Axiom 1.1.1 : Properties of Equivalence
The following is assumed to be true for all $\alpha, \beta,$ and, γ in the real numbers.

$$\forall \alpha, \beta, \gamma \in \mathbb{R}$$

An element α is equal to itself, this property is called reflexivity. If an element α is equal to an element β then β is also equal to α, this property is called symmetry. If and element α is equivalent to an element β, and β is also equivalent γ then α is equivalent to γ, this property is called transitivity.

$$\begin{aligned} &\text{(i) } \alpha = \alpha, \text{(relexivity)} \\ &\text{(ii) } \alpha = \beta \Rightarrow \beta = \alpha, \text{(symmetry)} \\ &\text{(iii) } \alpha = \beta \wedge \beta = \gamma \Rightarrow \alpha = \gamma, \text{(transitivity)} \end{aligned}$$

Remark 1.1.2 : Well Defined Operation
An operation is said to be well defined if given the same inputs, the operation will always give the same outputs. Or symbolically,

$$\forall \alpha, \beta \in \mathbb{R}, f \text{ is an operation}, \alpha = \beta \Rightarrow f[\alpha] = f[\beta]$$

at the moment we will take for granted the fact that the normal operations of numbers is well defined, that is addition, subtraction, multiplication, and division of 2 numbers.

Example 1.1.3 : Isolate x
We can apply the remark about well defined operations to manipulate our equations in the following manner.

$$x + 2 = 3 \tag{1}$$
$$x + 2 - 2 = 3 - 2 \tag{2}$$
$$x = 1 \tag{3}$$

Remark 1.1.4 : Interpretation of Well Defined Operation in Terms of Equation Manipulation
For those that are no convinced by the manipulation above, or how the definition of a well defined operation is related to it. Another way to interperet a well defined operation is as an operation where if you apply the operation to a single number, you should always get the same result.

In step (1) we know that we have two equivalent objects $x + 2$ and 3, the well defined operation that we apply to the two equivalent objects is "minus 2". Since minus is well defined, we conclude that $x + 2 - 2$ is equivalent to $3 - 2$, we collect the terms and conclude that x is equivalent to 1.

Example 1.1.5 : Isolate x

$$\begin{aligned} 2x &= 6 \\ \frac{1}{2} \cdot 2x &= \frac{1}{2} \cdot 6 \\ x &= 3 \end{aligned}$$

Example 1.1.6 : Isolate x

$$\begin{aligned} \frac{x}{3} &= 4 \\ \frac{x}{3} \cdot 3 &= 4 \cdot 3 \\ x &= 12 \end{aligned}$$

Example 1.1.7 : Isolate y

$$\begin{aligned} \frac{y+2}{3} &= 4x \\ 3 \cdot \frac{y+2}{3} &= 3 \cdot 4x \\ y+2 &= 12x \\ y &= 12x - 2 \end{aligned}$$

Example 1.1.8 : Isolate y

$$\begin{aligned} \frac{3-2y}{5} - y &= x \\ \frac{3-2y}{5} &= x + y \\ 3 - 2y &= 5x + 5y \\ 3 - 5x &= 7y \\ y &= \frac{3-5x}{7} \end{aligned}$$

Remark 1.1.9 : Simplest Form
When we are dealing with equations with more than one variable, isolating one variable will result creating an equation in the form of a variable written in terms of another variable. However that form is not simple like a single number, there is nothing wrong with that. It just happens that the simpler form cannot contain all of the information of the original equation.

Axiom 1.1.10 : Distribution
For α, β, γ the reals, left and right multiplication is distributed over addition.

$$\forall \alpha, \beta, \gamma \in \mathbb{R}$$
$$\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$$

Remark 1.1.11 : Geometric Demonstration of Distribution * this is not proof
If we accept that the the area of a rectangle is its length multiplied by its width, and that its area is also the sum of the parts its made of, then it follows that multiplication is distributed over addition.

$$\text{Area} = \alpha(\beta + \gamma), \text{ length by width}$$
$$\text{Area} = \alpha\beta + \alpha\gamma, \text{ sum of area of parts}$$

Remark 1.1.12 : Number Line (optional)
A number line is a visual tool that is able to visually represent a variable that takes on some value. It consists of a single axis, with regularly spaced markings of numbers that increase towards the right with a point on the marking on which its the variable's value.

Example 1.1.13 : $x = 4$

Remark 1.1.14 : Cartesian Plane (optional)
A Cartesian plane is an extended version of a number line that is able to visually represent two variables which take on some value. It consists of two perpendicular axes, both with markings like the number line. There is a point on the intersection of the two markings that take the values of the two variables intersect.

Example 1.1.15 : $x = 3 \land y = 4$
Often abbreviated as $(x, y) = (3, 4)$
Remark 1.1.16 : Linear Equation
A linear equation is an equation of the form $\alpha x + \beta y + \gamma = 0,$ where α, β, γ are in the reals.

Remark 1.1.17 : Slope Intercept Form of a Linear Equation
From the original form of the linear equation, we can derive the following alternate form of a linear equation.

$$\begin{aligned}\alpha x + \beta y + \gamma &= 0\\ \beta y &= -\alpha x - \gamma\\ y &= -\frac{\alpha}{\beta}x - \frac{\gamma}{\beta}\end{aligned}$$

Remark 1.1.18 : Linear Equation Constructor (optional)
Consider a contructor for linear equations

$$\text{texttt}\mathrm{LinearEquation} : \mathbb{R}^3 \mapsto \text{texttt}\mathrm{Equation}$$
$$(\alpha, \beta) \to \alpha x + \beta y + \gamma = 0$$

Remark 1.1.19 : Linear Equation Represented by Cartesian Plane
On a cartesian plane, a point requires the value of two variables. A linear equation is an equation of two variables, and thing to note about equations of two variables is that if you know the value of one variable, you can find out the value of the other.

Suppose: `LinearEquation`$(\alpha = -1, \beta = 1, \gamma = 0)$ which gives $-x+y+0 = 0$, is our equation

Infer: