

Lab 1

In this lab we will study root finding using `scipy.optimize`. We will also learn more about making good plots by creating our first, real one! Finally we will explore how to use plots (at least non-inlined plots) to quickly learn about the problem we are trying to solve. As always you should look up the documentation on new commands that are mentioned and ask questions when things do not make sense.

1. Throughout this lab we will study the washboard potential given by $V(r) = A_1 \cos(r) + A_2 \cos(2r) - F r$. Here A_1 , A_2 , and F are parameters. We will use $A_1 = 5$ and $A_2 = 1$ and initially consider $F = 1.5$ but will allow it to vary later. The washboard potential (though I will call it a potential you can think of it as a potential energy if you prefer) shows up in many physical problems particularly in solid state physics and condensed matter; for example in the study of superconducting Josephson junctions and in the motion of defects in an crystal.
 - a. Use `scipy.optimize.bisection` and `scipy.optimize.brentq` to find a zero of $V(r)$ in the interval $r \in [-3, 2]$. Write out the commands you use to find the zero and give your results.

Commands and values for the zero for using the two methods:	
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- b. You should have found a surprising result; you do **not** get the same answer from both methods! What does this mean? What we should have done was first plot the function. For your own uses do this now. Note that it is convenient to draw in the $y=0$ line in such plots. You can do this using `axhline()`. Do not include the plot but use it to explain why different answers were found when different methods were used in the previous part.

Explanation of different results from previous part:	
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2. In physics the magnitude of the potential is unimportant; only changes in the potential matter. Thus what we are really interested in is where the minima and maxima of the potential occur. From our quick plot in the previous part we should see that there is a maximum near $r = 0$.
 - a. Find the position of the maximum near $r = 0$ and the first minimum at $r < 0$.
 - . Recall from calculus that extrema occur where $dV/dr = 0$ thus this really is a case of finding zeros. I recommend using `scipy.optimize.brentq` though you can use any method you prefer. Provide the commands and results below.



Commands and values for the maximum and minimum:	
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- b. To confirm our results from the previous part let us make a figure. This will be our first “real” figure! Here are some rough guidelines. There are, of course, exceptions to these suggestions but in general they should be followed.
 - i. Each line in a figure should be distinct and easily recognizable. This means using different colors, styles, or both. In matplotlib the `plot` command provides many options for changing the look of lines. When you look at the documentation the most basic way of calling plot that allows changes is to include a format specifier. For example `plot(x, y, 'k-')` will plot a solid, black line. For purely online plots using different colors is often sufficient. If the plot may appear in print then it will most likely appear in black and white. In this case it is best to use different line styles instead of or in addition to changing color. For example, specifying the format `'k--'` will produce a dashed, black line.
 - ii. Each line in a figure should be labeled. This can be done using the `label` keyword. It should provide a short description of what the line is. The label is used to create a legend in the figure. This can be included by, not surprisingly, using the `legend()` command.
 - iii. All figures must include labels on the axes and a title. Any figure you create for a lab must have these! In general it is a good idea to include them in any figure you end up saving as an image file. When you later come back and wonder what this file is having clear labels and legends

will help.

- iv. There are many other things that can be done to polish figures. You may notice that by default only the major tick marks are plotted. This choice was originally made for speed reasons. A default set of minor ticks can be included by using `minorticks_on()`. Another minor polishing is that I often find the default line thickness to be too small. This can be changed using the `linewidth` or `lw` keyword to `plot()`.
 - c. With this advice in mind create a single plot of $V(r)$ and dV/dr versus r . You should pick a “good” range for r , one that clearly shows the range we are interested in including enough to see the behavior of functions but not too large so as to distract us from the region of interest. We definitely want to include the $y=0$ line. We also want to include vertical lines at the locations of the minimum and maximum; this can be accomplished using `axvline()`. Note that `axhline()` and `axvline()` accept many of the same keywords as `plot()`, in particular we can control the color (`color`), linestyle (`linestyle` or `ls`), and add a label (`label`). Include the plot below.
3. In an experiment the parameter F is often something we can control; for example, it may be the strength of an electric field. Thus it is something we want to tune. In the washboard potential as we change the value of F the slope of the potential changes. For your own purposes it is worth plotting the potential for a few choices of F to see this change. We should find as we increase F the potential near zero flattens out. In fact, there is a particular value for F where the minimum and maximum found in the part 2 merge. This is known as a saddle point bifurcation. Here we want to find this point.
 - a. The first question is what it means to find this point! We really have a two dimensional problem now, we want to find the value for F for which the minimum and maximum merge and the value of r at which this happens. We have not talked about how to solve two dimensional problems. Though we can come up with numerical techniques we will instead take a different approach; we will do it graphically.
 - b. Find F to 5 decimal places such that there is barely a minimum and maximum. You should find $F = 6.xxxxx$, determine the values of the x . To do this we can plot dV/dr and look to see for what value F the two places where it equals zero (the points we found in part 2) merge into a single point. For this purpose it will be useful to plot for a given value of F , see how close they are to merging, try a new F , and repeat. This will involve changing the range of r as you hone in and zooming in on the region of interest. To this end the plot window makes this easier. You can zoom in (left mouse button) or zoom out (right mouse button) of a rectangular region by using the “zoom rectangle” mode accessible by the button



Similarly the “pan and zoom” mode (pan with the left mouse button, zoom with the right) accessible by the button . Both of these buttons toggle the mode. The bottom right of the plot window should tell you what mode you are in. If you “get lost” the home button, , will return the plot window to its original scale when created. By iterating we can quickly find the desired value of F . Once you have this value use it to find the position of the minimum and maximum using a procedure similar to part 2, and the difference between them, Δr .

Value of F to 5 decimal places, the locations of the minimum and maximum, and the distance between these locations:	
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Enter the names of all people who worked on this lab as a group. By including a name here you are verifying that said person was actively involved in the work and understands the material:

Names or ids	
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