Inference for State Space Models

Objective

Explore the link between climate and Dengue in Thailand

3 steps

Intro to epidemic modeling and simulation with SSM

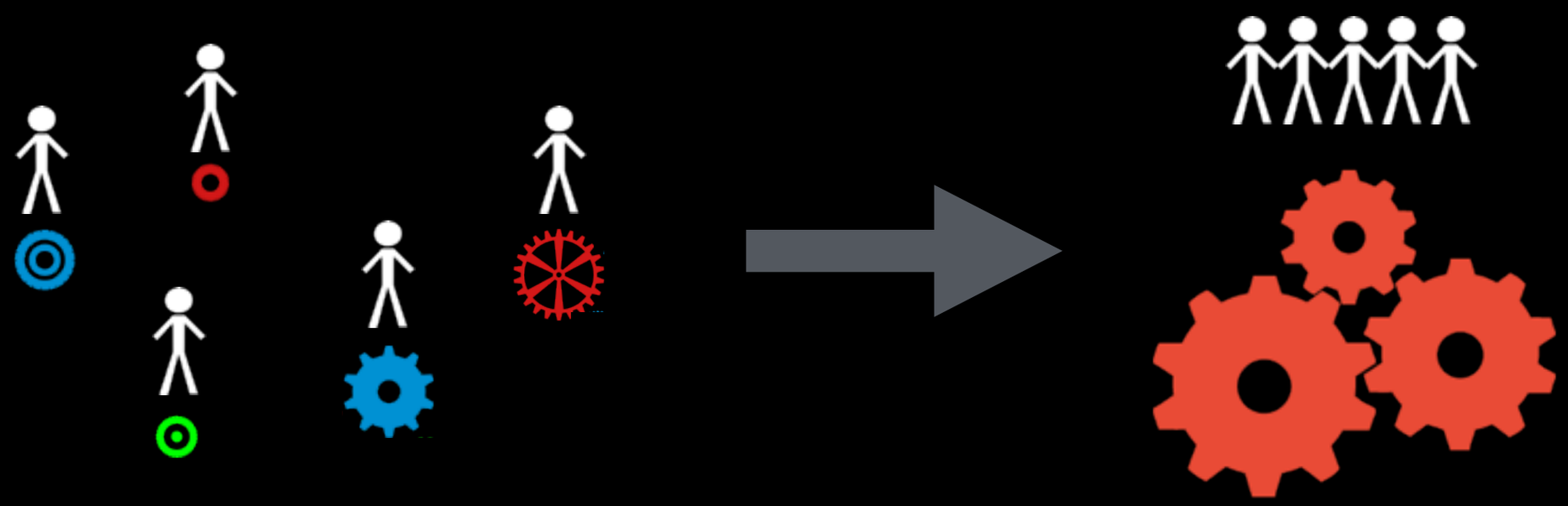
Inference: parameter estimation and model selection

Dengue and climate in Thailand

Intro to epidemic modeling and simulation with SSM

Modelling complex & dynamic quantities:

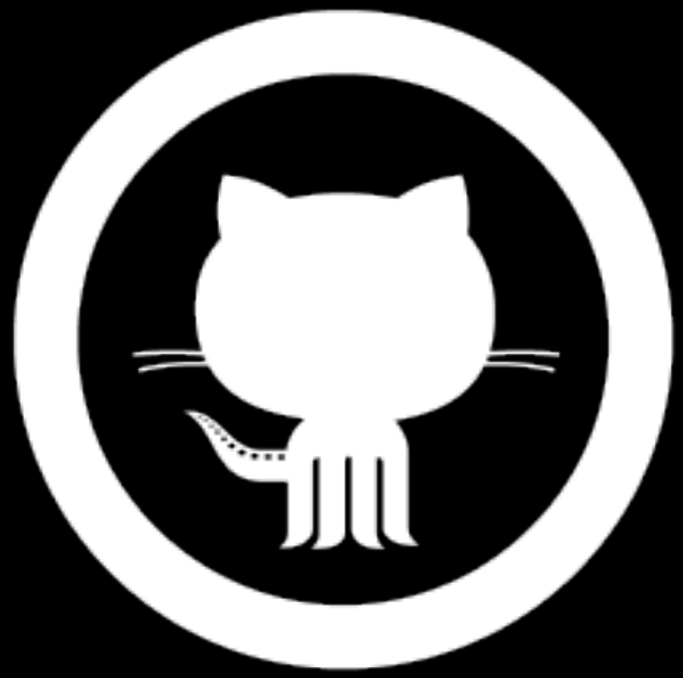
reorganising to meet up with technical challenges.



https://github.com/standard-analytics/ssm

Open Source community

+ you



Sign up for GitHub

https://github.com/

WHY?

Question?
Raise issues

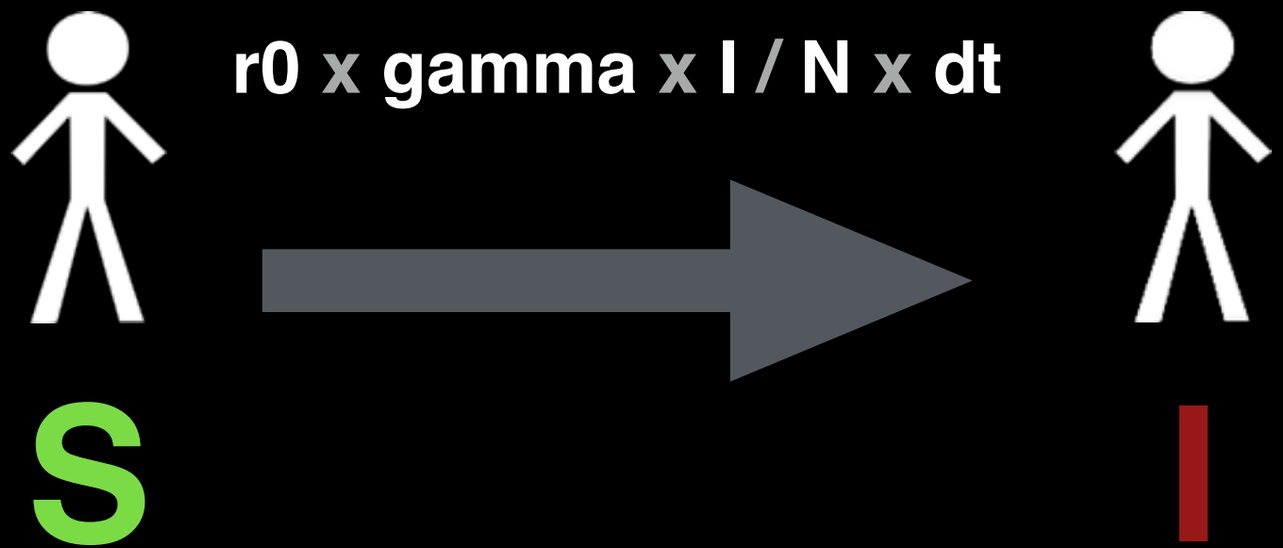
Support!

Keep informed

Is SSM properly installed on your machines?

In your terminal: `> ssm -V`

SI Intro



$$dS_t = -r0 \times gamma \times I / N \times S \times dt$$
$$dI_t = r0 \times gamma \times I / N \times S \times dt$$

SIR Intro

$$dS_t = -r0 \times gamma \times I / N \times S \times dt$$
$$dI_t = r0 \times gamma \times I / N \times S \times dt - v \times dt$$
$$dI_t = v \times dt$$

SIR First steps

change directory

```
> cd SIR-city
> ls
```

list content

Saint-Fuscien
Amiens
Paris

package.json???

JSON: JavaScript Object Notation

JSON

double-quotes only!

Lists: `[ ‘‘a’’, 3, … ]`

Objects: `{ ‘‘a’’: ‘‘A’’, ‘‘b’’: 3, …}`

Flexible, readable format.

Popular open standard.

SIR Let’s get to it

Closer look at the “model” object.

SIR Simulation

```
> ssm install package.json
> cd bin
> ./simul --help
> cat ../package.json | ./simul --traj
```

→ X_0.csv

SIR Plot X_0.csv

Open TD.R in R

!!! Set TD-STRU to be Working Directory

Plot X_0.csv with ssm.plot.X

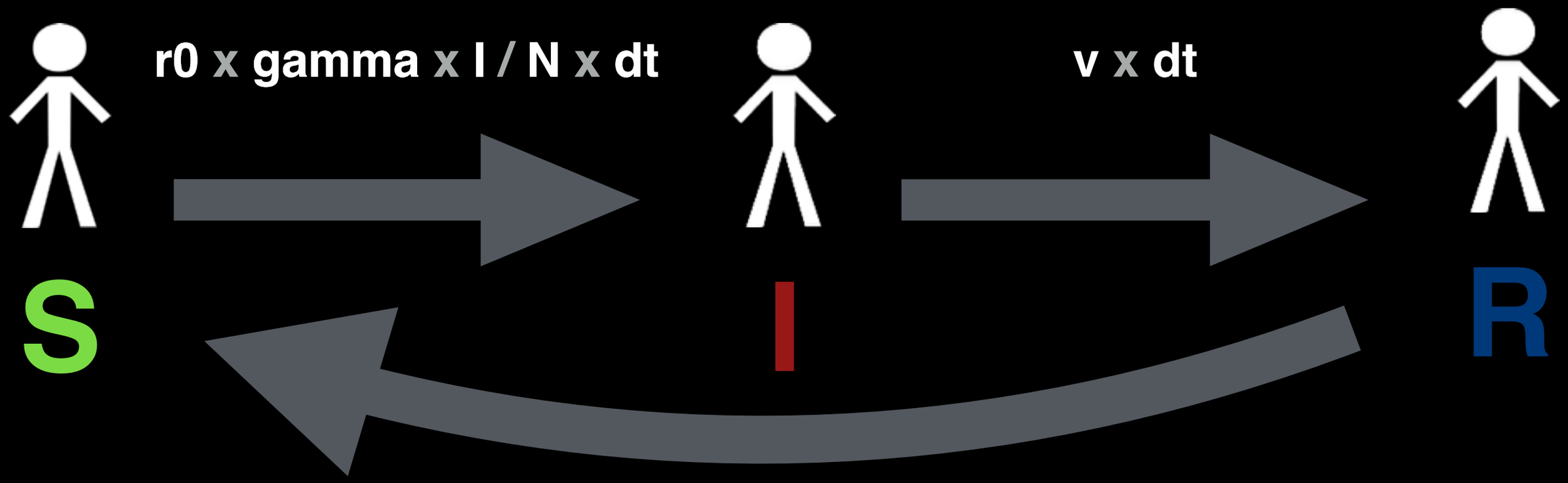
SIR Explore

Explore different values of R0 and d

Under which minimal conditions does an epidemic burst?

According to the SIR model, what will be the number of susceptibles on December 13th, 2012?

SIRS Make your own model



$$dS_t = -r0 \times gamma \times I / N \times S \times dt + k \times dt$$
$$dI_t = r0 \times gamma \times I / N \times S \times dt - v \times dt$$
$$dI_t = v \times dt - k \times dt$$

SIRS Make your own model

What should qualitatively be the impact of immunity loss on the number of susceptibles on December 13th, 2012?

SIRS Make your own model

Duplicate and rename SIR.
Modify it to obtain an SIRS model.

reactions **+** …,
{"from": "R", "to": "S", "rate": "k", "description":"recovery"}

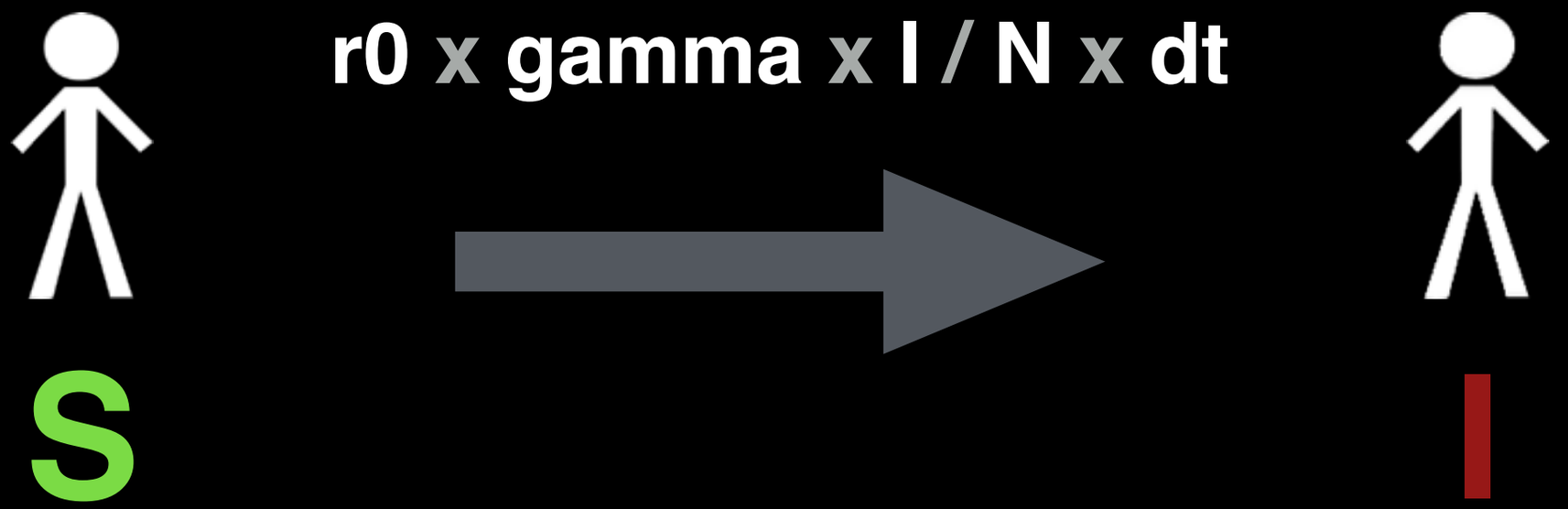
inputs **+** …,
{ "name": "k", "description": “rate of immunity loss",
"data": {"resource": "pr_k"}, "transformation": "1/pr_k",
"to_resource": "1/k" },

resources **+** { "name": "pr_k", "description": “duration of immunity",
"data": { "distribution": "fixed", "value": 250.0 } },

The role and influence of demographic stochasticity

Noise Poisson process formalism

For every individual:



infection is a random process

Noise Poisson process formalism

For S_t individuals:

$$\text{p(n infections)} \approx \binom{S_t}{n} \text{r0} \times \text{gamma} \times \text{I} / \text{N} \times \text{dt}$$

→ tractable stochastic model

For more details, see: Breto et al (2009). *Time series analysis for mechanistic models.*

Noise SDE formalism

Going further, following Ethier & Kurtz 1986:

drift

volatility

dSt = - r0 x gamma x I / N x St x dt - sqrt(r0 x gamma x I / N x St) dBt

dIt = r0 x gamma x I / N x St x dt + sqrt(r0 x gamma x I / N x St) dBt

→ ***Diffusion approximation***

For more details, see: Dargatz (2007). *A diffusion approximation for an epidemic model.*

To remember

psr best tractable approximation

sde continuous approximation
classical mathematical object
theory only for large populations

SIR Let's try with noise

```
> cat ../package.json | ./simul psr --traj
```

```
> cat ../package.json | ./simul sde --traj -I 1
```

Run id

Plot and compare X_0.csv and X_1.csv

SIR Let's try with noise

Number of particles

```
> cat ../package.json | ./simul psr --traj -J 3
```

```
> cat ../package.json | ./simul sde --traj -I 1 -J 3
```

Run id

More particles

SIR Let's try with noise

Number of particles

```
> cat ../package.json | ./simul psr --traj -J 1000
```

```
> cat ../package.json | ./simul sde --traj -I 1 -J 1000
```

Run id

Even more particles

SIR Let's try with noise

Number of particles

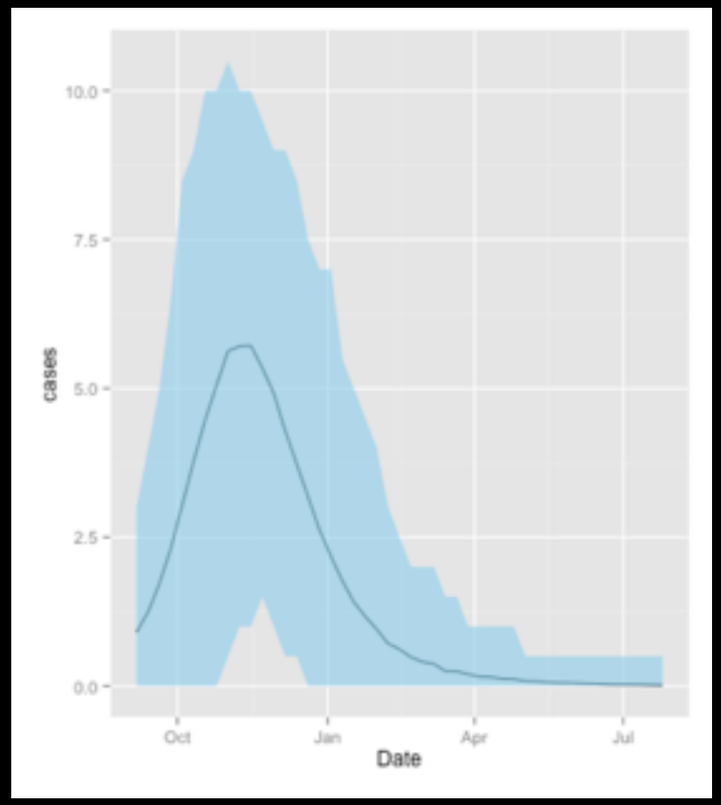
```
> cat ../package.json | ./simul psr --hat -J 1000
```

```
> cat ../package.json | ./simul sde --hat -I 1 -J 1000
```

Run id

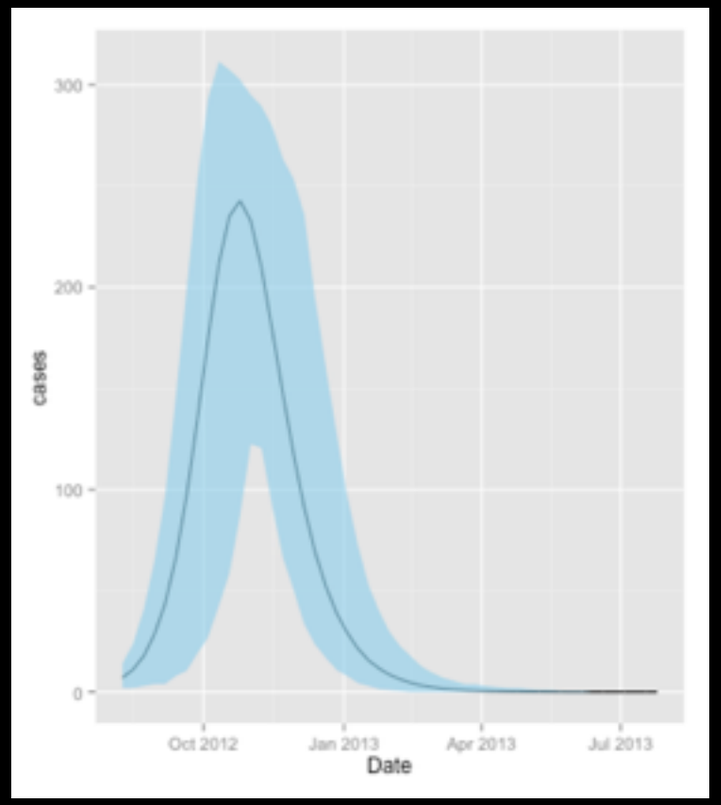
Generate trajectory confidence intervals

→ use ssm.plot.hat



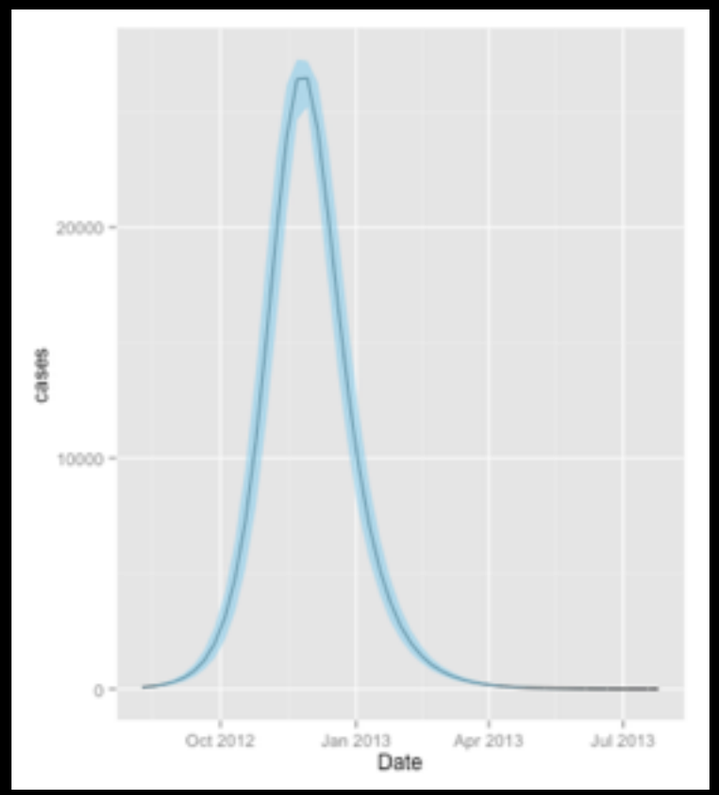
Saint-Fuscien

1’000 inhabs.



Amiens

100’000 inhabs.

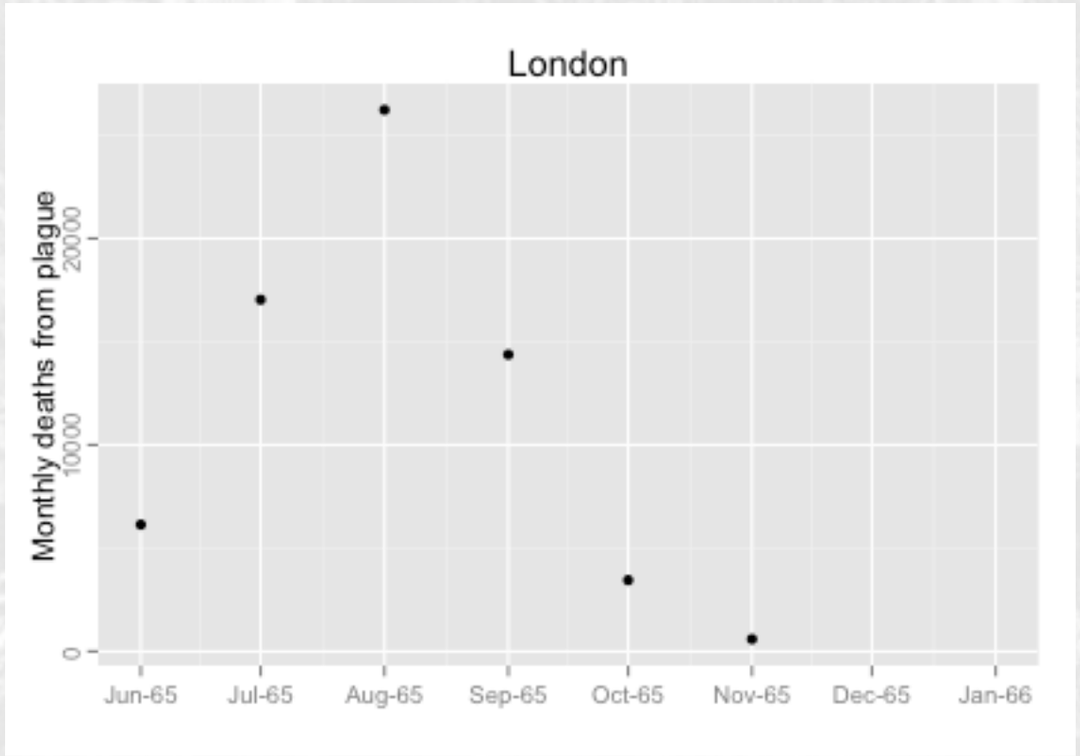


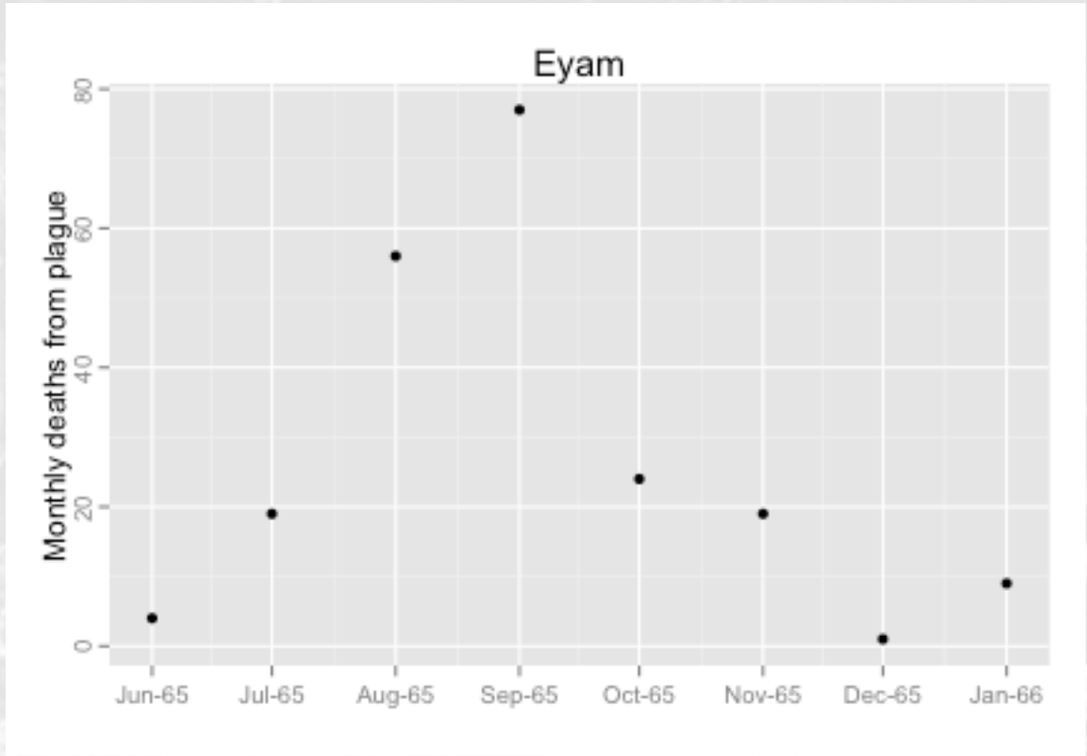
Paris

10M inhabs.

Inference: parameter estimation and model selection

A first real example

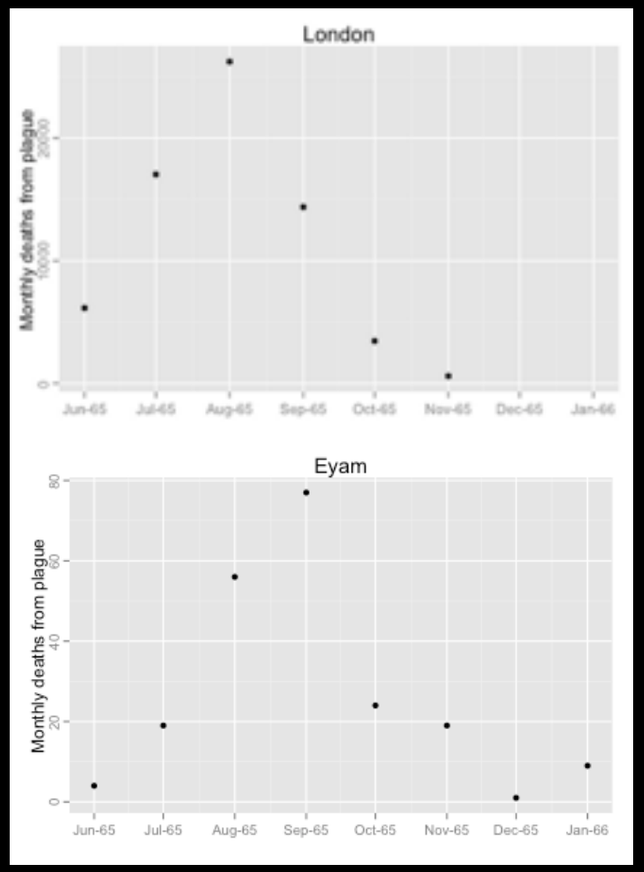




What was life expectancy with plague?
Were infection patterns similar in London and Eyam?

life_exp? r0_London? r0_Eyam?

p(y|Θ) Likelihood



p(| life_exp
r0_London
r0_Eyam)

Bayes Formula

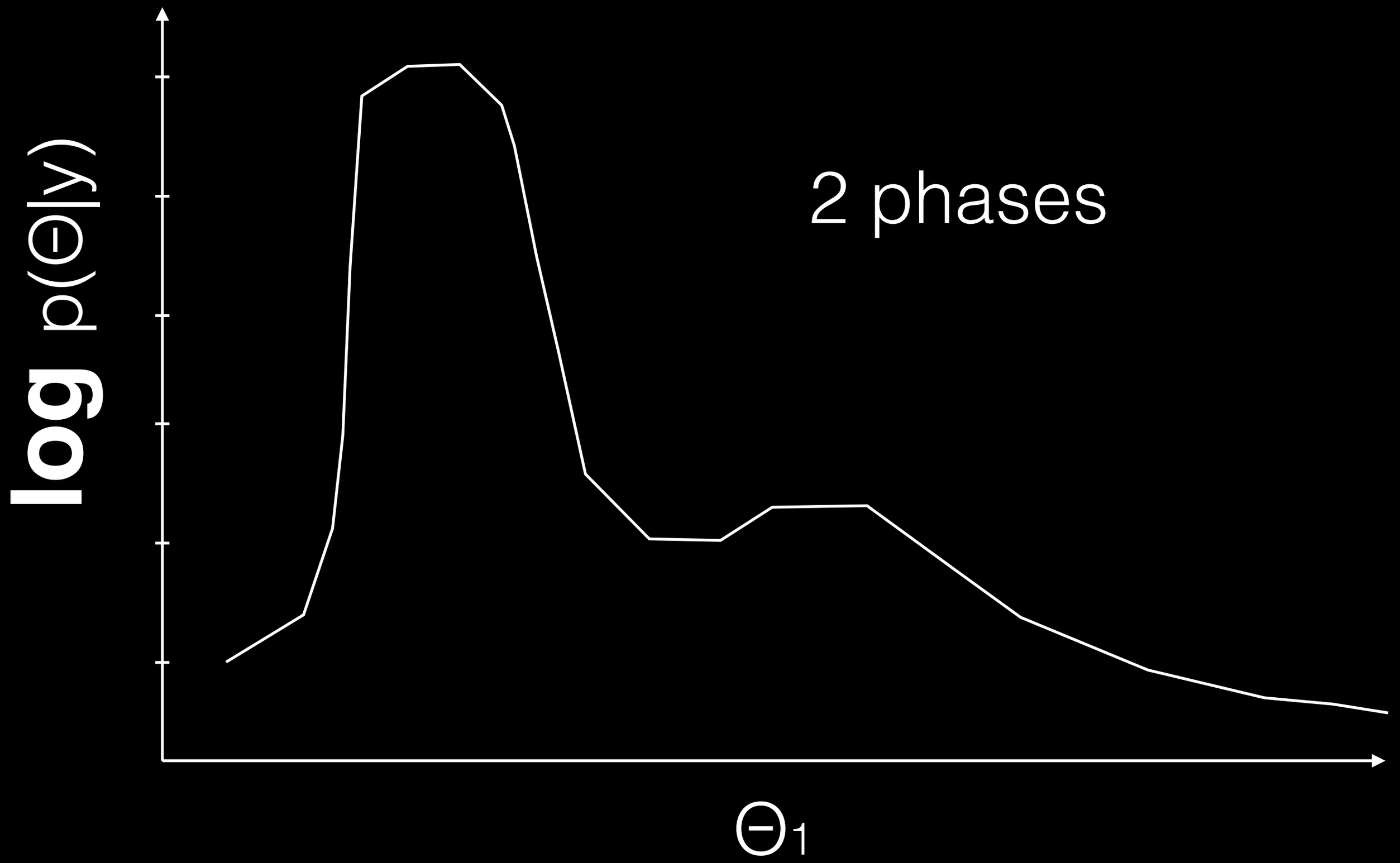
Posterior

Likelihood

Prior

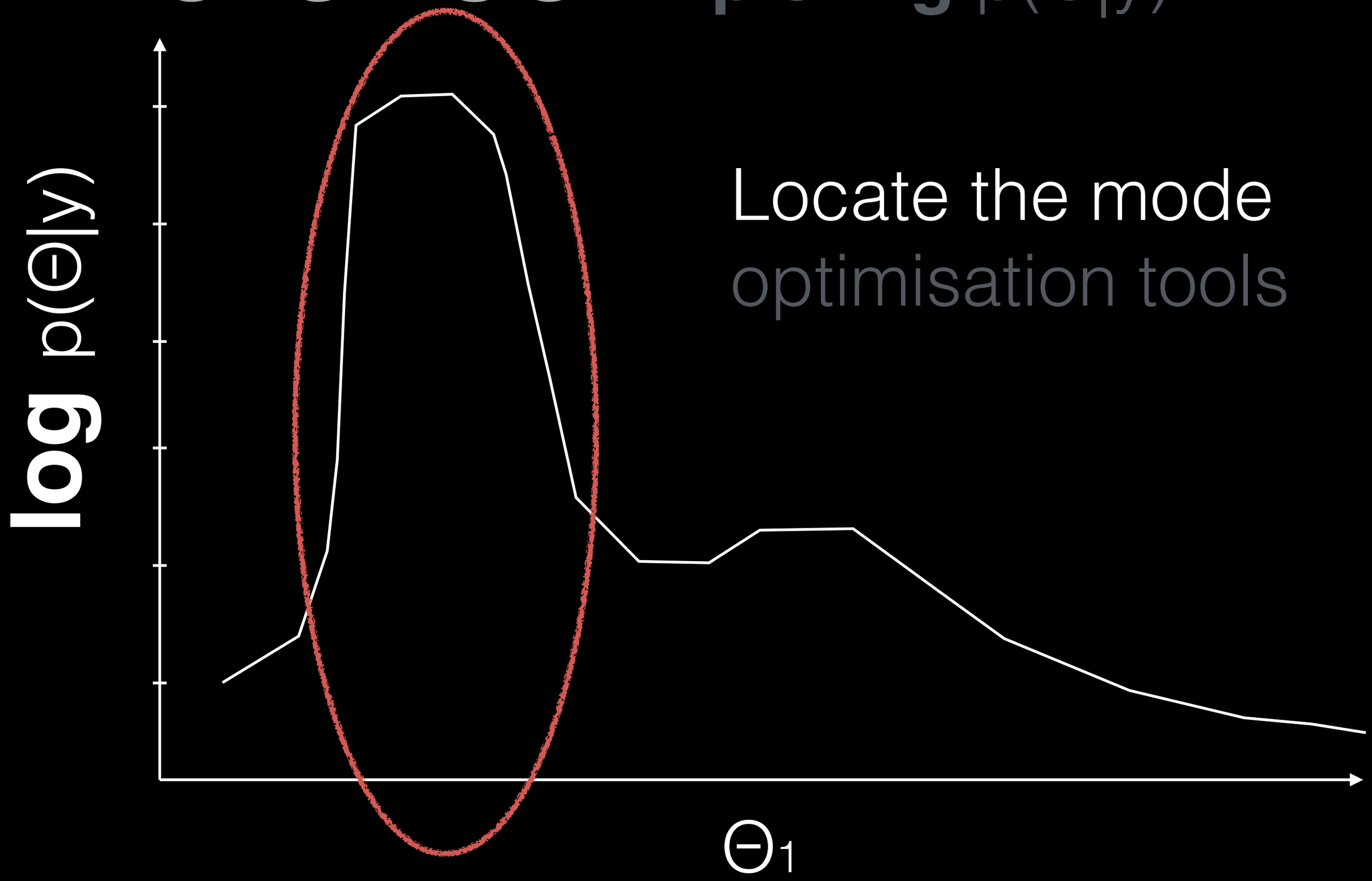
$$p(\Theta|y) \propto p(y|\Theta)p(\Theta)$$

Inference **Exploring** p(Θ|y)



2 phases

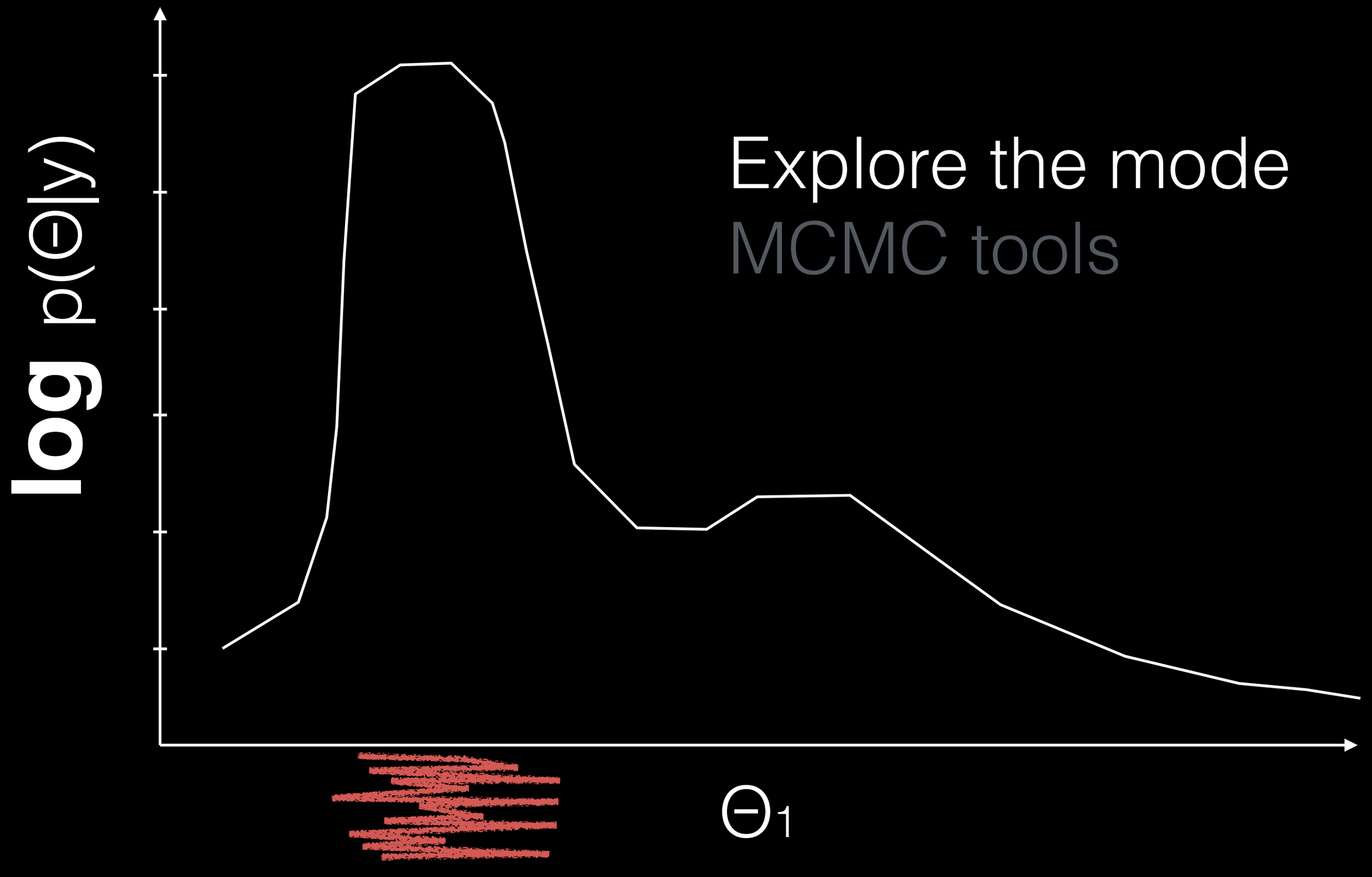
Inference **Exploring** p(Θ|y)



Locate the mode

optimisation tools

Inference **Exploring** p(Θ|y)



Explore the mode

MCMC tools

Illustration Manual inference

Install the model, and in the bin directory:

```
> cat ../package.json | ./simplex -M 1000 --trace > mle.json
```

```
> cat mle.json | ./pmcmc -M 1000 --trace > mle.json
```

use ssm.plot.trace, and ssm.plot.post

Illustration Automated pipeline

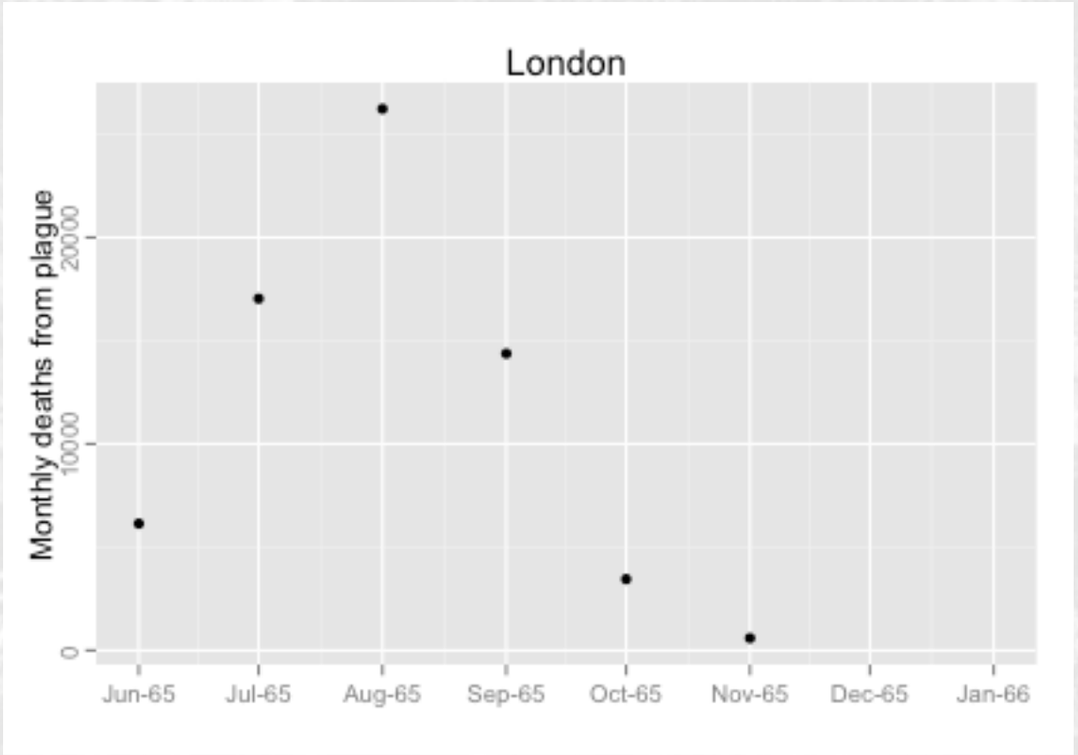
In the parent directory:

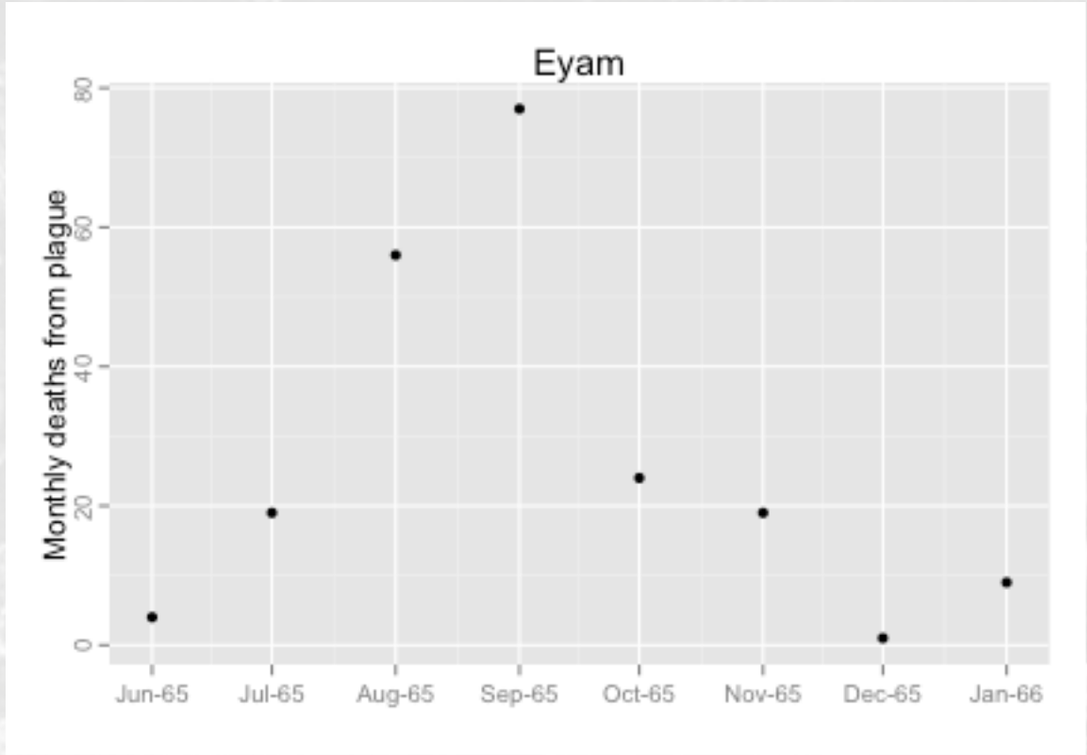
```
> ssm bootstrap package.json
```

```
> ssm run
```

use ssm.plot.trace, and ssm.plot.post

A first real example

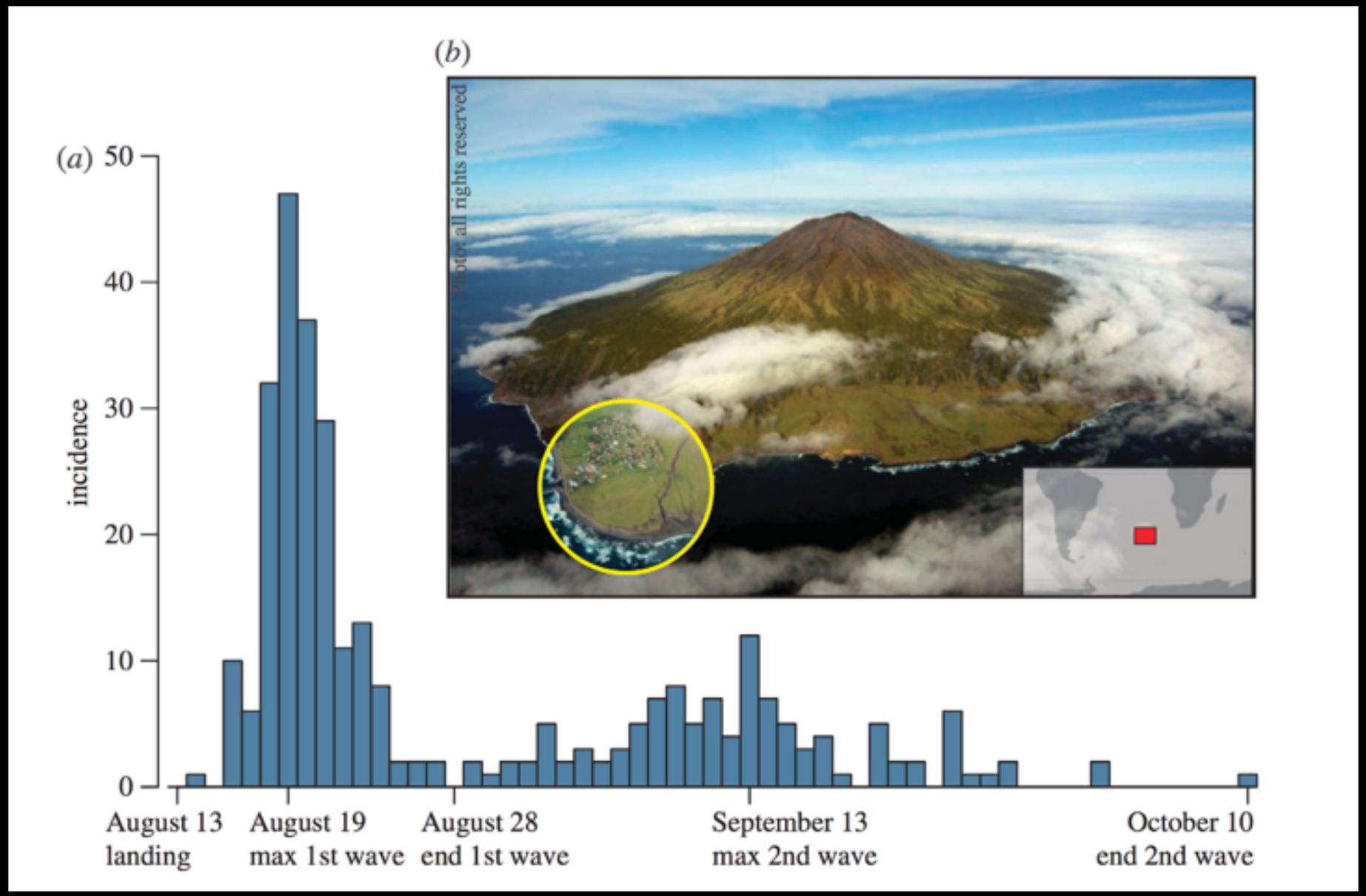




What was life expectancy with plague?

Were infection patterns similar in London and Eyam?

A second real example

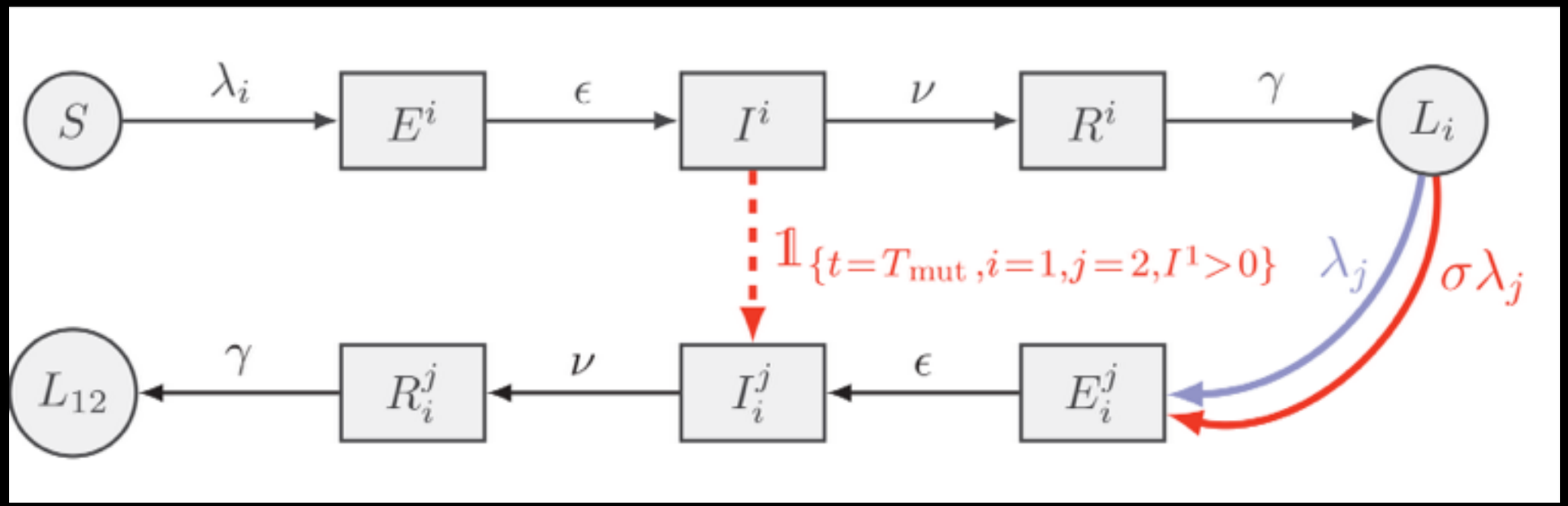


Explaining rapid reinfections in multiple- wave influenza outbreaks: Tristan da Cunha 1971 epidemic as a case study

Two theories

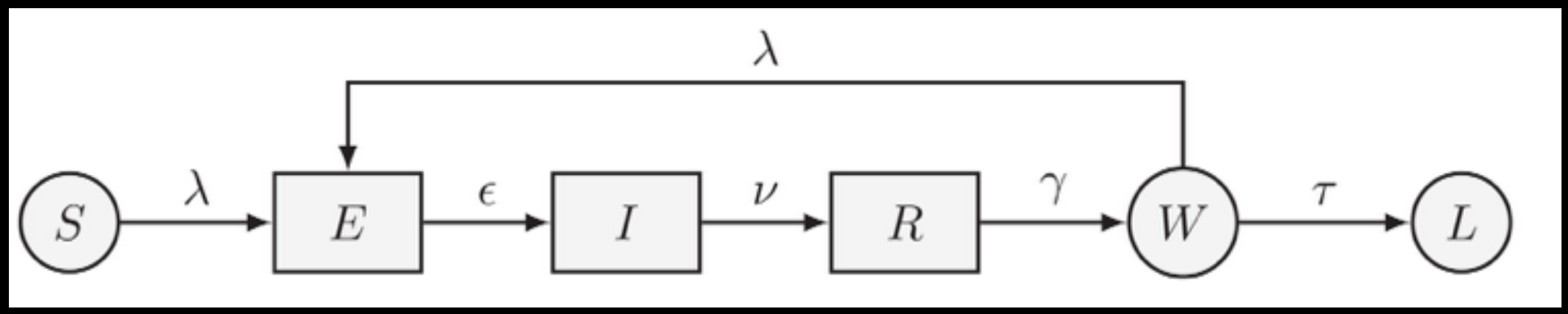
Mut

Strain mutation



Win

Reinfection window



Fit both models. Conclude.