M351K Homework 1

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1: Two Dice

a)

The sample space for the two dice rolls is $\{1, ..., 6\} \times \{1, ..., 6\}$, or explicitly:
$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

b)

If A is the set of events where the number of dots in first toss is not less than number of dots in second toss, then
$A = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(4,4),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$.

c)

If B is the event where the number of dots in first toss is 6, then
$B = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$.

d)

$A \cap B^c = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),$
$(4,4),(5,1),(5,2),(5,3),(5,4),(5,5)\}$.
This is the event that the first die roll has more dots facing up than the second die roll but the first roll was not 6.

e)

Let C be the event that the mangitude of the difference of dots between the two die rolls is two.
Then $A \cap C = \{(3,1),(4,2),(5,3),(6,4)\}$.

2: Names in a Hat

a)

The sample space is the permutation of their names:
$\{(Al, Bob, Chris), (Al, Chris, Bob), (Bob, Al, Chris),$
$(Bob, Chris, Al), (Chris, Al, Bob), (Chris, Bob, Al)\}.$

b)

$A = \{(Al, Bob, Chris), (Al, Chris, Bob)\}.$
$B = \{(Al, Bob, Chris), (Chris, Bob, Al)\}.$
$C = \{(Al, Bob, Chris), (Bob, Al, Chris)\}.$

c)

$\{(Bob, Chris, Al), (Chris, Al, Bob)\}$

d)

$\{(Al, Bob, Chris)\}$

e)

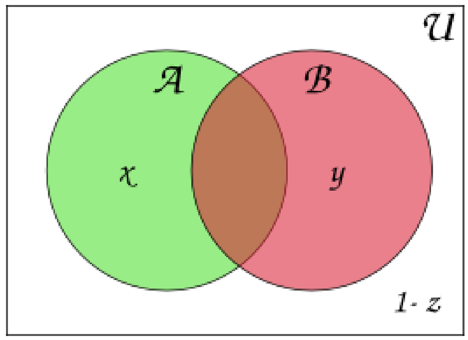
$A \cup B \cup C =$
$\{(Al, Bob, Chris), (Al, Chris, Bob), (Chris, Bob, Al), (Bob, Al, Chris)\}.$

3: Set Algebra

We want to show that
$A \cup B \cup C = A + B + C - (A \cap B) - (B \cap C) - (C \cap A) + (A \cap B \cap C).$

$A \cup B \cup C =$
$(A \cup B) \cup C =$
$(A + B - (A \cap B)) \cup C =$
$(A + B - (A \cap B)) + (C) - ((A + B - (A \cap B)) \cap C) =$
$A + B + C - (A \cap B) - ((A \cap C) + (B \cap C) - ((A \cap B) \cap C)) =$
$A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C).$
Which is what we sought to show.

4: Venn Diagrams



$P[A \cap B] = x + y - z.$
$P[A^c \cap B^c] = 1 - P[A \cup B] = z.$
$P[A^c \cup B^c] = 1 - P[A \cap B] = 1 - x - y + z.$
$P[A \cap B^c] = P[A] - P[A \cap B] = x - (x + y - z) = z - y.$
$P[A^c \cup B] = 1 - P[A \cap B^c] = 1 - z + y.$

5: Counting

a)

There are $100 \cdot 100 \cdot 100$ or $1,000,000$ unique lock positions.

b)

There are $6 \cdot 6 \cdot 2 \cdot 2 \cdot 52$ or 7488 unique outcomes.

6: Pizza

With repeats, there are $15 \cdot 15 \cdot 15 \cdot 15 \cdot$ or 50625 unique pizza types. With no repeats, there are $\binom{15}{4}$ or 1365 unique pizza types.

7: Passwords

There are $14+10+26+26$ or 76 choices for each character. Passwords may be 8, 9, or 10 characters long. Then there are $76^8+76^9+76^{10}$ or $6514592610973974528 \approx 5 \cdot 2^{60}$ unique passwords. Brute forcing at 1 password per microsecond sequentially would take just under 206500 years, at worst. Note however, that with proper brute forcing technique and modern machines, this password is not necessarily secure.