

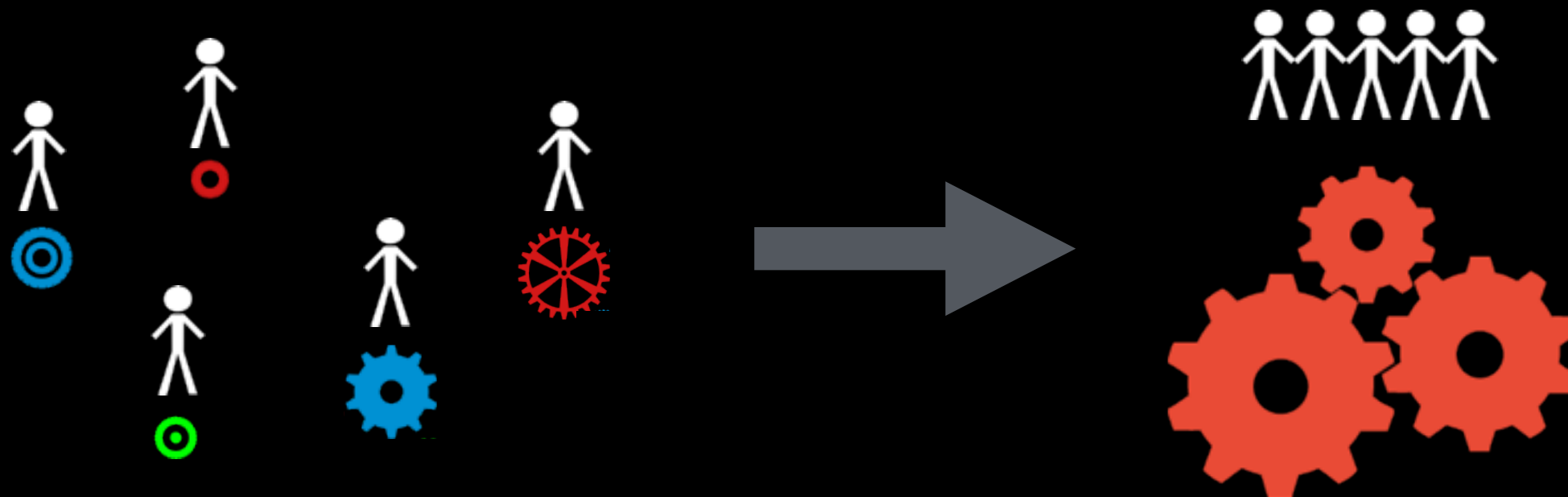
S | S | M

Inference for State Space Models





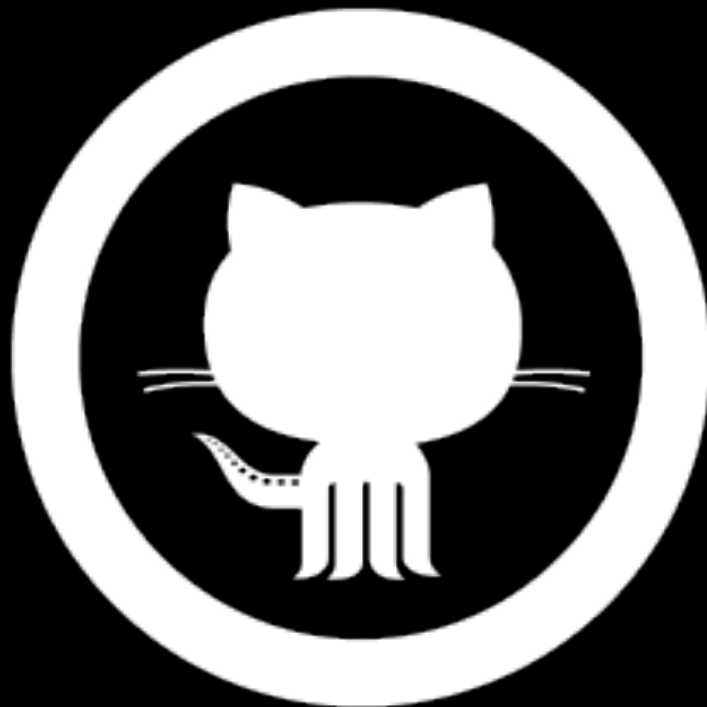
Modelling complex & dynamic quantities:
reorganising to meet up with technical challenges.



<https://github.com/standard-analytics/ssm>

**Open Source
community**

+ you



Sign up for GitHub

<https://github.com/>

WHY?

New Issue

**Question?
Raise issues**



Star

4

Support!



Watch ▼

8

Keep informed

Objectives 5x3 hours

Build your own compartmental models

Calibrate them against simple and complex data

Study an open problem in epidemiology

Step 0

**Is SSM properly installed
on your machines?**

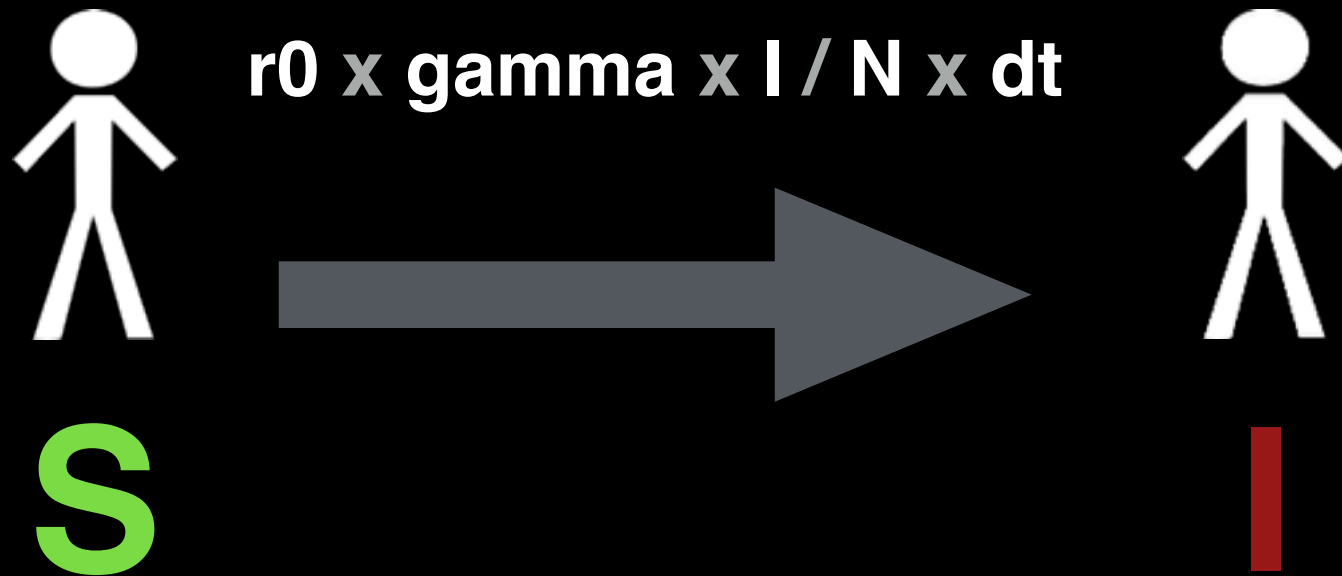
In your terminal: `> ssm -V`

Step 1

Build simple models.

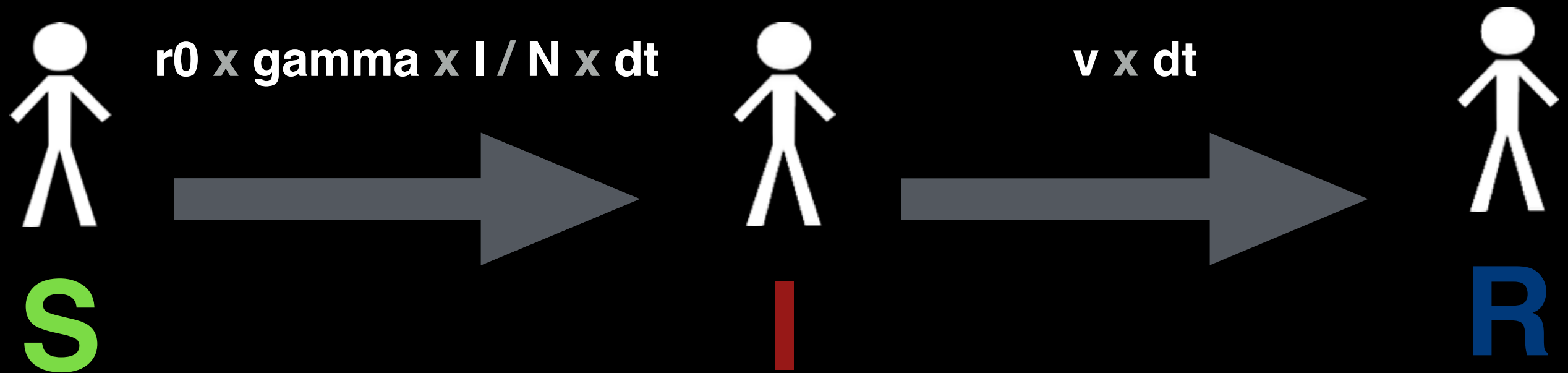
Simulate deterministic and stochastic scenarios from them.

SI Intro



$$dS_t = -r_0 \times \text{gamma} \times I / N \times S \times dt$$
$$dI_t = r_0 \times \text{gamma} \times I / N \times S \times dt$$

SIR Intro



$$dS_t = -r_0 \times \text{gamma} \times I / N \times S \times dt$$

$$dI_t = r_0 \times \text{gamma} \times I / N \times S \times dt - v \times dt$$

$$dR_t = v \times dt$$

SIR First steps

change directory



```
> cd SIR-city
```

```
> ls
```



Saint-Fuscien

Amiens

Paris

list content

package.json???

JSON: JavaScript Object Notation

JSON

double-quotes only!



Lists: [`“a”`, 3, ...]

Objects: { `“a”`: `“A”`, `“b”`: 3, ... }

Flexible, readable format.

Popular open standard.

package.json

Used to distribute code & data through
the node package manager (npm)

Standard keys:

`'name'`, `'description'`, `'keywords'`,
`'licenses'`, `'version'`, `'resources'`

SIR

Let's get to it

Closer look at the “model” object.

SIR Simulation

```
> ssm install package.json  
> cd bin  
> ./simul --help  
> cat ../package.json | ./simul --traj
```

→ X_0.csv

SIR Plot X_0.csv

Open TD.R in R

!!! Set TD-STRU to be Working Directory

Plot X_0.csv with ssm.plot.X

SIR Explore

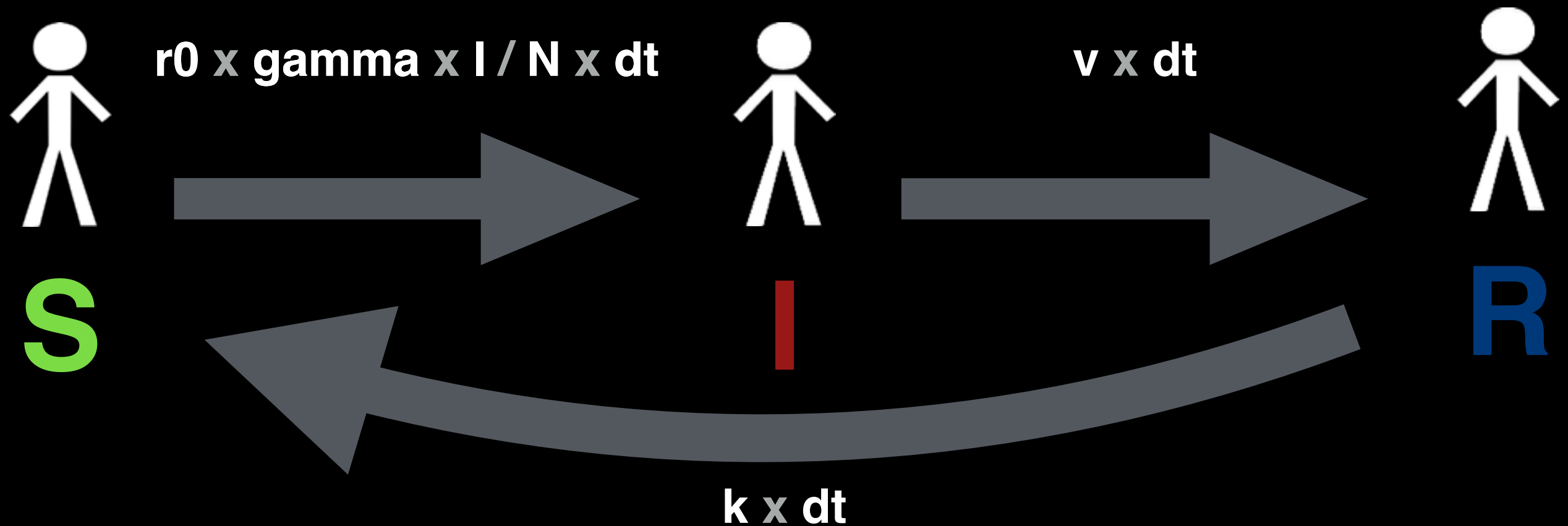
Explore different values of R_0 and d

Under which minimal conditions does an epidemic burst?

According to the SIR model, what will be the number of susceptibles on December 13th, 2012?

SIRS

Make your own model



$$dS_t = -r_0 \times \text{gamma} \times I / N \times S \times dt + k \times dt$$

$$dI_t = r_0 \times \text{gamma} \times I / N \times S \times dt - v \times dt$$

$$dR_t = v \times dt - k \times dt$$

SIRS

Make your own model

What should qualitatively be the impact of immunity loss on the number of susceptibles on December 13th, 2012?

SIRS Make your own model

Duplicate and rename SIR.

Modify it to obtain an SIRS model.

reactions **+** `...,
{"from": "R", "to": "S", "rate": "k", "description": "recovery"}`

inputs **+** `...,
{"name": "k", "description": "rate of immunity loss",
"data": {"resource": "pr_k", "transformation": "1/pr_k",
"to_resource": "1/k" },`

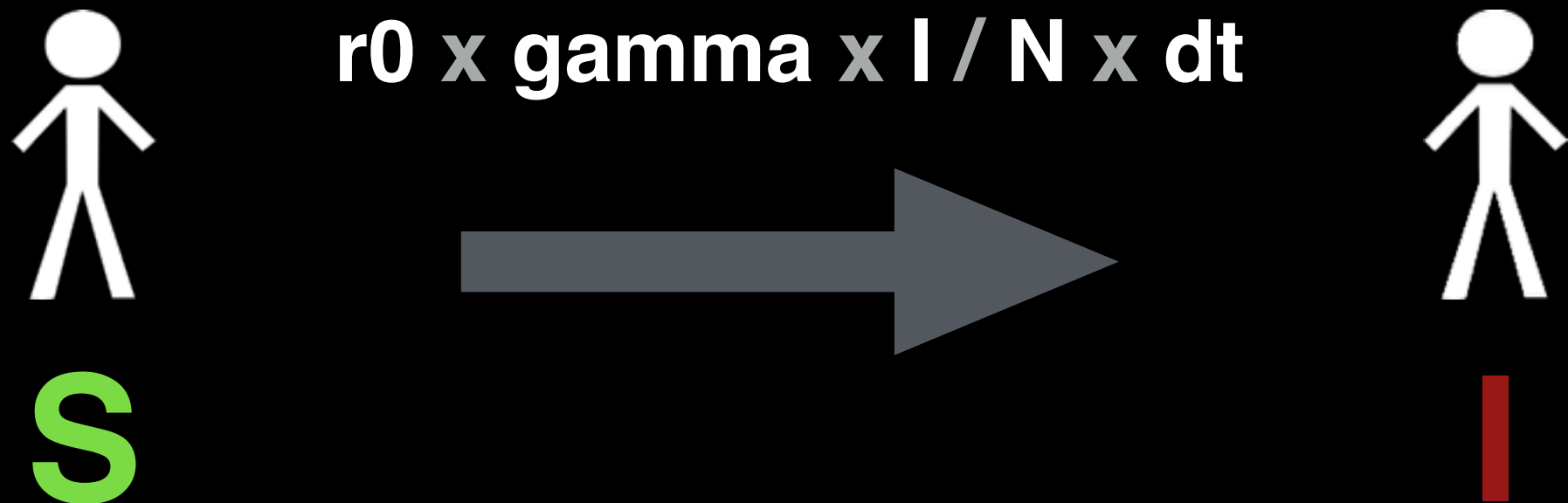
resources **+** `{ "name": "pr_k", "description": "duration of immunity",
"data": { "distribution": "fixed", "value": 250.0 } },`

Ok, but...

Aren't these deterministic scenarios
a bit too simplistic?

Noise Poisson process formalism

For every individual:



infection is a random process

Noise Poisson process formalism

For S_t individuals:

$$p(n \text{ infections}) \approx \binom{S_t}{n} r_0 \times \text{gamma} \times I / N \times dt$$

→ tractable stochastic model

Noise SDE formalism

Going further, following Ethier & Kurtz 1986:

drift

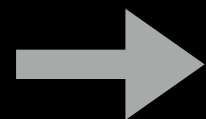


volatility



$$dS_t = -r_0 \times \text{gamma} \times I / N \times S_t \times dt - \text{sqrt}(r_0 \times \text{gamma} \times I / N \times S_t) dB_t$$

$$dI_t = r_0 \times \text{gamma} \times I / N \times S_t \times dt + \text{sqrt}(r_0 \times \text{gamma} \times I / N \times S_t) dB_t$$



Diffusion approximation

For more details, see: Dargatz (2007). *A diffusion approximation for an epidemic model.*

To remember

psr best tractable approximation

sde continuous approximation
classical mathematical object
theory only for large populations

SIR

Let's try with noise

```
> cat ../package.json | ./simul psr --traj
```

```
> cat ../package.json | ./simul sde --traj -I 1
```



Run id

Plot and compare X_0.csv and X_1.csv

SIR Let's try with noise

Number of particles



```
> cat ../package.json | ./simul psr --traj -J 3
```

```
> cat ../package.json | ./simul sde --traj -I 1 -J 3
```



Run id

More particles

SIR Let's try with noise

Number of particles



```
> cat ../package.json | ./simul psr --traj -J 1000
```

```
> cat ../package.json | ./simul sde --traj -I 1 -J 1000
```



Run id

Even more particles

SIR Let's try with noise

Number of particles



```
> cat ../package.json | ./simul psr --hat -J 1000
```

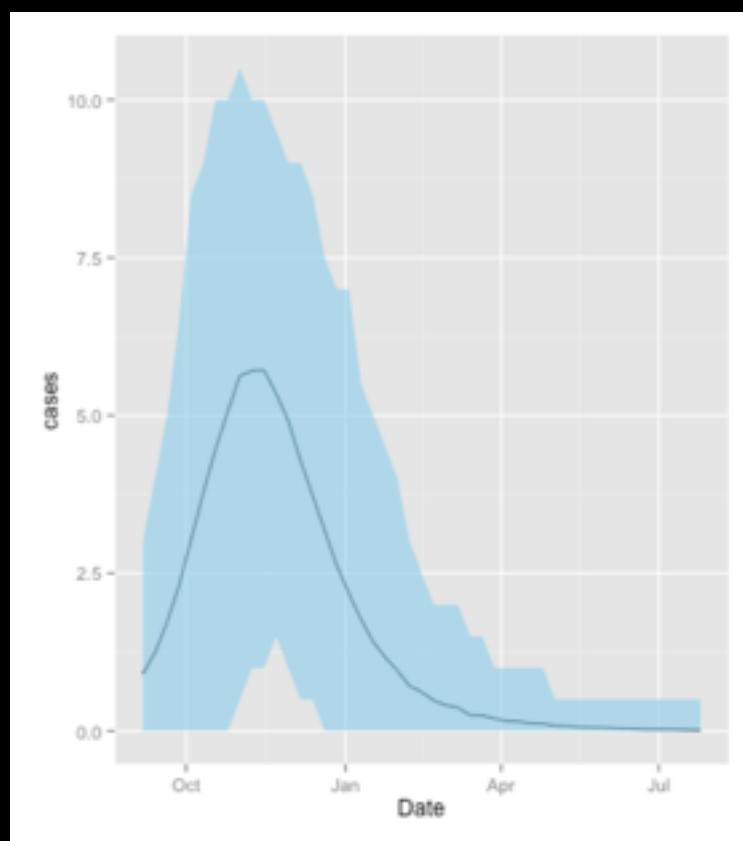
```
> cat ../package.json | ./simul sde --hat -I 1 -J 1000
```



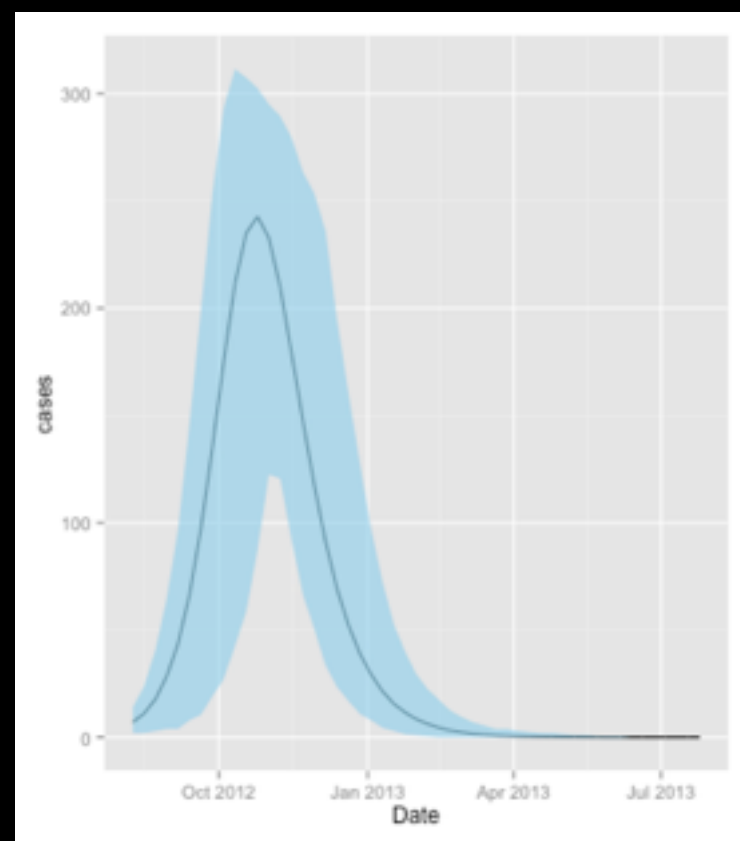
Run id

Generate trajectory confidence intervals

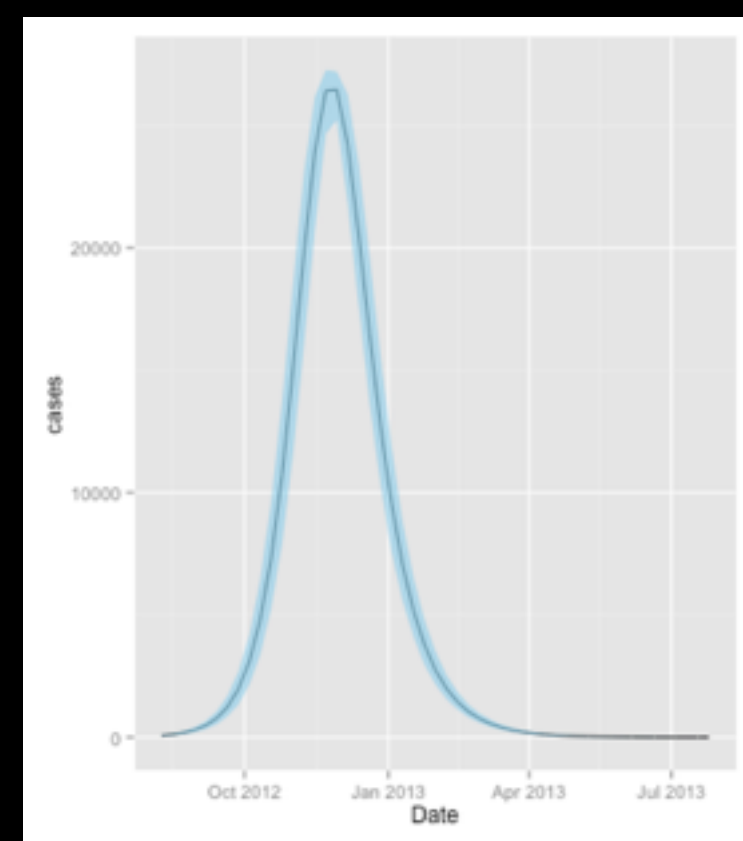
→ use `ssm.plot.hat`



Saint-Fuscien
1'000 inhabs.



Amiens
100'000 inhabs.



Paris
10M inhabs.