M351K Homework 3

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1: Conditional Probability

Let $P(W)$ be the probability that a white ball is selected and $P(B_1)$ the probability that the first box is chosen by the coin.
$P(B_1 \mid W) = \frac{P(B_1)P(W|B_1)}{P(W)}$.

$$\frac{P(B_1)P(W|B_1)}{P(W)} = \frac{(\frac{3}{4})(\frac{7}{13+7})}{(\frac{3}{4})(\frac{7}{13+7}) + (\frac{1}{4})(\frac{8}{2+8})} = \frac{1}{1 + (\frac{8}{40})(\frac{80}{21})} = \frac{1}{1+\frac{16}{21}} = \frac{21}{37}.$$

2: PMF

$$\sum_{-\infty}^{\infty} p_X(x) = 1.$$

$$\sum_{-\infty}^{\infty} p_X(x) = \sum_{-\infty}^{1} p_X(x) + p_X(0) + \sum_{1}^{\infty} p_X(x)$$

$$= p_X(0) + 2\sum_{x=1}^{\infty} 0.4(1-q)q^{x-1}$$

$$= b + 0.8(1-q)\sum_{x=1}^{\infty} q^{x-1}$$

$$= b + 0.8(1-q)(\sum_{x=1}^{\infty} q^{x} + 1)$$

$$= b + 0.8(1-q)(\frac{p}{1-p} + 1)$$

$$= b + (\frac{4}{5})(\frac{3}{4})(\frac{4}{3}).$$

$$1 - (\frac{4}{5}) = b.$$

$$b = \frac{1}{5}.$$

3: Coins

a)

The sample space is $\{(x_1, x_2, ..., x_{150}) \in \{H, T\}^{150} \mid x_n \in \{H, T\}\}$.

b)

Let h be the number of heads. Then $h \sim \mathcal{B}(150, 0.75)$.

4: Infinite sums

Let d be the number of games played. Then $d \sim \mathcal{NB}(1, 0.4)$.
The probability that Alice wins the game is

$$0.2 + 0.2(0.4)^1 + 0.2(0.4)^2 + ... = 0.2 + 0.2\left(\sum_{n=1}^{\infty} 0.4^n\right) = 0.2 + 0.2(\tfrac{0.4}{1-0.4}) = \tfrac{1}{3}.$$

The probability that Bob wins is the probability that Alice loses: $1 - \frac{1}{3} = \frac{2}{3}$.

5: Negative binomial

$X \sim \mathcal{NB}(n, p)$, where X is the random variable given.

6: Uniform distribution

I am quite convinced that Y, Z, and R are discrete random variables.

a)

$$p_Y(x) = \begin{cases} \frac{3}{8} & \text{if } x = 0 \\ \frac{1}{16} & \text{if } 1 \leq x \leq 10, x \in \mathbb{Z} \\ 0 & \text{if } x < 0 \text{ or } 10 < x \end{cases}$$

b)

$$p_Z(x) = \begin{cases} \frac{5}{8} & \text{if } x = 1 \\ \frac{1}{16} & \text{if } -5 \leq x \leq 0, x \in \mathbb{Z} \\ 0 & \text{if } x < -5 \text{ or } 1 < x \end{cases}$$

c)

$$p_R(x) = \begin{cases} \frac{1}{8} & \text{if } 1 \leq x \leq 5, x \in \mathbb{Z} \\ \frac{1}{16} & \text{if } x = 0 \text{ or } 5 < x \leq 10, x \in \mathbb{Z} \\ 0 & \text{if } x < 0 \text{ or } 10 < x \end{cases}$$