EE313 Homework 1

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September 3, 2014

0.1 Complex Numbers

$z_1 = 4 - j3$, $z_2 = 1 + j$.
a) $z_1^* = 4 + j3 = 5\angle 0.64$
b) $z_2^2 = 2j = 2\angle\frac{\pi}{2}$
c) $z_1 + z_2^* = 5 - j4 = \sqrt{41}\angle{-0.67}$
d) $\frac{jz_2}{z_1^2} = \frac{j-1}{7-24j} = \frac{(j-1)(7+24j)}{(49+576)} = \frac{7j-7-24-24j-7-24}{625} = \frac{-31-17j}{625} =$
$-\frac{31}{625} - j\frac{17}{625} = 0.57\angle 0.50$
e) $z_1^{-1} = \frac{1}{4-j3} = \frac{4+j3}{16-9} = \frac{4}{7} + j\frac{3}{7} = \frac{5}{7}\angle 0.64$
f) $\frac{z_1}{z_1+z_2} = \frac{4-j3}{5+j2} = \frac{(4-j3)(5+j2)}{29} = \frac{20-15j+8j+6}{29} = \frac{26-7j}{29} = \frac{26}{29} - \frac{7}{29}j = 0.93\angle{-0.26}$
g) $e^{z_2} = e \cdot e^j = e \cdot (\cos 1 + j \sin 1) = 1.47 + 2.29j = e\sqrt{2}\angle 1$
h) We use difference of squares to show $z_1 z_1^* z_2 z_2^* = (16+9)(1+1) =$
$50 + 0j = 50\angle 0$
i) $z_1 z_2 = 7 + j = 5\sqrt{2}\angle 0.14$

0.2 Simplify

We use Euler's formula.
a)$e^{4j\pi} = \cos 4\pi + j \sin 4\pi = 1$
b)$e^{5j\pi} = \cos 5\pi + j \sin 5\pi = -1$
c)$e^{2014j\pi} = \cos 2014\pi + j \sin 2014\pi = 1$
d)$e^{2015j\pi} = \cos 2015\pi + j \sin 2015\pi = -1$

0.3 Proofs

Show:
a)$zz^* = r^2$
Let $z = a + bi$, where $a, b \in \mathbb{R}$. Then $z^* = a - bi$, by definition.
$zz^* = (a + bi)(a - bi) = a^2 + b^2$.
By defininiton, $r = \sqrt{a^2 + b^2}$.
$\therefore r^2 = (\sqrt{a^2 + b^2})^2 = a^2 + b^2 = zz^*$.
$\therefore zz^* = r^2$, by direct proof.

b)$z + z^* = 2Re(z)$
Let $z = a + bi$, where $a, b \in \mathbb{R}$. Then $z^* = a - bi$, by definition.
$z + z^* = (a + bi) + (a - bi) = 2a$.
By defininiton, $Re(z) = a$.
$\therefore z + z^* = 2Re(z)$, which was what we sought to show.

c)$(e^z)^* = e^{z^*}$
Let $z = a + bi$, where $a, b \in \mathbb{R}$.
$(e^z)^* = (e^{a+bi})^* = (e^a(e^{bi}))^* = (e^a(\cos b + i \sin b))^* = e^a(\cos b - i \sin b) = e^a(\cos -b + i \sin -b) = e^a(e^{-bi}) = e^{a-bi} = e^{z^*}$
$\therefore (e^z)^* = e^{z^*}$, by direct proof.

d)A man was killed by a bear and the temperature was $T = \sum_{k=1}^{5} e^{4jk}$ degrees Fahrenheit. What colour was the bear?
The first statement is unprovable given the axioms of mathematics. The second sentance is not a statement. However, mildly related data follows:
$T = \sum_{k=1}^{5} e^{4jk} = e^{4j\cdot 1} + e^{4j\cdot 2} + e^{4j\cdot 3} + e^{4j\cdot 4} + e^{4j\cdot 5} = 1 + 1 + 1 + 1 + 1 = 5$, by Euler's formula.

0.4 Using Octave

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octave:1> dt = 1/100;
octave:2> t = -1 : dt : 1;
octave:3> Fo = 4;
octave:4> x = 100 * real(exp(j*(2*pi*Fo*(t - 0.75))));
octave:5> subplot(2,1,1);
XOpenIM() failed
octave:6> plot(t, x), grid
octave:7> title( 'Section of a sinusoid'), xlabel('time(sec)')
octave:8>
```