M351K Homework 4

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1: Workers

a) joint pmf

b) conditional pmf

2: Money

k is either 1 or 50.
X is either 2 or 51.
$P(X = 2|k = 1) = \frac{8}{9}$.
$P(X = 51|k = 1) = \frac{1}{9}$.
$P(X = 2|k = 50) = 0$.
$P(X = 51|k = 50) = 1$.

3: Negative Binomial

An m-bit password has a space of size 2^m.
Then $P(X = n) = \frac{2^m-1}{2^m} \cdot \frac{2^m-2}{2^m-1} \cdot \ldots \frac{2^m-(n-1)}{2^m-(n-2)} \cdot \frac{1}{2^m-(n-1)} = \frac{1}{2^m}$.
Then $P(X = n|k$ incorrect tries$) = \frac{1}{2^m-k}$.

4: Dice

5: Chocolate

$\frac{0.9}{0.9+0.02} \approx 0.9783$.

6: Bayes

Let A be the event that a randomly chosen child among two is a boy.
Let B be the event that a given sibling is a boy.
We want to find $P(B|A)$.
We use total probability theorem on A.
$P(A) = P(\text{boy and not boy})P(A|\text{boy and not boy})$
$+ P(\text{two boys})P(A|\text{two boys})$
$+ P(\text{no boys})P(A|\text{no boys})$
$\approx \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 + 0 = \frac{1}{2}$.
Then we use this to find $P(B|A)$
$= P(\text{boy and not boy})P(B|A, \text{boy and not boy}).+P(\text{two boys})P(B|A, \text{two boys})$
$= \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1$
$= \frac{2}{3}$.