

EE313 Lecture 13

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Recall definition of CTFS:

Synthesis equation:

$$x(t) = \sum a_n \cdot e^{jkw_0 t}$$

And analysis equation $x(t) \rightarrow a_k$:

$$\frac{1}{T} \int_{-T}^T x(t) \cdot e^{-jk \cos t dt}$$

Conditional convergence of series can be rearranged to sum to any number (Reimann's theorem)

Understand absolute and conditional convergence

Example:

Square pulse

$$\begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T}{2} \end{cases}$$

Find Fourier coefficients $a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t) \cdot e^{-jk \cos t dt}$.

Find (DC coefficient):

$$a_0 = \frac{1}{T} \int_{-T_1}^{T_1} (1) \cdot (1) dt = \frac{2T_1}{T}.$$

Find a_1 :

...

$$a_k = \frac{1}{T} \frac{-1}{jkw_0} [e^{-jkw_0 T_1} - e^{+jkw_0 T_1}]$$

Use tricky Euler magic to get:

$$= \frac{1}{T} \frac{2}{kw_0} \cdot \sin(kw_0 T_1)$$

$$= \frac{2T_1}{T}$$

Note that the resultant transform is $a_k = \text{sinc}[k]$ and $k \in \mathbb{Z}^+$ such that $k = \frac{T}{2T_1} \rightarrow a_k = 0$.

Heisenberg Uncertainty Principle

$P(s)$ = probability of particle at positions, $s \in \mathbb{R}$. FP = probability of momentum.

Properties of Fourier

Linearity f, g, x+g, af+bg (Why?)

Time Shift $x(t)$, a_k $y(t) = x(t - t_0)$ shift by t_0 (phase shift) $x(t - t_0) \xrightarrow{\text{phaseshift}}$
 $e^{-jkw_0 t_0} \cdot a_k$. (Why? Prove it.)