\left(-1+\sqrt{1+2\frac{p_\pm}{b}}\right) \geq 0$$

within the pressure domain $p_\pm \in [0,P]$.

Then, since we have

$$\nabla_{\tilde{x}}\phi_2(\tilde{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\tilde{p}_+}{b}+1 & 0 \\ 0 & 0 & 0 & \frac{\tilde{p}_-}{b}+1 \end{bmatrix} = T_2,$$

the physical form (10) is transformed to the following:

$$\left\{\begin{aligned} \begin{bmatrix} \dot{s} \\ \dot{p_m} \\ \dot{p}_+ \\ \dot{p}_- \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & -D & J_{23} & J_{24} \\ 0 & -J_{23} & 0 & 0 \\ 0 & -J_{24} & 0 & 0 \end{bmatrix}}_{T_2F_BT_2^{\mathrm{T}}=F_c}\nabla H+\underbrace{\begin{bmatrix} 0 \\ 0 \\ g_3Q_+ \\ g_4Q_- \end{bmatrix}}_{T_2g_Bu=g_cu} \\ y &= g_c^{\mathrm{T}}\nabla H \end{aligned}\right. \tag{12}$$

with

$$\left\{\begin{aligned} J_{23}(s,p_+) &= -\frac{A_+(1+2p_+/b)}{V_+(s)}, \\ J_{24}(s,p_-) &= +\frac{A_-(1+2p_-/b)}{V_-(s)}, \\ g_3(s,p_+) &= +\frac{b\sqrt{1+2p_+/b}}{V_+(s)}, \\ g_4(s,p_+) &= -\frac{b\sqrt{1+2p_-/b}}{V_-(s)} \end{aligned}\right.$$

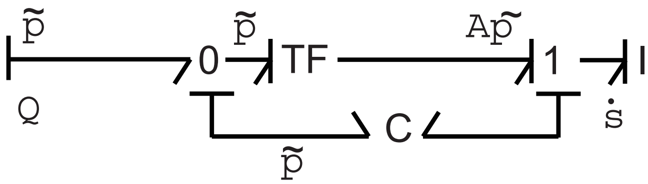


Fig. 4. The proposed nominal model in a bond graph representation.

and the energy

$$H = \frac{p_m^2}{2M} + \frac{V_+(s)}{2b}\left(-b+b\sqrt{1+\frac{2p_+}{b}}\right)^2 + \frac{V_-(s)}{2b}\left(-b+b\sqrt{1+\frac{2p_-}{b}}\right)^2.$$

■

The physical form (9) makes the proposed nominal model (8) (2) decomposed into the several elements as shown in Figure 3 in the bond graph representation (I element is omitted here):

$$\mathrm{C}_+ : \begin{cases} \dfrac{ds}{dt} &= -f_1 \\ \dfrac{d\tilde{V}_+}{dt} &= -f_3 (= \tilde{Q}_+ = -A_+\dot{s} + Q_+) \\ e_1 &= \nabla_s U_+ \\ e_3 &= \nabla_{\tilde{V}_+} U_+ \ (= \tilde{p}_+) \end{cases} \quad (13)$$

with the energy $U_+(s,\tilde{V}_+) = b/(2V_+(s))\tilde{V}_+(s)^2$ and

$$\mathrm{C}_- : \begin{cases} \dfrac{ds}{dt} &= -f_1 \\ \dfrac{d\tilde{V}_+}{dt} &= -f_4 (= \tilde{Q}_- = +A_-\dot{s} - Q_-) \\ e_1 &= \nabla_s U_- \\ e_4 &= \nabla_{\tilde{V}_-} U_- \ (= \tilde{p}_-) \end{cases} \quad (14)$$

with the energy $U_-(s,\tilde{V}_-) = b/(2V_-(s))\tilde{V}_-(s)^2$, via the corresponding Dirac structure:

$$\mathscr{D} \ni \left(-\frac{d\hat{x}}{dt}, f_c, \nabla H, e_c\right) \quad (15)$$

where $f_c$ and $e_c$ are the input $u$ and the output $y$, respectively.

Now, there is no paradox issue discussed in Section 2 of the paper in the sense that the physical dimension of the outputs $e_3$ and $e_4$ of the $\mathrm{C}_\pm$ element is the pressure.

Furthermore, some properties ([15] [16] [18]) as simplicity of the conventional nominal model are found in the proposed model. Especially, the following two properties are useful in the validation in the next section.

**Proposition 7 (Property 4)** *Let the notation $\phi[\theta, x(0), u(t)]$ denote the state $x(t)$ of the proposed nominal model (8) (2) of $\theta = (M, D, L, A_+, A_-, b, C, P)$ at time $t$ starting from the initial state $x(0)$ in the presence of the input signal $u(\tau)$ $(0 \le \tau \le t)$. Suppose the state $x(t) = \phi[\theta, x(0), u(t)]$ exists. Then the non-dimensional state $x^*(t^*) = X_s^{-1}\phi[\theta, X_s x^*(0), U_s u^*(T_s t^*)]$ in the special non-dimensional representation:*

$$\begin{cases} 1 \cdot \ddot{s}^* = -D^*\dot{s}^* + 1p_+^* - A_-^* p_-^* \\ \dot{p}_+^* = \tilde{b}^*(p_+^*)V_+^*(s)^{-1}\left[-1\dfrac{\tilde{b}^*(p_-^*)}{1}\dot{s}^* + Q_+(p_+^*, u^*)^*\right] \\ \dot{p}_-^* = \tilde{b}^*(p_-^*)V_-^*(s)^{-1}\left[+A_-^*\dfrac{\tilde{b}^*(p_-^*)}{1}\dot{s}^* - Q_-^*(p_-^*, u^*)\right] \end{cases} \quad (16)$$

*with*

$$\tilde{b}(p_\pm) = 1\sqrt{1+(2p_\pm/1)}.$$

*at non-dimensional time $t^* = (1/T_s)t$ starting from the nondimensional initial state $x^*(0) = X_s^{-1}x(0)$ in the presence of the non-dimensional input $u^*(t^*) = (1/U_s)u(t^*)$ is given as*

$$x^*(t^*) = \phi[\theta^*_{\mathrm{special}}(\theta), x^*(0), u^*(t^*)] \quad (17)$$

*of* $\theta^*_{\mathrm{special}}(\theta) = (1, \underbrace{D\sqrt{L/(MbA_+)}}_{0<D^*}, 1, 1, \underbrace{A_-/A_+}_{0<A^*\le 1}, 1, 1, \underbrace{P/b}_{0<P^*}).$

**Proof of Proposition 7.** See the paper [18] about the information of $X_s$, $U_s$, and so on and apply the similar techniques. ■

As in the conventional nominal model, the 8-dimensional parameter space of the proposed nominal model is reduce to the 3-dimensional parameter space $(D^*, A^*, P^*) \in \mathbb{R}_+ \times (0,1] \times \mathbb{R}_+ \subset \mathbb{R}_+^3$. The result will reduce the computational time in the comprehensive numerical validation since we can reduce the parameter updates.

**Proposition 8 (Property 5)** *Consider the proposed nominal model (8) (2). Then the physical form (10) of the proposed nominal model with the zero input has the following two Casimir functions:*

$$C_+ = A_+ s + \tilde{V}_+, \quad C_- = A_- s - \tilde{V}_-,$$

**Proof of Proposition 8.** See the paper [16] about the information of Casimir functions ([20] [5]) in the presence of the energy dissipation and apply the similar techniques. ■

As in the conventional nominal model, again, the result will reduce the computational time in the comprehensive numerical validation since we can make the direct dynamics computation faster.

## IV. Validation

It is needless to say that the accuracy of the proposed nominal model is one of the most important aspects. In this section, we discuss a comprehensive numerical validation of the proposed nominal model as well as a non-comprehensive experimental validation.

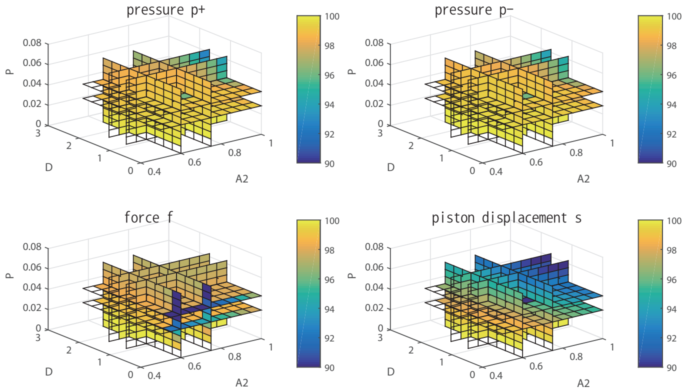


Fig. 5. The conventional nominal model outputs vs. the proposed nominal model outputs (every case).

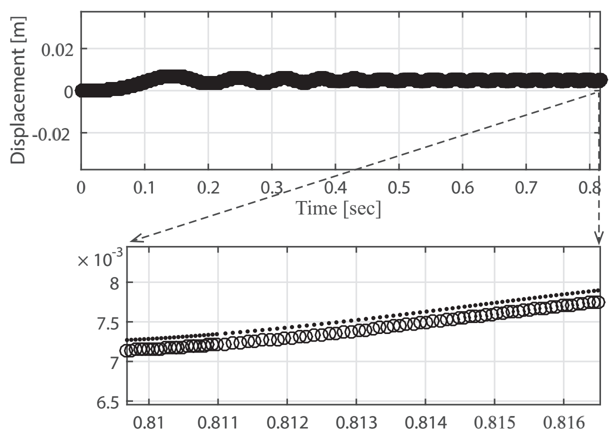


Fig. 6. The conventional nominal model outputs vs. the proposed nominal model outputs (the worst case).

## A. Comprehensive numerical validation

Unfortunately, many experimental validations are often restrictive in terms of the scale and can not be comprehensive. On the other hand, the usual numerical validations are also restrictive when the parameter space is too large. Indeed, the conventional nominal model and proposed nominal model have the 8-dimensional parameter space $(M,D,L,A_+,A_-,b,C,P) \in \mathbb{R}^8_+$. However, this parameter space is reduced due to Proposition 7. Furthermore, instead of the original state, the Casimir functions reduce the computational time due to Proposition 8.

Figure 4 shows the FIT ratio between the conventional nominal model and the proposed nominal model in the 3-dimensional parameter space $(D^*,A^*,P^*)$ by the direct search to repeat the nonlinear dynamics computations and the parameter updates in the special non-dimensional representation. In the nonlinear dynamics computations, the equation (17) is applied to compute the non-dimensional state $x^*(t^*)$ starting from the initial state $x^*(0) = (0,0,P^*/2,A^*P^*/2)^{\mathrm{T}}$ in the presence of the test signal $A_u^* \sin(2\pi f_u^* t^*)$ with the amplitude $A_u^* := A_u/U_s = 0.01$ and the frequency $f_u^* := T_s f_u \in \{0.002, 0.15\}$. The test period is defined as $[0,T_u^*] := [0, 5/f_u^*]$ and the Adams-Moulton method with the variable step is applied (MATLAB 2014b, Simulink ver.8.4, 64-bit CPU 2.60 GHz, Memory 8.0 GB). In almost every case of Figure 4, the FIT ratios are very high and over 90 % ! This is not expected result.

Figure 5 shows the time responses at the worst case in which the FIT was around 65.1 % at $(D^*,A^*,P^*) = (0.00056.0.79, 0.0049)$. The dots denote the conventional nominal model outputs and the circles denote the proposed nominal model outputs. In order to improve the readability, every outputs are non-dimensional ones but we display them in the experimental setup scale which is given later. As shown in the bottom figure (the enlarged version of the top figure) in Figure 5, the maximal error in the piston displacement level was less than 0.002 mm. This result justifies the fact that both models generate the similar behaviors.

## B. Non-comprehensive experimental validation

*1) Experimental condition:* Figure 6 shows an appearance of the experimental setup. The experimental setup consists of a manipulator, a pump (NDR081-071H-30, 11.7 [L/min], $p_s = 7\,[\mathrm{MPa}]$), a tank 7 [L]), four pipes (SWP70-6, 1/4 inch), a valve (LSVG-01EH-20-WC-A1-10 (Yuken Kogyo)), two cylinders (asymmetric cylinder (KW-1CA30×75, $L = 75\,[\mathrm{mm}]$), a filter (UM-03-20U-1V) and an oil (ISO VG32, $860\,[\mathrm{kg/m^3}]$ $40 \pm 2\,[°C]$).

The output signals $p_\pm$ are measured by a pressure sensor (AP-15S), the output $s$ is measured by a potentio-meter (LP-100F-C) and the identification input $u_s$ are measured by the LVDT (1 [mm]/1.4 [V]). These output and input signals are detected by an AD converter (PCI-3155, 16 bit) and recorded by a control PC (LX7700, Linux, 2.53 [GHz]) by a sampling time 1 [ms]).

In order to identify the parameters, a chirp inputs is applied via DA converter (PCI-3325, 12 bit). The identified and measured parameters are $(M,D,L,A_+,A_-,b,C,P) = (14, 3200, 0.075, 7.0\times10^{-4}, 5.4\times10^{-4}, 5.3\times10^{8}, 1.6\times10^{-4}, 7\times10^{6})$ [17]. Since the parameter

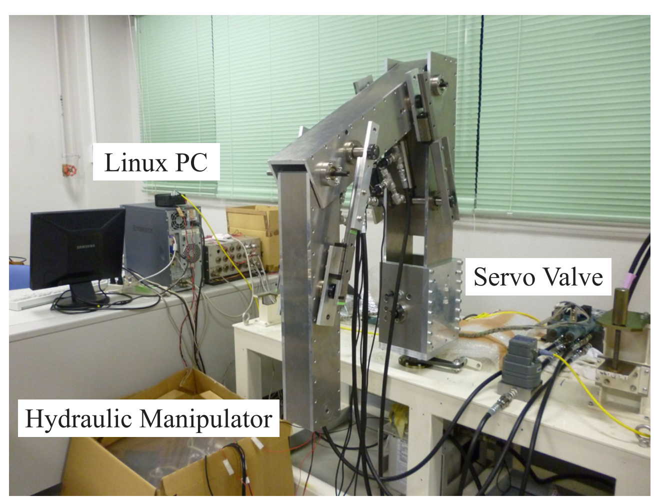


Fig. 7. The experimental setup.

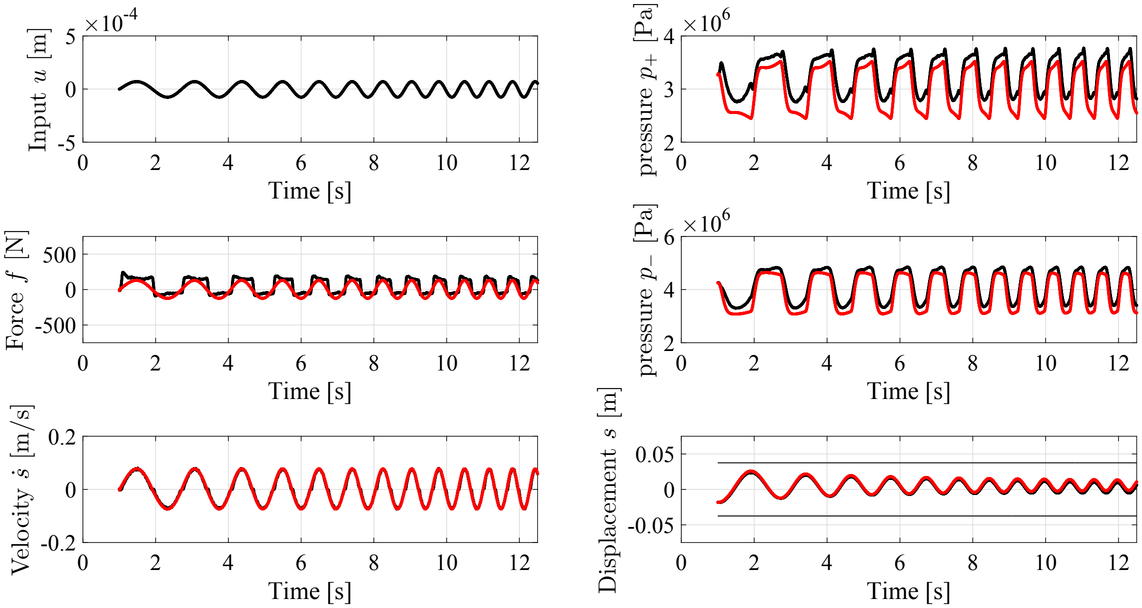


Fig. 8. The experiment (Red) vs. the model (Black).

identification is in the off-line world, the time derivative of the piston displacement $s$ and the pressures $p_\pm$ are sufficiently calculated via the first or second difference approximation and the standard moving average (20-order, cut-off 50 [Hz]).

*2) Experimental results and discussion:* Figure 7 displays a long time cross validation in which the experimental outputs (the red dots) were never used in the parameter identification procedure for the proposed nominal model outputs (the black curves). Nevertheless, with respect to the nonlinear responses in the pressures and displacement, the proposed nominal model has an accuracy that any linearized model (transfer function) cannot have.

Table 1 shows the difference as the FIT ratio [11]:

$$\mathrm{FIT}(y_0^i) = \left( 1 - \frac{\sqrt{\sum_{t=0}^{T_e} (\hat{y}_0^i(t) - y_0^i(t))^2}}{\sqrt{\sum_{t=0}^{T_e} (y_0^i(t) - \bar{y}_0^i)^2}} \right) \times 100$$

where $\bar{y}_0^i$ is the mean value of the $i$-th element $y_0^i(t)$ ($i = 1, \cdots, 4$) of the outputs $y_0(t) := (p_+(t), p_-(t), f(t), s(t))^{\mathrm{T}}$ of the experimental setup and $\hat{y}_0^i$ is the $i$-th element of the corresponding outputs $\hat{y}_0(t) := (\hat{p}_+(t), \hat{p}_-(t), \hat{f}(t), \hat{s}(t))^{\mathrm{T}}$ of the proposed nominal model. The results on the velocity $v(t)$ can be discussed by that on the displacement $s(t)$. If the numerical existence is achieved, $T_e := T_u$, otherwise $T_e := t_e \in [0, T_u]$.

Note that the value of $\mathrm{FIT}(y_0^i)$ can be negative in general ! At a glance, one may think that such asymmetric displacements were generated by a nonlinear friction effect. This conjecture is not true because the proposed nominal model ignores the nonlinear friction effect. The asymmetric

TABLE I

FIT RATIO (CAN BE NEGATIVE)

| Signal | Notation | Fit ratio [%] |
|---|---|---|
| displacement | $s(t)$ | +78 |
| velocity | $v(t)$ | +79 |
| cap pressure | $p_+(t)$ | +52 |
| rod pressure | $p_-(t)$ | +62 |
| driving force | $f(t)$ | +72 |

displacements is nothing but a nonlinearity. Even the piston velocity, the proposed nominal model can have a good accuracy for this experimental scale at least.

## V. CONCLUSION

The paper provides analysis and validation for a new nominal model of hydraulic cylinder dynamics. Via a power preserving interconnection as a port-based approach ([2] [3] [5]), it is shown that the links to fruitful results (e.g., robust stabilization, learning) can be lost in the conventional nominal model. This motivates us to propose a new nominal model. It is shown that some properties such as simplicity of the conventional nominal model are found in the proposed nominal model successfully. One of the members in the proposed nominal model: $\tilde{b}(p_\pm)$ may justify a practical modification for the conventional nominal model, that is, a pressure dependent bulk modulus in many industrial reports. The effectiveness of the proposed nominal model is confirmed by non-comprehensive experimental validation as well as comprehensive numerical validation based on our previous result. All these results are unexpected and economical. In our next works. the clarified properties will be used as the links in the controller design procedure at least.

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