PHYS 250 Homework 5

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Problem 1

(i)

To derive the leap-frog approximation to y_{n+1}, we start with the center differencing formula for the first derivative,

$$\frac{dy}{dt} = \frac{y(t+\Delta t) - y(t-\Delta t)}{2\Delta t}$$

From this, we simply solve for the next term in our solution function, $y(t+\Delta t)$, in terms of the known values, the differential equation $-dy/dt$ and the initial value $y(t-\Delta t)$.

$$2\Delta t\frac{dy}{dt} = y(t+\Delta t) - y(t-\Delta t)$$

$$y(t+\Delta t) = 2\Delta t\frac{dy}{dt} + y(t-\Delta t)$$

(ii)

The error in y_{n+1} can be found from the expression,

$$y_{n+1} + \delta y_{n+1} = 2\Delta t\frac{dy}{dt} + y_{n-1} + \delta y_{n-1}$$

and the fact that the error in each term is,

$$\delta y_n = \beta y_{n-1}$$

So we have estimates for the error in,

(iii)

We can determine whether the method is stable or unstable for different cases of differential equations by determinig whether the value of β is below or above 1.0, respectively.

Problem 2

(i)

To find the maximum height of the projectile ignoring air resistance we define an equation for its position in time, assuming $y = 0$ is its starting height,

$$y(t) = y_0 + v_0 t + \frac{1}{2} g t^2$$

Then we take its derivative with respect to t,

$$\frac{dy}{dt} = v_0 + gt$$

Setting this equal to 0, we can calculate the exact time t at which the proctile's velocity is zero and it reaches its apex,

$$0 = v_0 + gt$$
$$-gt = v_0$$
$$t = \frac{v_0}{-g}$$

Knowing that $v_0 = 8m/s$ and $g = 9.8m/s^2$, we calculate t to be approximately 0.816 seconds. So the maximum height of the projective is $y(0.816) = 3.262$ meters.

(ii)

If air resistance is modeled as a force quadratic in velocity,

$$F = -bv|v|$$

We can define the motion of the projectile as a system of first order differential equations by considering the acceleration as just the derivative of the unknown function of the velocity, v. So from the equation,

$$m\frac{d^2y}{dt^2} = -bv|v|$$

We have the set of differential equations,

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = \frac{-bv|v|}{m}$$

(iii)

Now we can numerically calculate our answer to question (i) with the assumption $b = 0.002$ kg/m, So we use the scipy module 'odeint' like so,

(iv)

Problem 3

See attached.

Problem 4

(i)

Find the analytic solution of

$$\frac{dy}{dt} + 50y = 50t^2 + 2t$$

(ii)

Evaluate using rk2 with $\delta t = 0.005$ at $y(0.2s)$.

(iii)

Repeat (iii) with rk4.