M328K Homework 6

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2(e), 14(b,d), 16, 18

0.1 4.2.2.e

Find all solutions to $128x \equiv 833 \pmod{1001}$

$(128, 1001) = (128, 1001 - 8(128)) = (128, -23) = 1$
$\therefore$ there is exactly one unique solution mod 1001.
$128x - 1001q = 1$
$1001 + (-8)128 = -23$
$128 + (5)((1)1001 + (-8)128) = 13$
$(1)1001 + (-8)128 + (2)((1)128 + (5)((1)1001 + (-8)128)) = 3$
$(128+(5)((1)1001+(-8)128))+(-4)((1)1001+(-8)128+(2)((1)128+(5)((1)1001+(-8)128))) = 1$
$128+(5)1001+(-40)128+(-4)1001+(32)128+(-8)128+(-40)1001+(320)128 = 1$
$(-39)1001 + (305)128 = 1$
$\therefore 1\bar{2}8 = 305.$
$1\bar{2}8(128x) \equiv 1\bar{2}8(833) \pmod{1001}.$
$\therefore x \equiv 305(833) \equiv 812 \pmod{1001}$
$\therefore x \in S$ such that $S = \{n \mid n = 812 + 1001k \ \ \forall k \in \mathbb{Z}\}$.

0.2 4.2.14.b

$2x + 4y \equiv 6 \pmod{8}$

$x + 2y \equiv 3 \pmod{4}$
$(1, 2, 4) \mid 3$
$4n = (x + 2y - 3)$ for some $n \in \mathbb{Z}$.
$(x + 2y - 4n) = 3.$
Let k be some integer where $2k = (2y - 4n)$.
Then $(x + 2k) = 3$.
By observation, we see that one solution is $x = 3$, $k = 0$.
All solutions to the previous equation then are expressible in the form:
$x = 3 - 2t, k = t \ \ \forall t \in \mathbb{Z}$.

Now we solve $2k = (2y - 4n)$.
$k = (y - 2n)$.
First we solve $y - 2n = 1$
By observation, we see that one solution is:
$y = 3, n = 1$
So the solution to $k = (y - 2n)$ is:
$y = 3k + 2s, n = k - s \ \ \forall s \in \mathbb{Z}$.
$\therefore x = 3 - 2t, y = 3t + 2s \ \ \forall s, t \in \mathbb{Z}$ represent all the solutions to $x + 2y \equiv 3 \pmod 4$.

0.3 4.2.14.d

$10x + 5y \equiv 9 \pmod{15}$

$(10, 5, 15) \nmid 9$
Thus, there are no solutions.

0.4 4.2.16

Show $x^2 \equiv 1 \pmod{2^k}$ has exactly four unique solutions: $\pm 1, \pm(1 + 2^{k-1}$

0.5 4.2.18

a