M328K Homework 10

Joshua Dong

April 4, 2014

0.1 7.1.6

Find all $n \in \mathbb{Z}^+$ such that $\varphi(n) = 24$.

n must be either prime or composite.
Case 1:
$n \in \mathbf{P} \to n = \varphi(n) + 1$
$\therefore n = 24 + 1 = 25$
But 25 is not prime.
Therefore, n is not prime.
Case 2:
$\exists a, b \in \mathbb{Z}^+$ such that $ab = n$ and $a \perp b$.
This implies that $\varphi(n) = \varphi(a)\varphi(b)$
Therefore (a, b) must be in the set of factors of 24:
$(a, b) \in \{(1, 24), (2, 12), (3, 8), (4, 6)\}$ (without loss of generality)
We can eliminate (1,24) because any solutions will be duplicate.
We can eliminate (2,12) because 2 and 12 are not coprime.
We can eliminate (3,8) because no number has a Euler totient value of 3.
Calculating the values of the totient function yeilds:

$$1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16, 6, 18, 8, 12, 10, 22, 8, 20, 12, 18, 12, 29, 8$$

$\therefore n = ab \quad \forall (a, b) \in \{5, 8, 10, 12\} \times \{7, 9, 14, 18\}$.
Explicitly enumerated:

$$35, 45, 56, 70, 70, 72, 84, 90, 90, 108, 112, 140, 144, 168, 180, 216$$

0.2 7.1.14

Find all $n \in \mathbb{Z}^+$ such that $\varphi(n) \mid n$.

0.3 7.1.20

Let $p \in \mathbf{P}$. Show that $p \nmid n$, where $n \in \mathbb{Z}^+$ iff $\varphi(np) = (p-1)\varphi(n)$.

$\varphi(p) = (p-1)$.
$\varphi(np) = (p-1)\varphi(n) \to n \perp p$.
$\therefore p \nmid n$.

0.4 7.1.32

Show that $m, n \in \mathbb{Z}^+, m \mid n \to \varphi(m) \mid \varphi(n)$.