EE313 Lecture 6

Joshua Dong

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0.1 LTI Systems

Properties of systems:

1. Memory
2. Invertability
3. Causality
4. Stability
5. Time Invariance
6. Linearity

e.g. $y[n] = nx[n]$ Recall:
$x_1[n] \xrightarrow{S} y_1[n] \to x_1[n-\delta] \xrightarrow{S} y_1[n-\delta]$.

$x_1[n] = \delta[n]$
$S[\delta[n]] = 0 \ \ \forall n.$

$x_1[n-5] = \delta[n-5]$
$S[\delta[n-5]] = 5\delta[n-5]$.

$\therefore$ Not time invariant.

e.g. $y(t) = x(2t)$
Not time invariant. Time is compressed by a factor of 2.
i.e Input $x_1 = u(t+2) - u(t-2)$.
$y_1(t) = u(2t+2) - u(2t-2)$.

0.2 Recognizing Linear Systems

Linearity: Rescaling + Superposition

Any linear system has the zero-in-zero-out property.

$x[n] = 0 \ \forall n \in \mathbb{Z} \to g[n] = 0 \ \forall n \in \mathbb{Z}$.

e.g. $x[n-7] + 2 \cdot x[n-3] + 7$
This is not linear because it violates zero-in-zero-out property.

Causality example:
$y(t) = x(t-7) + x(t-31) + 2t \cdot x(t-1) + \int_{t-35}^{t-12} x(t) \log(t^{2\cos t}) dt$
This system is causal.

0.3 Chapter 2: Linear and Time Invariant Systems (LTI)

Very amazing but perhaps obvious is retrospect fact:
For discrete time,
I can use $\delta[n]$ to construct any discrete-time signal.

For any signal x[n] I write it as a linear combination of shifted $\delta[n]$'s:

$$y[n] = \sum_{k=-\infty}^{\infty}$$

0.3.1 Assuming Linearity

Assume I have a system that is linear (but not yet Time Invariant).
Impulse Response: $x[n] = \delta[n] \to h_0[n]$. h_0 is the impulse response.
Shifted Impulse Response: $x[n] = \delta[n-k] \to h_k[n]$.
If the system is Time Invariant, then
$h_1[n] = h_0[n-1]$.
$h_k[n] = h_0[n-k]$.
$\therefore y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h_0[n-k]$.