2 Pipes and Cisterns

Tip: 1) → Invert $N \rightleftarrows \frac{1}{N}$

eg: Pipe fills tank in 8 hours

in 1 hour → $\frac{1}{8}$ tank is filled

2) when tap fill → water added (+)

when outlet tap open → water subtract (−)

1) Two pipes M and N can fill a tank in 22 hours and 33 hours respectively. In how much time will tank be full, if both the pipes are opened simultaneously?

Solⁿ:- M → 22 hr → 1 hour → $\frac{1}{22}$

N → 33 hr → 1 hour → $\frac{1}{33}$

both M+N → $\frac{1}{22} + \frac{1}{33} \rightarrow \frac{55\ 5}{22\ (33)_3} \Rightarrow \frac{5}{66}$

↓

in 1 hour

M+N completely fill → $\frac{66}{5}$ hours //

2) Pipe P can fill a tank in 38 hours. Pipe Q alone can fill it in 19 hours. Pipe R can empty full tank in 133 hours. If all the pipes are opened together, how much time will be needed to make the tank full?

Sol: $P \Rightarrow \frac{1}{38}$ (1 hour)

$Q \Rightarrow \frac{1}{19}$ (1 hour)

$R \Rightarrow -\frac{1}{133}$ (1 hour)

$P+Q-R \Rightarrow \frac{1}{38} + \frac{1}{19} - \frac{1}{133}$

$= \frac{35+7-1}{133} \Rightarrow \frac{9.5}{133}$ (1 hour)

$\Rightarrow \frac{1}{14}$ (1 hour)

P+Q−R complete tank full ⇒ 14 hours //

3) A tap can fill a tub in 24 hours. Due to a leak at the bottom of tub, the tap fills the tub in 36 hours. If tub is full, how much time will leak take to empty it?

Sol: tap $\Rightarrow \frac{1}{24}$ (1 hour)

leak $\Rightarrow x \Rightarrow \frac{1}{x}$ (1 hour)

$\frac{1}{24} - \frac{1}{x} = \frac{1}{36} \Rightarrow \frac{1}{x} = \frac{1}{24} - \frac{1}{36}$

$= \frac{3-2}{72} \Rightarrow \frac{1}{72}$

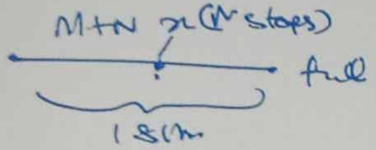
$x = 72$ hours

4) Two pipes M, N can fill a tank in 22.5 and 15 mins, respectively. If both pipes are opened simultaneously, after how much time should N be closed so the tank is full in 18 mins?

<u>Soln</u> >

$N \rightarrow 15 \rightarrow \frac{1}{15}$ per min

$M \rightarrow 22.5 \rightarrow \frac{1}{22.5}$ p/m



→ in this N is closed but M works at all time

so whole work = work by M ~~and~~ + work (N)

M work per min $\times 18 \Rightarrow \frac{1}{22.5} \times 18$

$= \frac{18}{22.5}$

whole work = 1

so N work $= 1 - \frac{18}{22.5}$

$= \frac{4.5}{22.5}$ → (N alone)

~~whereas~~ two N work $\frac{1}{15}$ per min

$\frac{1}{15} \times x \text{ min} = \frac{4.5}{22.5}$

$x = \frac{4.5}{22.5} \times 15 = 3$

Time > $\boxed{x = 3}$

So N closes after 3 minutes

5) Pipe A can fill a tank 5 times faster than pipe B and takes 32 mins less than Pipe B to fill tank. If both pipes are opened together, then in how much time the tank would be full?

Soln: Time:

$B = 5A$

also $B - 32 = A$

$5A - 32 = A$

$4A = 32$

$A = 8$ min $\Rightarrow$ time taken by Pipe A

$B = 5(A) \Rightarrow 40$ mins Pipe B.

Pipe B $\rightarrow \frac{1}{40}$ per min

A $\rightarrow \frac{1}{8}$ per min

Together B+A $\rightarrow \frac{1}{8} + \frac{1}{40} \rightarrow \frac{6}{40} \Rightarrow \frac{3}{20}$ per min

Time take to fill tank $(A+B) = \frac{20}{3}$ mins //

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6) A tank has 3 types, P, Q, R. Tap P and Q can fill tank in 1.5 and 2hr respectively. Tap R can empty filled tank in $\frac{1}{2}$ half hour. Tap P is opened at 8am., tap Q opened at 9 am, R at 10 am what time tank is empty?

Soln. Let T be the hours after tank is empty.

As P is started from start-8am $\Rightarrow$ Time of P = T

(9am) Q one hour later $\rightarrow$ Q = T-1

(10am) R $\rightarrow$ R = T-2

P - 15 hrs → $\frac{1}{15}$ → $\frac{1}{3/2}$ → $\frac{2}{3}$ in 1 hr

Q → 2 hrs → $\frac{1}{2}$ in 1 hr

R → $\frac{1}{2}$ hr → $\frac{1}{1/2}$ → 2 in 1 hr

At last tank is empty so total work → $W_P + W_Q + (-W_R) = 0$ (leak)

$W_P - W_Q, W_R$ is total work

total work is work per hour × time taken

→ Time × work/hr

$W_P + W_Q - W_R = 0$

$\left(\frac{2}{3}T\right) + \frac{1}{2}\times(T-1) - 2(T-2) = 0$

$\frac{2T}{3} + \frac{T-1}{2} - \frac{2(T-2)}{1} = 0$

$\frac{4T + 3T - 3 - 12T + 24}{6} = 0$

$4T + 3T - 3 - 12T + 24 = 0 \Rightarrow -5T + 21 = 0$

$-5T + 21 = 0$

$11T = 21$

$T = \frac{21}{11}$

$T = \frac{21}{5} = 4.2$ hrs

$T = 4.2$ hrs

4.2 hrs = 4 hrs 12 mins

start time → 9 am

9 am + 4.2 hrs → 12 pm 12 mins

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7) A cistern can be filled in 6 hrs by taps P, Q. If tap R also joins them, then cistern is filled in 5 hours. Tap P can fill the cistern at twice rate of tap Q. In what time tap Q and R fill the cistern?

Sol:-

Let P take phours to fill $\Rightarrow \frac{1}{P}$ in 1hr

Q " Q $\Rightarrow \frac{1}{Q}$ in 1hr

R " R $\Rightarrow \frac{1}{R}$ in 1hr

P+Q = $\frac{1}{P} + \frac{1}{Q}$

given P+Q completes in 6hrs $\Rightarrow$ some one hour $\Rightarrow \frac{1}{6}$ work

P+Q = $\boxed{\frac{1}{P} + \frac{1}{Q} = \frac{1}{6}}$ —①

P, Q, R take 5hrs $\Rightarrow \frac{1}{5}$ work per hour

$\boxed{\frac{1}{P} + \frac{1}{Q} + \frac{1}{R} = \frac{1}{5}}$ —②

① in ②

$\frac{1}{6} + \frac{1}{R} = \frac{1}{5} \Rightarrow \frac{1}{R} = \frac{1}{5} - \frac{1}{6}$

$\boxed{\frac{1}{R} = \frac{1}{30}}$

R completely fill in 30hours. //

Now also given P = 2Q

$\frac{1}{P} = 2\left(\frac{1}{Q}\right)$

w.k.t $\frac{1}{P} + \frac{1}{Q} = \frac{1}{6} \Rightarrow \frac{2}{Q} + \frac{1}{Q} = \frac{1}{6}$

$\frac{3}{Q} = \frac{1}{6} \Rightarrow \boxed{\frac{1}{Q} = \frac{1}{18}}$

To find work done time taken to fill by both Q, R together

one hour $\Rightarrow \frac{1}{Q} + \frac{1}{R} \Rightarrow \frac{1}{18} + \frac{1}{30} \Rightarrow \frac{4}{90}$

one hour $= \frac{4}{45}$

No of hour to fill = $\frac{45}{4}$ hours //

8) A cistern is filled by Pipe A and Pipe B together in 2.4 hours. Pipe A alone can fill the cistern at rate of 100 litres per hours. Pipe B alone can fill cistern in 4 hours. What is capacity of cistern?

Solu: let A → A hours → one hour → $\frac{1}{A}$

B → 4 hrs → one hour → $\frac{1}{4}$

$A+B = \frac{1}{A}+\frac{1}{4}$

A, B take 2.4 hrs so one hour → $\frac{1}{2.4}$

$\frac{1}{A}+\frac{1}{4}=\frac{1}{2.4}$

$\frac{1}{A}=\frac{1}{2.4}-\frac{1}{4}$

$= \frac{4-2.4}{2.4(4)} \Rightarrow \frac{1.6}{9.6} \rightarrow \frac{1}{6}$.

$\frac{1}{A}=\frac{1}{6}$ in one hour

A only → 6 hours.

B → 4 hours

A → 100 l/hr rate

Time taken for A to fill cis → $\Rightarrow 6 \times 100 \Rightarrow 600$ litres

Too So cistern capacity = 600 litres.

9) Pipe R can empty full tank in 30 hours. But 2 pipes P and Q can fill a tank in 15 hours and 10 hours respectively. Ram unknowingly opened all 3 taps. After 2 hours Shyam realised it and closed Pipe R. Due to this mistake how much time more would it take to fill tank?

Soln:- $P \rightarrow 15hr \Rightarrow \frac{1}{15}$

$Q \rightarrow 10hr \Rightarrow \frac{1}{10}$

$R \rightarrow 30 \Rightarrow \frac{1}{30}$ (-)

if P, Q opened $P+Q = \frac{1}{15}+\frac{1}{10} \Rightarrow \frac{5}{30}$

so total hour $\Rightarrow \frac{30}{5} \Rightarrow 6$ hours to fill

Ram mistake for 2 hour

$P+Q-R \Rightarrow \frac{1}{15}+\frac{1}{10}-\frac{1}{30} \Rightarrow \frac{2+3-1}{30} \Rightarrow \frac{4}{30}$

$\Rightarrow \frac{1}{5}$ hr

2 hr? $\Rightarrow 2 \times \frac{4}{30} \Rightarrow \frac{8}{30}$ work //

how much remaining work $\Rightarrow 1-\frac{8}{30}$

$\Rightarrow \frac{22}{30}$

P, Q need to do this $\Rightarrow \frac{22}{30}$

P, Q in 1 hr $\Rightarrow \frac{5}{30}$

$\frac{5}{30}x = \frac{22}{30} \Rightarrow x = \frac{22 \times 30}{30 \times 5}$

$x = 4.4$ hrs

More time taken $\Rightarrow 4.4$ hrs //

total time $\Rightarrow 2+4.4$ hrs $\Rightarrow 6.4$ hrs //

How much extra = Real time taken − P, Q time takes ~~without~~ ~~2 hours for R~~

= ~~6 – 6.4~~ 6.4 − 6

= 0.4 hours

= 24 mins //

extra needed to fill tank //