M361K Homework 1

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0.1 1.2.1

Prove that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}\forall\, n\in\mathbb{N}$
Let $P(x)$ be the statement: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{x(x+1)}=\frac{x}{x+1}$.
$\frac{1}{1\cdot 2}=\frac{1}{1+1}$.
$\therefore P(0)$ holds.
$\therefore P(n)$ for some $n\in\mathbb{N}$.
$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}+\frac{1}{(n+1)(n+2)}=$
$\frac{n}{n+1}+\frac{1}{(n+1)(n+2)}=\frac{n(n+2)}{(n+1)(n+2)}+\frac{1}{(n+1)(n+2)}=\frac{n(n+2)+1}{(n+1)(n+2)}=\frac{n^2+2n+1}{(n+1)(n+2)}=\frac{n+1}{n+2}$.
$\therefore P(n)\rightarrow P(n+1)$.
$\therefore P(n)\ \forall\, n\in\mathbb{N}$.

0.2 1.2.4

Prove that $1^3+2^3+\ldots+(2n-1)^2=\frac{4n^3-n}{3}\ \forall\, n\in\mathbb{N}$
Let $P(n)$ be the statement: $1^3+2^3+\ldots+(2n-1)^2=\frac{4n^3-n}{3}$.
$1^3=\frac{4\cdot 1^3-1}{3}$
$\therefore P(0)$ holds.
$\therefore P(n)$ for some $n\in\mathbb{N}$.
$1^3+2^3+\ldots+(2n-1)^2+(2n+1)^2=\frac{4n^3-n}{3}+(2n+1)^2=$
$\frac{4n^3-n}{3}+\frac{3(4n^2+4n+1)}{3}=\frac{4n^3-n+12n^2+12n+3}{3}=\frac{4n^3+12n^2+11n+3}{3}=\frac{4(n^3+3n^2+3n+1)-(n+1)}{3}=$
$\frac{4(n+1)^3-(n+1)}{3}$.
$\therefore P(n)\rightarrow P(n+1)$.
$\therefore P(n)\ \forall\, n\in\mathbb{N}$.

0.3 1.2.8

Prove that $1^3+2^3+\ldots+(2n-1)^2=\frac{4n^3-n}{3}\ \forall\, n\in\mathbb{N}$
Let $P(n)$ be the statement: $1^3+2^3+\ldots+(2n-1)^2=\frac{4n^3-n}{3}$.
$1^3=\frac{4\cdot 1^3-1}{3}$
$\therefore P(0)$ holds.
$\therefore P(n)$ for some $n\in\mathbb{N}$.
$1^3+2^3+\ldots+(2n-1)^2=\frac{4n^3-n}{3}$.

$\therefore P(n) \to P(n+1)$.
$\therefore P(n) \; \forall \; n \in \mathbb{N}$.

0.4 1.2.13

0.5 1.2.18

0.6 1.2.20