Digital Logic Design: Homework 2

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Unless otherwise specified, all numbers are decimal.

3.11

$(A+B'+C+E')(A+B'+D'+E)(B'+C'+D'+E')$
$=((A+B')+(C+E')(D'+E))(B'+C'+D'+E')$
$=((A+B')+(D'E'+CE))(B'+C'+D'+E')$
$=(A+B'+D'E'+CE)(B'+C'+D'+E')$
$=(A+B'+D'E'+CE)B'+(A+B'+D'E'+CE)C'+(A+B'+D'E'+CE)D'+(A+B'+D'E'+CE)E'$
$=AB'+B'+B'D'E'+B'CE+AC'+B'C'+D'E'+AD'+B'D'+D'E'+CED'+AE'+B'E'+D'E'$
$=AC'+AE'+B'+CD'+D'E'$

3.12

$A'CD'E+A'B'D'+ABCE+ABD=A'B'D'+ABD+BCD'E$
$A'CD'E+ABCE+A'B'D'+ABD=A'B'D'+ABD+BCD'E$
$CE(A'D'+AB)+A'B'D'+ABD=A'B'D'+ABD+BCD'E$
$CE(A'D'+AB+BD')+AB(CE+BD)+A'B'D'+ABD=A'B'D'+ABD+BCD'E$
$A'CD'E+ABCE+BCD'E+A'B'D'+ABD=A'B'D'+ABD+BCD'E$
$A'CD'E+A'B'D'+(ABD+BCD'E+ABCE)=A'B'D'+ABD+BCD'E$
$A'CD'E+A'B'D'+(ABD+BCD'E)=A'B'D'+ABD+BCD'E$
$A'B'D'+ABD+BCD'E=A'B'D'+ABD+BCD'E$
Thus the two sides are equal.

3.12

a)

$K'L'M + KM'N + KLM + LM'N'$
$= K'L'M + KM'N + KLM + LM'N'$
$= (K'L'M + K)(K'L'M + M')(K'L'M + N) + L(KM + M'N')$
$= (K'+K)(K+L')(M+K)(K'+M')(M'+L')(M+M')(N+K')(N+L')(M+N) + L(KM + M'N')$
$= (K + L' + L(KM + M'N'))(M + K + L(KM + M'N'))(K' + M' + L(KM + M'N'))(M' + L' + L(KM + M'N'))(M + M' + L(KM + M'N'))(N + K' + L(KM + M'N'))(N + L' + L(KM + M'N'))(M + N + L(KM + M'N'))$
$= (K + L' + M')(K' + L + M')(K + L + M)(L + M + N)(K + M + N')$

e)

$WXY + WX'Y + WYZ + XYZ' = (W + X)(W + Y)(W + Z')$

3.15

a)

$(K' + M' + N)(K' + M)(L + M' + N')(K' + L + M)(M + N)$
$= K'M'N + K'MN' + LMN$

3.16

b)

$(KL \oplus M) + M'N'$
$= (KLM + (K' + L')M')' + M'N'$
$= (K + M + N')(L + M + N')(K' + L' + M')$

3.19

a)

$x + y = x \oplus y \oplus xy$
$x + y = (xy + x'y')' \oplus xy$
$x + y = ((xy + x'y')'xy + (xy + x'y')(x' + y'))'$
$x + y = ((xy + x'y')'xy)'((xy + x'y')(x' + y'))'$
$x + y = ((x' + y')(x + y)xy)'((xy + x'y')(x' + y'))'$
$x + y = ((x' + y')' + (x + y)' + (x' + y'))((xy + x'y')(x' + y'))'$
$x + y = (xy + x'y' + x' + y')((xy + x'y')' + (x' + y')')$
$x + y = (xy + x'y' + x' + y')((xy)'(x'y')' + xy)$
$x + y = (xy + x' + y')((x' + y')(x + y) + xy)$
$x + y = (xy + x' + y')((x' + y' + xy)(x + y + xy))$
$x + y = (xy + x' + y')(x' + y' + xy)(x + y)$
$x + y = (x' + y' + xy)(x + y)$
$x + y = (x' + 1y' + xy + 1x)(x + y)$
$x + y = (1)(x + y)$
$x + y = x + y$
Thus the two sides are equal.

3.21

a)

$BC'D' + ABC' + AC'D + AB'D + A'BD' = A'BD' + AB'D + ABC'$

b)

$W'Y' + WYZ + XY'Z + WX'Y = WX'Y + W'Y' + WXZ$

3.25

f)

$A'BCD + A'BC'D + B'EF + CDE'G + A'DEF + A'B'EF = CDE'G + B'EF + A'BD$

3.32

c)

If $A + B = C$, then $A + B + D = C + D$, since the OR operation is a well-defined function.

3.38

a)

If $x(y + a') = x(y + b')$, then $a = b$ does not hold in general.
For example, take $x = 0$. No matter what a and b are, both sides will be equal.

4.9

a)

$F(a, b, c) = abc' + ab'c + ab'c' + a'b'c + a'b'c' = m_0 + m_1 + m_2 + m_3 + m_6$

b)

$F(a, b, c) = M_4M_5M_7$

c)

$F'(a, b, c) = m_4 + m_7$

d)

$F'(a, b, c) = M_0M_1M_2M_3M_6$

4.13

$ABCD + ABCD' + A'BCD + AB'C'D' + A'B'C'D + A'B'C'D' //* = (ABCD + ABCD') + (ABCD + A'BCD) + (A'B'C'D' + AB'C'D') + (A'B'C'D + A'B'C'D')$, by associativity and the Idempotent law of boolean addition $//* = ABC + A'B'C' + BCD + B'C'D'$, by the Uniting Theorem of Addition and commutativity.

This problem was already done to great detail in Lab 1 (pdf attached).

4.16

a)

$z = \sum m(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$
$y_0 = \sum m(0, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15)$
$y_1 = \sum m(0, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

b)

$z = \prod M(0)$
$y_0 = \prod M(1, 4, 5, 6, 7)$
$y_1 = \prod M(1, 2, 3)$

4.23

$F_1F_2 = \prod M(0, 4, 5, 6, 7)$, since the product of products is the product of the terms.
Proof: $\prod M(a_0, a_1, ..., a_n) \cdot \prod M(b_0, b_1, ..., b_m) = M_{a_1}Ma_2...Ma_nMb_1Mb_2...Mb_m$.
$M_xM_y = Mx$ when $x = y$, so we can simplify like terms of the product.

4.24

$F_1 + F_2 = \sum m(1, 2, 3, 7) + \sum m(1, 2, 3, 5, 6) = \sum m(1, 2, 3, 5, 6) = \prod M(0, 4)$, since the sum of sums is a sum of the terms.
A general rule is that we can take the intersection of the terms of the maxterm product as the resulting product.
The proof is similar to the one provided prior. $\prod M(a_0, a_1, ..., a_n) + \prod M(b_0, b_1, ..., b_m) = m_{c_1} + mc_2 + ... + mc_x + md_1 + mb_2 + ... + md_y$, where c_i is the i^{th} term not in the series of a, and d_j is the j^{th} term not in the series of b.
$m_x + m_y = mx$ when $x = y$, so we can simplify like terms of the sum.

4.35

a)

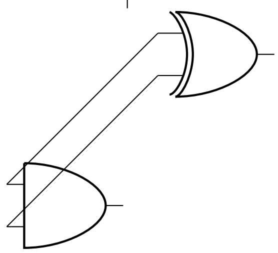
$X = \sum m(1, 2, 4, 7, 8, 11, 13, 14) Y = \sum m(3, 5, 6, 7, 9, 10, 11, 12, 13, 14) Z = \sum m(15)$

b)

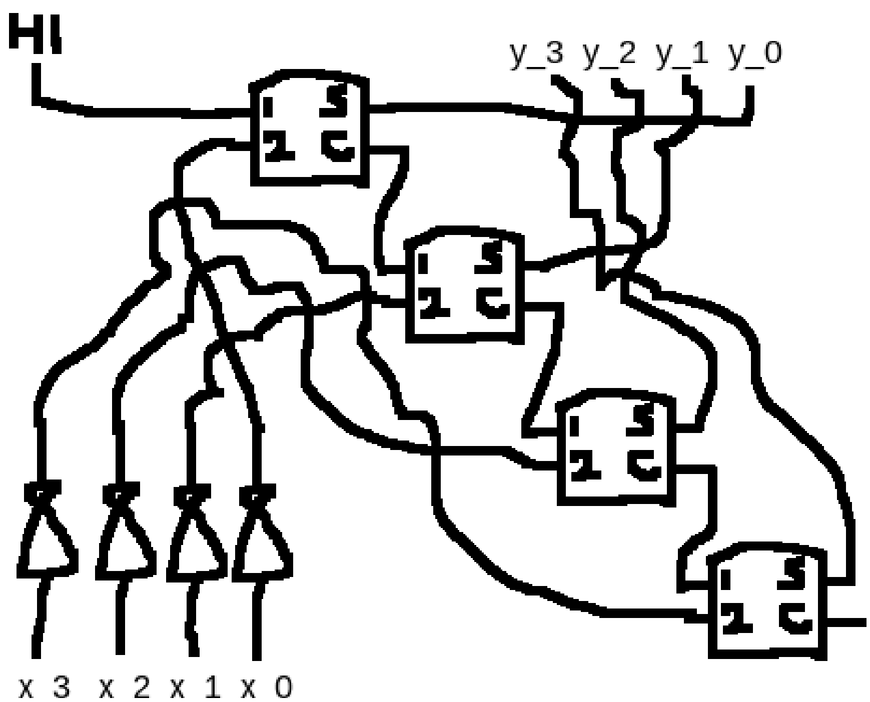
$X = \prod M(0, 3, 5, 6, 9, 10, 12, 15) Y = \prod M(0, 1, 2, 4, 8, 15) Z = \prod M(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)$

4.40

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



The output of the AND gate is carry bit and the output of the XOR gate is the sum bit. The inputs are symmetric.



The unused carry bit could be grounded perhaps.