Digital Logic Design: Homework 1

Joshua Dong

August 31, 2016

Unless otherwise specified, all numbers are decimal.

1.4

a)

1457.11
$= 1457 + 0.11$
$= (16 \cdot 91 + 1) + 0.11$
$= (16 \cdot (16 \cdot 5 + 11) + 1) + 0.11$
$= (16 \cdot (16 \cdot 5 + 11) + 1) + (\frac{1}{16} \cdot 1.76)$
$= (16 \cdot (16 \cdot 5 + 11) + 1) + (\frac{1}{16} \cdot (1 + \frac{1}{16} \cdot 12.16))$
Thus $1457.11_{10} = \text{5B1.1C}_{16}$.

1.7

b)

$-14 = 110010_2,\ -32 = 100000_2$ in 2's complement representation.

$$\begin{array}{r} 1\,1\,0\,0\,1\,0 \\ +\,1\,0\,0\,0\,0\,0 \\ \hline 1\,0\,1\,0\,0\,1\,0 \end{array}$$

The result, after discarding the carry from the sign bit, is positive. It is clear that there is an overflow.

c)

$-25 = 100111_2,\ 18 = 010010_2$ in 2's complement representation.

$$\begin{array}{r} 1\,0\,0\,1\,1\,1 \\ +\,0\,1\,0\,0\,1\,0 \\ \hline 1\,1\,1\,0\,0\,1 \end{array}$$

There is no carry from the sign bit and there is no overflow.
Using 1's complement representation, $-25 = 100110_2,\ 18 = 010010_2$.

$$\begin{array}{r} 1\,0\,0\,1\,1\,0 \\ +\,0\,1\,0\,0\,1\,0 \\ \hline 1\,1\,1\,0\,0\,0 \end{array}$$

There is no carry from the sign bit and there is no overflow.

1.8

If 2's complement is used, the system's range is -2^7 to $+(2^7-1)$, or -128 to $+127$. If 1's complement is used, the system's range is $-(2^7-1)$ to $+(2^7-1)$, or -127 to $+127$.

1.10

a)

1305.375
$= 1305 + 0.375$
$= (16 \cdot 81 + 9) + 0.375$
$= (16 \cdot (16 \cdot 5 + 1) + 9) + 0.375$
$= (16 \cdot (16 \cdot 5 + 1) + 9) + \frac{1}{16} \cdot 6$
Thus $1305.375_{10} = 519.6_{16}$.
Conversion to decimal is easy by converting each hexidecimal digit to its binary representation:
0101 0001 1001 . 0110
That is, $519.6_{16} = 010100011001.0110_2$.

d)

1644.875
$= 1644 + 0.875$
$= (16 \cdot 102 + 12) + 0.875$
$= (16 \cdot (16 \cdot 6 + 6) + 12) + 0.875$
$= (16 \cdot (16 \cdot 6 + 6) + 12) + \frac{1}{16} \cdot 14$
Thus $1644.875_{10} = 66\mathrm{C.E}_{16}$.
Conversion to decimal is easy by converting each hexidecimal digit to its binary representation:
0110 0110 1100 . 1110
That is, $66\mathrm{C.E}_{16} = 011001101100.1110_2$.

1.19

b)

Shown below is division with binary numbers:

```
        11011
1110)110000001
     1110
     10100
      1110
       11000
        1110
        10101
         1110
          111
```

$14 \cdot 27 + 7 = 385$,

$1110_2 \cdot 11011_2 + 111_2 = 110000001_2$,

The answer of 11011_2 is correct, with remainder 111_2.

1.37

a)

$$\begin{array}{r} 0\,1\,0\,0\,1 \\ -1\,1\,0\,1\,0 \\ \hline \end{array}$$

We interpret this as 1's complement:

$$\begin{array}{r} 0\,1\,0\,0\,1 \\ +0\,0\,1\,0\,1 \\ \hline 0\,1\,1\,1\,0 \end{array}$$

And as 2's complement:

$$\begin{array}{r} 0\,1\,0\,0\,1 \\ +0\,0\,1\,1\,0 \\ \hline 0\,1\,1\,1\,1 \end{array}$$

No carry occurs on the sign bits, so there is no overflow.

c)

$$\begin{array}{r} 1\,0\,1\,1\,0 \\ -\,0\,1\,1\,0\,1 \\ \hline \end{array}$$

We interpret this as 1's complement:

$$\begin{array}{r} 1\,0\,1\,1\,0 \\ +\,1\,0\,0\,1\,0 \\ \hline 1\,0\,1\,0\,0\,0 \\ \hline 0\,1\,0\,0\,1 \end{array}$$

And as 2's complement:

$$\begin{array}{r} 1\,0\,1\,1\,0 \\ +\,1\,0\,0\,1\,1 \\ \hline 1\,0\,1\,0\,0\,1 \\ \hline 0\,1\,0\,0\,1 \end{array}$$

There is overflow with both methods.

e)

$$\begin{array}{r} 1\,1\,1\,0\,0 \\ -\,1\,0\,1\,0\,1 \\ \hline \end{array}$$

We interpret this as 1's complement:

$$\begin{array}{r} 1\,1\,1\,0\,0 \\ +\,0\,1\,0\,1\,0 \\ \hline 1\,0\,0\,1\,1\,0 \\ \hline 0\,0\,1\,1\,1 \end{array}$$

And as 2's complement:

$$\begin{array}{r} 1\,1\,1\,0\,0 \\ +\,0\,1\,0\,1\,1 \\ \hline 1\,0\,0\,1\,1\,1 \\ \hline 0\,0\,1\,1\,1 \end{array}$$

There is no overflow.

2.3

a)

$X'Y'Z + (X'Y'Z)' = 1X'Y'Z + 1(X'Y'Z)' = 1$, by the uniting theorem of boolean addition.

f)

$(A + BC) + (DE + F)(A + BC)' = (A + BC) + (DE + F)$, by the elimination theorem of boolean addition.

2.5

a)

$(A+B)(C+B)(D'+B)(ACD'+E)$
$= (A(C+B)+B(C+B))(D'+B)(ACD'+E)$
$= (AC+AB+BC+B^2)(D'+B)(ACD'+E)$
$= (AC+B)(D'+B)(ACD'+E)$
$= (AC+B)(D'(ACD'+E)+B(ACD'+E))$
$= (AC+B)(AC(D')^2+D'E+ABCD'+BE)$
$= (AC+B)(ACD'+BE+D'E)$
$= AC(ACD'+BE+D'E)+B(ACD'+BE+D'E)$
$= A^2C^2D'+ACBE+ACD'E+BACD'+B^2E+BD'E)$
$= ACD'+ACBE+ACD'E+BACD'+BE+BD'E)$
$= ACD'+BE$

b)

$(A'+B+C')(A'+C'+D)(B'+D')$
$= (A'(B'+D')+B(B'+D')+C'(B'+D'))(A'+C'+D)$
$= (A'B'+A'D'+BB'+BD'+C'B'+C'D')(A'+C'+D)$
$= (A'B'+A'D'+BD'+B'C'+C'D')(A'+C'+D)$
$= A'(A'B'+A'D'+BD'+B'C'+C'D')+C'(A'B'+A'D'+BD'+B'C'+$
$C'D')+D(A'B'+A'D'+BD'+B'C'+C'D')$
$= A'A'B'+A'A'D'+A'BD'+A'B'C'+A'C'D'+C'A'B'+C'A'D'+C'BD'+$
$C'B'C'+C'C'D'+DA'B'+DA'D'+DBD'+DB'C'+DC'D'$
$= A'B'+A'D'+A'BD'+A'B'C'+A'C'D'+A'B'C'+A'C'D'+BC'D'+$
$B'C'+C'D'+A'B'D+0+0+B'C'D+0$
$= A'B'+A'D'+B'C'+C'D'$

2.6

a)

$AB+C'D'$
$= (AB+C')(AB+D')$
$= (A+C')(B+C')(A+D')(B+D')$

f)

$A+BC+DE$
$= (A+B)(A+C)+DE$
$= ((A+B)(A+C)+D)((A+B)(A+C)+E)$
$= ((A+B+D)(A+C+D))((A+B+E)(A+C+E))$
$= (A+B+D)(A+C+D)(A+B+E)(A+C+E)$

2.8

a)

$[(AB)' + C'D]' = AB(C'D)' = AB(C + D') = ABC + ABD'$

2.9

a)

$F = ((A + (A + B)')' + (A + B)')(A + (A + B)')'$
$= ((A'(A + B)) + (A'B'))(A + A'B')'$
$= ((A'(A + B)) + (A'B'))(A' + (A'B')')$
$= ((A'(A + B)) + (A'B'))(A' + A + B)$
$= ((A'(A + B)) + (A'B'))(1)$
$= (A'A + A'B + (A'B'))$
$= 0 + A'B + A'B'$
$= A'$

b)

$G = (((R + S + T)'P((R + S)'T))'T)'$
$= (R + S + T)'P((R + S)'T) + T'$
$= R'S'T'PR'S'T + T'$
$= PR'S'TT' + T'$
$= 0 + T'$
$= T'$

2.12

a)

$(X + Y'Z) + (X + Y'Z)' = 1(X + Y'Z) + 1(X + Y'Z)' = 1$, by the unification theorem of boolean addition.

f)

$(V' + U + W)[(W + X) + Y + UZ'] + [(W + X) + UZ' + Y]$
$= (V' + U + W)[(W + X) + UZ' + Y] + [(W + X) + UZ' + Y]$
$= V' + U + W$, by the absorption theorem of boolean addition.

2.15

a)

$f(A,B,C,D) = [A + (BCD)'][(AD)' + B(C' + A)]$
$dual(f) = [A(B + C + D)'] + [(A + D)'(B + C'A)]$
$complement(f) = [A'(B' + C' + D')'] + [(A' + D')'(B' + CA')]$
$= A'BCD + AD(B' + CA')$

2.16

a)

$f(A,B,C,D) = [A + (BCD)'][(AD)' + B(C' + A)]$
$dual(f) = [A(B + C + D)'] + [(A + D)'(B + C'A)]$

2.21

A	B	C	F	G	H
1	1	1	1	**X**	1
1	1	0	0	**0**	0
1	0	1	0	**1**	1
1	0	0	0	**0**	0
0	1	1	1	**X**	1
0	1	0	0	**1**	1
0	0	1	1	**X**	1
0	0	0	0	**0**	0

X represents an unspecified value in the above table.

2.22

a)

$A'B' + A'CD + A'DE'$
$= A'(B' + CD + DE')$
$= A'((B' + C)(B' + D) + DE')$
$= A'((B' + C + DE')(B' + D + DE'))$
$= A'(B' + C + DE')(B' + D)$
$= A'(B' + C + D)(B' + C + E')(B' + D)$
$= A'(B' + D)(B' + C + E')$

2.24

a)

$[(XY')' + (X' + Y)'Z] = (X' + Y) + (XY')Z = X' + Y + XY'Z$