Axiom 1.1.1 : Properties of Equivalence

The following is assumed to be true for all sets S, for all $\alpha, \beta, \text{and}, \gamma$ in S.

$$\forall S, S \text{ is a set}, \forall \alpha, \beta, \gamma \in S$$

An element α is equal to itself, this property is called reflexivity. If an element α is equal to an element β then β is also equal to α, this property is called symmetry. If and element α is equivalent to an element β, and β is also equivalent γ then α is equivalent to γ, this property is called transitivity.

$$\begin{aligned}
\text{(i) } & \alpha = \alpha, \text{(relexivity)} \\
\text{(ii) } & \alpha = \beta \Rightarrow \beta = \alpha, \text{(symmetry)} \\
\text{(iii) } & \alpha = \beta \wedge \beta = \gamma \Rightarrow \alpha = \gamma, \text{(transitivity)}
\end{aligned}$$

Remark 1.1.2 : Well Defined Operation

An operation is said to be well defined if given the same inputs, the operation will always give the same outputs. Or symbolically,

$$\forall \alpha, \beta \in S, f \text{ is an operation}, \alpha = \beta \Rightarrow f[\alpha] = f[\beta]$$

at the moment we will take for granted the fact that the normal operations of numbers is well defined, that is addition, subtraction, multiplication, and division of 2 numbers.

Example 1.1.3 : Isolate x

We can apply the remark about well defined operations to manipulate our equations in the following manner.

$$x + 2 = 3 \tag{1}$$
$$x + 2 - 2 = 3 - 2 \tag{2}$$
$$x = 1 \tag{3}$$

Remark 1.1.4 : Interpretation of Well Defined Operation in Terms of Equation Manipulation
For those that are no convinced by the manipulation above, or how the definition of a well defined operation is related to it. Another way to interperet a well defined operation is as an operation where if you apply the operation to a single number, you should always get the same result.

In step (1) we know that we have two equivalent objects $x + 2$ and 3, the well defined operation that we apply to the two equivalent objects is "minus 2". Since minus is well defined, we conclude that $x + 2 - 2$ is equivalent to $3 - 2$, we collect the terms and conclude that x is equivalent to 1.

Example 1.1.5 : Isolate x

$$\begin{aligned} 2x &= 6 \\ \frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 6 \\ x &= 3 \end{aligned}$$

Example 1.1.6 : Isolate x

$$\begin{aligned} \frac{x}{3} &= 4 \\ \frac{x}{3} \cdot 3 &= 4 \cdot 3 \\ x &= 12 \end{aligned}$$

Example 1.1.7 : Isolate y

$$\begin{aligned} \frac{y+2}{3} &= 4x \\ 3 \cdot \frac{y+2}{3} &= 3 \cdot 4x \\ y + 2 &= 12x \\ y &= 12x - 2 \end{aligned}$$

Example 1.1.8 : Isolate y

$$\begin{aligned} \frac{3-2y}{5} - y &= x \\ \frac{3-2y}{5} &= x + y \\ 3 - 2y &= 5x + 5y \\ 3 - 5x &= 7y \\ y &= \frac{3-5x}{7} \end{aligned}$$

Remark 1.1.9 : Simplest Form
When we are dealing with equations with more than one variable, isolating one variable will result creating an equation in the form of a variable written in terms of another variable. However that form is not simple like a single number, there is nothing wrong with that. It just happens that the simpler form cannot contain all of the information of the original equation.

Remark 1.1.10 : lorem ipsum tater tots