INFERENTIAL STATISTICS PROJECT REPORT

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PROJECT REPORT

Contents

List of figures

List of Tables

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?

SOLUTION:

$$P(random\ chosen\ player\ is\ injured) = \frac{Total\ no.of\ injured\ players}{Total\ no.of\ players}$$

Based on the above given data,

Total number of injured players = 145

Total number of players = 235

$$\boxed{P(random\ chosen\ player\ is\ injured) = \frac{145}{235} = 0.617}$$

1.2 What is the probability that a player is a forward or a winger?

SOLUTION:
Since a player cannot be a forward or a winger simultaneously, this is a mutually exclusive event

$$P(A\ player\ is\ a\ forward\ or\ a\ winger) = P\left(Forward\ player \bigcup Winger\ player\right)$$

$$= P(Forward\ player) + P(Winger\ player)$$

Based on the above given data,

Total number of forward players = 94

Total number of winger players = 29

Total number of players = 235

$$P(A\ player\ is\ a\ forward\ or\ a\ winger) = \frac{Total\ no.\ of\ Forward\ players}{Total\ no.of\ players} + \frac{Total\ no.\ of\ Winger\ players}{Total\ no.of\ players}$$

$$= \frac{94}{235} + \frac{29}{235} = \frac{94+29}{235} = \frac{123}{235} = 0.523$$

$$\boxed{P(A\ player\ is\ a\ forward\ or\ a\ winger) = 0.523}$$

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

SOLUTION:
The probability that a randomly chosen player is a striker and has a foot injury is a **joint probability** because it is based on the value which is in the centre of the table

$$P(random\ chosen\ player\ is\ striker\ and\ injured) = \frac{Total\ no.of\ injured\ striker\ players}{Total\ no.of\ players}$$

Based on the above given data,

Total number of injured striker players = 45

Total number of players = 235

$$\boxed{P(random\ chosen\ player\ is\ striker\ and\ injured) = \frac{45}{235} = 0.191}$$

1.4 What is the probability that a randomly chosen injured player is a striker?

SOLUTION:
The probability that a randomly chosen player being a striker given that he is injured is a **conditional probability**

$$P(random\ chosen\ injured\ player\ is\ striker) = \frac{Total\ no.of\ injured\ striker\ players}{Total\ no.of\ injured\ players}$$

Based on the above given data,

Total number of injured striker players = 45

Total number of injured players = 145

$$P(random\ chosen\ striker\ player\ is\ injured) = \frac{45}{145} = 0.310$$

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

SOLUTION:

Based on the given information,

The breaking strength of gunny bags used for cement packaging is normally distributed, $X \sim N(\mu, \sigma)$, with mean, $\mu = 5\ \frac{kg}{cm^2}$ and standard deviation, $\sigma = 1.5\ \frac{kg}{cm^2}$

$$P(X < 3.17) = 0.1112$$

There is 11.12% chance that the breaking strength of the gunny bags will be less than 3.17 kg per sq cm

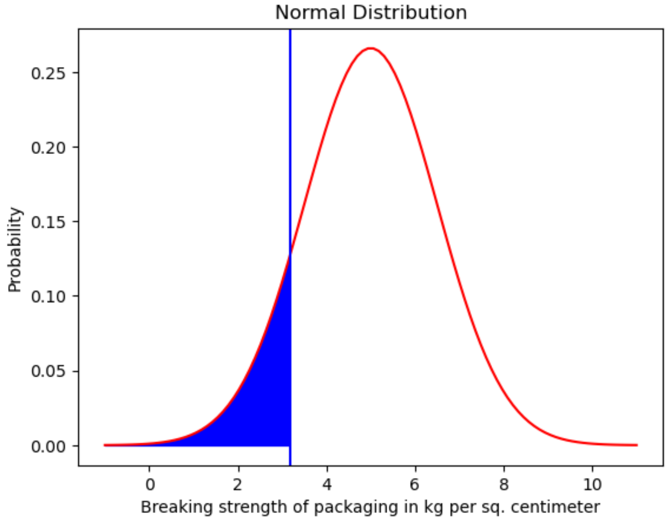

Figure 1: Visual representation of breaking strength of gunny bags less than 3.17 kg per sq.cm

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

SOLUTION:

Based on the given information,
The breaking strength of gunny bags used for cement packaging is normally distributed, $X \sim N(\mu, \sigma)$, with mean, $\mu = 5\ \frac{kg}{cm^2}$ and standard deviation, $\sigma = 1.5\ \frac{kg}{cm^2}$

$$P(X \geq 3.6) = 1 - P(X < 3.6) \ = \ 0.8247$$

There is 82.47% chance that the breaking strength of the gunny bags will be at least 3.6 kg per sq cm

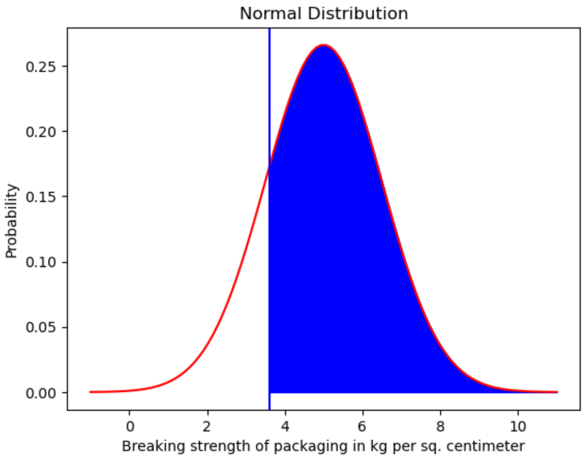

Figure 2: Visual representation of breaking strength of gunny bags of at least 3.6 kg per sq.cm

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

SOLUTION:

Based on the given information,
The breaking strength of gunny bags used for cement packaging is normally distributed, $X \sim N(\mu, \sigma)$, with mean, $\mu = 5\ \frac{kg}{cm^2}$ and standard deviation, $\sigma = 1.5\ \frac{kg}{cm^2}$

$$P(5 < X < 5.5) = P(X < 5.5) - P(X < 5) \ = \ 0.1306$$

There is 13.06% chance that the breaking strength of the gunny bags will be between 5 and 5.5 kg per sq cm

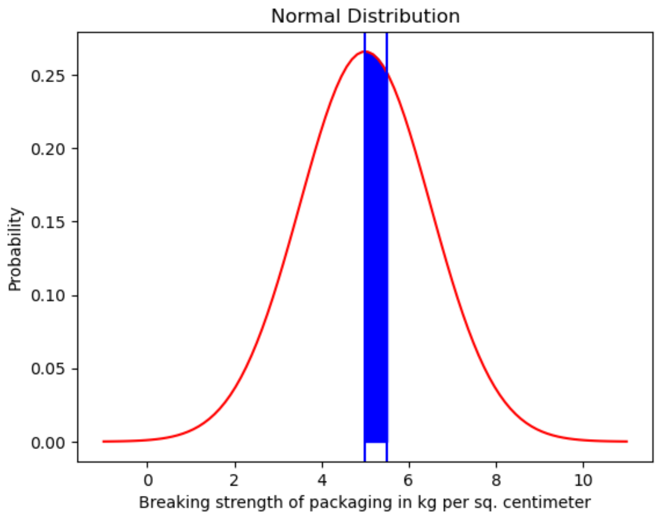

Figure 3: Visual representation of breaking strength of gunny bags of between 5 and 5.5 kg per sq.cm

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

SOLUTION:

Based on the given information,

The breaking strength of gunny bags used for cement packaging is normally distributed, $X \sim N(\mu, \sigma)$, with mean, $\mu = 5 \, \frac{kg}{cm^2}$ and standard deviation, $\sigma = 1.5 \, \frac{kg}{cm^2}$

$$\boxed{1 - P(3 < X < 7.5) = 1 - (P(X < 7.5) - P(X < 3)) \ = \ 0.139}$$

There is 13.90% chance that the breaking strength of the gunny bags will NOT be between 3 and 7.5 kg per sq cm

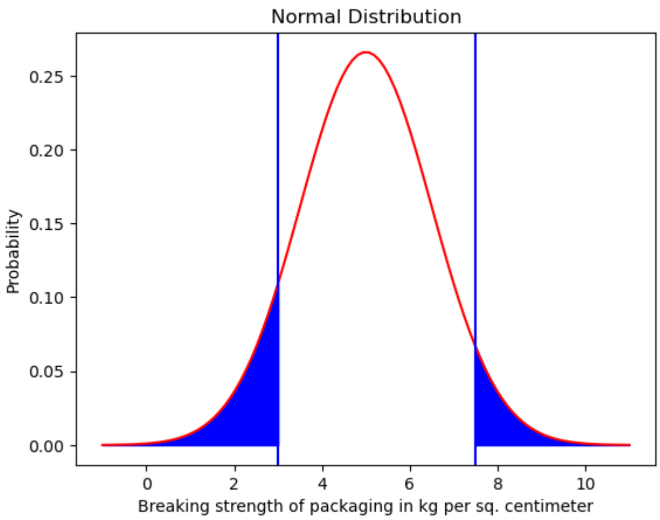

Figure 4: Visual representation of breaking strength of gunny bags of NOT between 3 and 7.5 kg per sq.cm

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

STEP 1: Loading of the data set

STEP 2: Getting the overview of the data

- The dataset contains the details of the **Brinell's hardness index of both the unpolished and polished stones**
- The dataset contains the information about a sample of **75 stones**
- The columns 'Unpolished', and 'Treated and Polished' are of float type (numerical)

STEP 3: Checking the statistical summary of the data

	Unpolished	Treated and Polished
count	75.000000	75.000000
mean	134.110527	147.788117
std	33.041804	15.587355
min	48.406838	107.524167
25%	115.329753	138.268300
50%	135.597121	145.721322
75%	158.215098	157.373318
max	200.161313	192.272856

Table 1: Statistical summary of the Zingaro company's data

- Overall, the average Brinell's hardness index for the sample of 75 stones have increased by 10.2% to approximately 147.79 with treating and polishing of stones
- The maximum Brinell's hardness index from polishing the stones is 192.27, which is 3.9% lesser than the maximum output from the not polishing the stones
- The minimum Brinell's hardness index from polishing the stones has an increase by 122.13%
- With polishing the stones, 50% of the stones have a hardness index of atleast 145.72, which has increased by 7.46% compared to not treating them
- It can be seen that there is more variation in treating the stones as compared to not treating them

STEP 4: Visual analysis of Brinell's hardness index

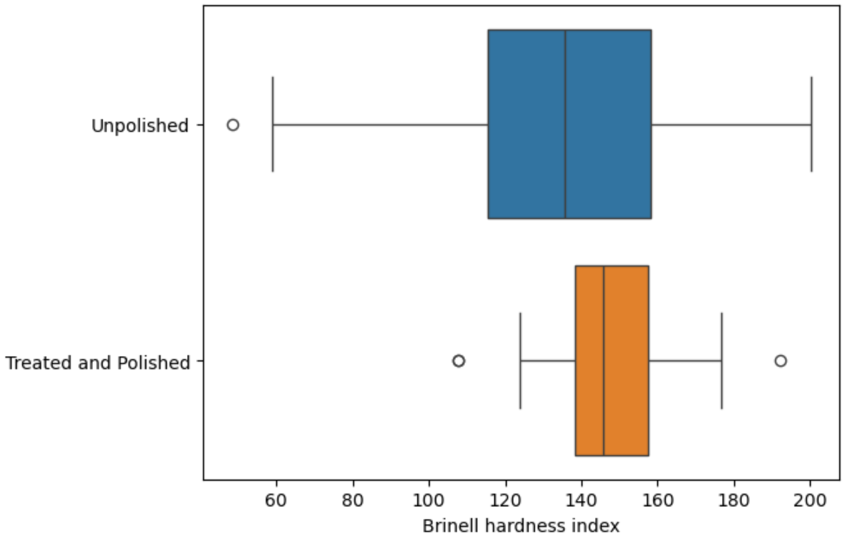

Figure 5: Visual analysis of Brinell's hardness index

- The distribution of outputs for the two varieties of stones are different and a slight increase is observed in the median value of treating and polishing the stones as compared to not polishing them
- The Brinell's hardness index of 75% of the stones remains the same both the two treatments, however the maximum value of unpolished is higher compared to the maximum value of polished varieties
- Outliers can be observed in both varieties of stones, suggesting that some stones had unusually better or worse Brinell's hardness index than the others
- A statistical test is used to justify that the unpolished stones are not suitable for printing

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

SOLUTION:

STEP 1: Null and Alternate Hypotheses

Zingaro stone printing company believes that the unpolished stones (one sample) may not be suitable for printing. The Brinell's hardness index of atleast 150 is considered to be the optimum level for printing of the image on the stone surface.

Let μ be the optimum level of Brinell's hardness index

The null hypothesis

$$H_0: \mu \geq 150$$

need to be tested against the alternate hypothesis

$$H_a: \mu < 150$$

The Brinell's hardness index is measured on a continuous scale. The population is normally distributed and the sample size is 75 > 30. The population standard deviation is not known. The population being normally distributed, is there enough evidence at $\alpha = 0.005$ to support that the Zingaro's claim is false?

We can use T-test for this problem

STEP 2: Find the p_value

The p-value is 4.171286997419652e-05
The test_stat is -4.164629601426757

STEP 3: Insight

As the p-value is much less than the level of significance, we can **reject the null hypothesis**, that is Brinell's hardness index is not greater or equal to 150 for **Unpolished stones**. Hence, we do not have enough significance to conclude that the unpolished stones have the optimal Brinell's hardness index.

Thus, the Zingaro's claim that the unpolished stones not being suitable for printing of images is ***TRUE***

3.2 Is the mean hardness of the polished and unpolished stones the same?

SOLUTION:

STEP 1: Find the mean and standard deviation of the unpolished and polished stones

The **mean** Brinell hardness index for **Unpolished stones** is **134.11052653373332**
The **mean** Brinell hardness index for **Treated and Polished stones** is **147.78811718133335**
The **standard deviation** of Brinell hardness index for **Unpolished stones** is **33.04**
The **standard deviation** of Brinell hardness index for **Treated and Polished stones** is **15.59**

STEP 2: Check for T-test assumptions

- Continuous data - Yes, the Brinell hardness index is measured on a continuous scale.
- Normally distributed populations - Yes, we are informed that the populations are assumed to be normal.
- Independent populations - As we are taking random samples for two different groups, the two samples are from two independent populations.
- Unequal population standard deviations - As the sample standard deviations are different, the population standard deviations may be assumed to be different.
- Random sampling from the population - Yes, we are informed that the collected sample a simple random sample.

We can use two sample T-test for this problem.

STEP 3: Null and Alternate Hypotheses

Let μ_1, μ_2 be he mean Brinell's hardness index values of **Unpolished** and **Treated and Polished** stones respectively.

We will test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

against the alternate hypothesis

$$H_a: \mu_1 \neq \mu_2$$

STEP 4: Find the p_value

The p-value is 0.001588379295584306

STEP 5: Insight

As the p-value (~0.0016) is less than the level of significance ($\alpha = 0.005$), we can **reject the null hypothesis**. Hence, we do not have enough evidence to support the claim that the unpolished and polished stones have the same population mean Brinell hardness index values. **So, the mean hardness of the polished and unpolished stones are not the same.**

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

STEP 1: Loading of the data set

STEP 2: Getting the overview of the data

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Table 2: First 5 rows of the loaded dataset

The dataset reflects the response of interests from various patients with respect to the hardness of metal implants in dental cavities. The hardness depends on multiple factors such as implant methods, alloys used, dentist's favour and the metal's temperature. However the metal's temperature is just an observation.

STEP 3: Checking the statistical summary of the data

	Dentist	Method	Alloy	Temp	Response
count	90.000000	90.000000	90.000000	90.000000	90.000000
mean	3.000000	2.000000	1.500000	1600.000000	741.777778
std	1.422136	0.821071	0.502801	82.107083	145.767845
min	1.000000	1.000000	1.000000	1500.000000	289.000000
25%	2.000000	1.000000	1.000000	1500.000000	698.000000
50%	3.000000	2.000000	1.500000	1600.000000	767.000000
75%	4.000000	3.000000	2.000000	1700.000000	824.000000
max	5.000000	3.000000	2.000000	1700.000000	1115.000000

Table 3: Statistical summary of the Dental implant dataset

- The dataset produces the response of interests from **90 patients** with dental cavities regarding the hardness level of metal implants
- There are a maximum of **5 dentists** following **3 methods of treatment** using either of the **2 alloys** as metal to implant at the dental cavity
- It is also seen that the method, alloy and dentist are taken as numbers whereas they are labels, they are not values but they are categories.

STEP 4: Checking the information of the data

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 90 entries, 0 to 89
Data columns (total 5 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Dentist   90 non-null     int64
 1   Method    90 non-null     int64
 2   Alloy     90 non-null     int64
 3   Temp      90 non-null     int64
 4   Response  90 non-null     int64
dtypes: int64(5)
memory usage: 3.6 KB
```

Table 4: Information about the columns of Dental implant dataset

It is clear that the Dentist, Method and Alloy are considered as integer Data types which need to be changed into Categorical variables.

STEP 5: Converting the label columns from integer to categorical variables

4.1 How does the hardness of implants vary depending on dentists?

RUBRIC - State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."

SOLUTION:

EXPLORATORY DATA ANALYSIS

Value counts of different types of dentists

```
Dentist
1    18
2    18
3    18
4    18
5    18
Name: count, dtype: int64
```

Table 5: Value counts of dentists

- Based on the information of the data, there are Five different Dentists: 1, 2, 3, 4 and 5
- In the sample of 90 people, each 18 were treated of every one of the five dentists

Value counts of different types of alloys

```
Alloy
1    45
2    45
Name: count, dtype: int64
```

Table 6: Value counts of alloys

- Based on the information of the data, there are 2 different alloys used to treat dental cavities: 1, and 2
- In the sample of 90 people, each 45 were treated with either one of the alloys

Visualize the data

Visual analysis of the responses for the five different dentists

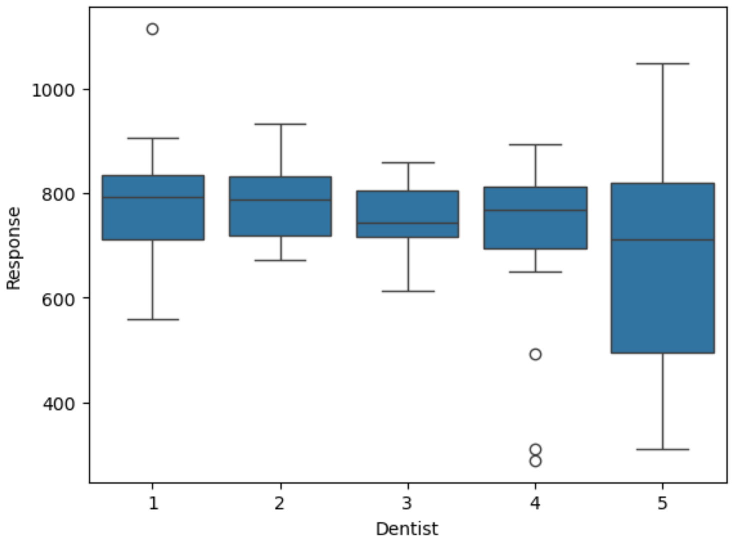

Figure 6: Visual analysis of responses for dentists

Visual analysis of the responses for the five different dentists with hue based on 'Alloy' column

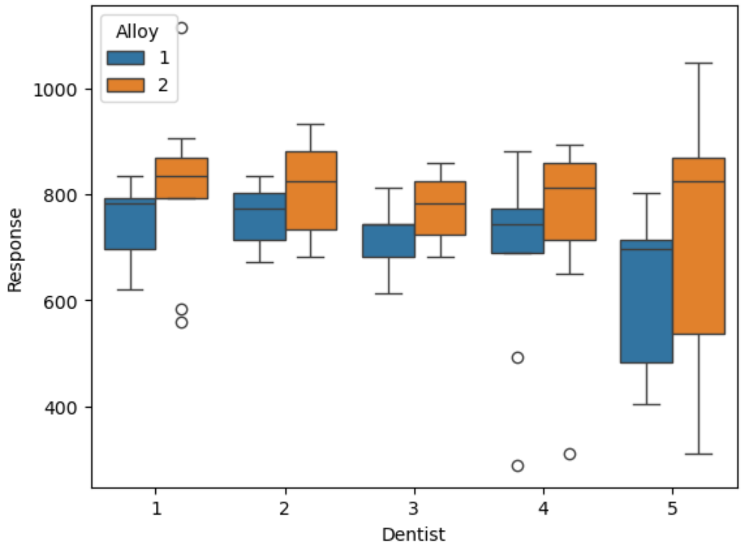

Figure 7: Visual analysis of responses for dentists based on alloys

- The distribution of responses seem to differ among 2 different types of alloys
- The response with alloy 2 treatment is high compared to the response with alloy1
- The median value of responses with alloy 2 looks similar with dentist 1, 2 and 4,5
- The difference is observed in responses among the dentists. To determine this we will test the differences using the statistical tests

STEP 1: Null and Alternate Hypotheses

Given that two different alloys say "Alloy-1" and "Alloy-2" are used by 5 different dentists to treat dental cavities through implants, the dental implant hardness can be hypothesized for both the alloys as follows:

The null and alternate hypothesis can be formulated as:

H_0: The mean responses with respect to **each dentist is equal**

$$H_0: \mu_{\text{dentist1}} = \mu_{\text{dentist2}} = \mu_{\text{dentist3}} = \mu_{\text{dentist4}} = \mu_{\text{dentist5}}$$

H_a: At least one of the means with respect to the **five dentist is different**

$$H_a: \mu_{\text{dentist1}} \neq \mu_{\text{dentist2}} \neq \mu_{\text{dentist3}} \neq \mu_{\text{dentist4}} \neq \mu_{\text{dentist5}}$$

STEP 2: Select the appropriate test

Check the assumptions of the hypothesis test

This is a problem, concerning five population means. ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

ANOVA test

In an ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.

- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene's test is applied to the response variable.

Shapiro-Wilk's test – Alloy1

We will test the null hypothesis

H_0: The responses follow a normal distribution

Against the alternative hypothesis

H_a: The responses do not follow a normal distribution

The p-value is 1.1945308699072215e-05

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene's test- – Alloy1

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

Shapiro-Wilk's test – Alloy2

We will test the null hypothesis

H_0: The responses follow a normal distribution

Against the alternative hypothesis

H_a: The responses do not follow a normal distribution

The p-value is 0.00040293129942514585

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene's test- – Alloy2

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

The test for ANOVA fails for two different alloys

- The response does not follow normal distribution
- All the population variances are not equal

We proceed with the test by designing the formula and using the Ordinary Least Squares algorithm

Considering the mean implant hardness response for both the alloys together

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	1.577946e+05	39448.638889	1.934537	0.112066
Residual	85.0	1.733301e+06	20391.776471	NaN	NaN

Table 7: P-value for mean implant hardness response for both alloys together

Considering two alloys together

The p-value is 0.112066

Since p-value of the test is greater than the 5% significance level, **we fail to reject the null hypothesis that mean response of five different dentists are same.**

Visualize the plot of mean response among different dentists with respect to alloys

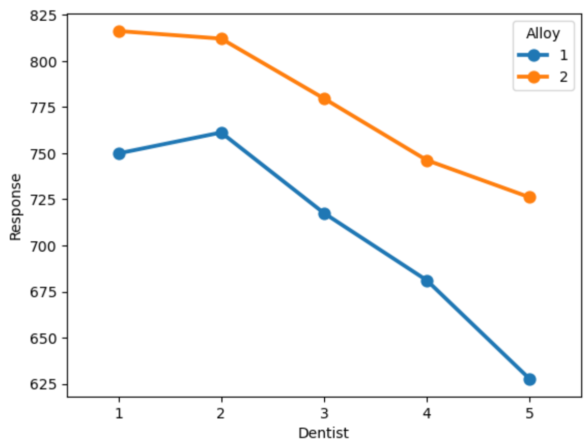

Figure 8: Visual analysis of mean responses for dentists based on alloys

STEP 3: Analysis of variation in hardness of implants depending on dentists while using ALLOY-1

<u>Considering the mean implant hardness response for dentists while treating with alloy-1</u>

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Table 8: P-value for mean implant hardness response for dentists while treating with alloy-1

<u>Considering the hardness implant response with respect to ALLOY-1</u>
The p-value is 0.116567
Since p-value of the test is greater than the 5% significance level, **we fail to reject the null hypothesis that mean implant hardness response for five different dentists are same while treating the dental cavities with alloy-1.**

STEP 4: Analysis of variation in hardness of implants depending on dentists while using ALLOY-2

<u>Considering the mean implant hardness response for dentists while treating with alloy-2</u>

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Table 9: P-value for mean implant hardness response for dentists while treating with alloy-2

<u>Considering the hardness implant response with respect to ALLOY-2</u>
The p-value is 0.718031
Since p-value of the test is greater than the 5% significance level, **we fail to reject the null hypothesis that mean implant hardness response for five different dentists are same while treating the dental cavities with alloy-2.**

Thus the hardness of implants does not vary with dentists irrespective of the alloys used to treat the dental cavities

4.2 How does the hardness of implants vary depending on methods?

RUBRIC - State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."

SOLUTION:

<u>EXPLORATORY DATA ANALYSIS</u>

<u>Value counts of different types of methods</u>

```
Method
1    30
2    30
3    30
Name: count, dtype: int64
```

Table 10: Value counts of methods

- Based on the information of the data, there are Three different Methods: 1, 2, and 3
- In the sample of 90 people, each 30 were treated with any one of the three methods

<u>Value counts of different types of alloys</u>

```
Alloy
1    45
2    45
Name: count, dtype: int64
```

Table 11: Value counts of alloys

- Based on the information of the data, there are 2 different alloys used to treat dental cavities: 1, and 2
- In the sample of 90 people, each 45 were treated with either one of the alloys

<u>Visualize the data</u>

<u>Visual analysis of the responses for the three different methods</u>

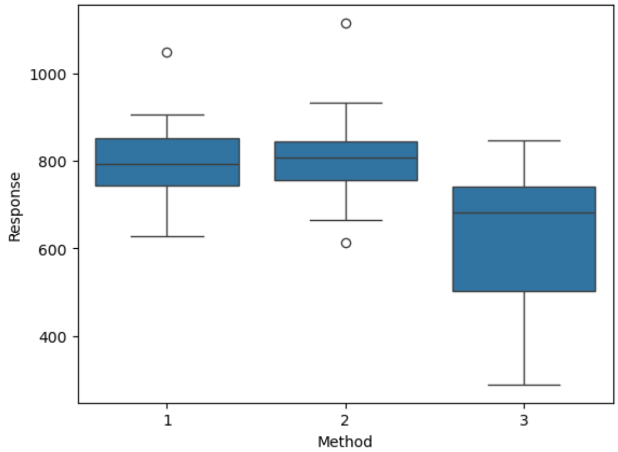

Figure 9: Visual analysis of responses for methods

Visual analysis of the responses for the three different methods with hue based on 'Alloy' column

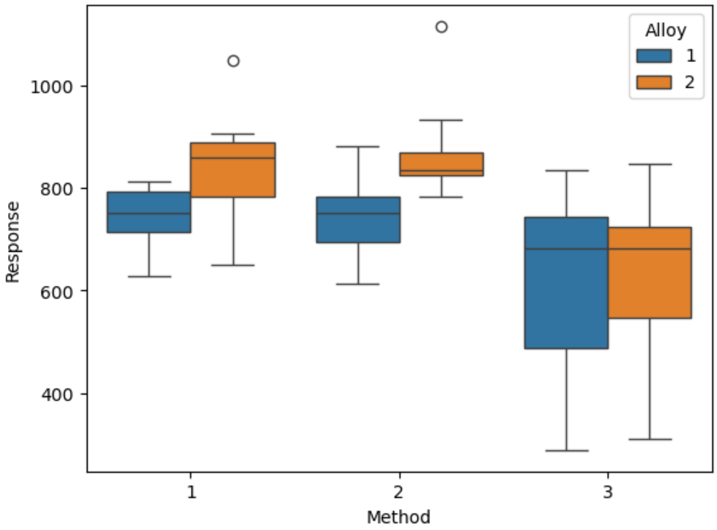

Figure 10: Visual analysis of responses for methods based on alloys

- The distribution of responses seem to differ among 2 different types of alloys
- The response with alloy 2 treatment is high compared to the response with alloy1 in methods 1 and 2, whereas with method 3 the response is similar
- The median value of responses with alloy 2 looks similar with method 3
- The difference is observed in responses among the methods. To determine this we will test the differences using the statistical tests

STEP 1: Null and Alternate Hypotheses

Given that two different alloys say "Alloy-1" and "Alloy-2" are used by 3 different methods to treat dental cavities through implants, the dental implant hardness can be hypothesized for both the alloys as follows:

The null and alternate hypothesis can be formulated as:

H_0: The mean responses with respect to **each method is equal**

$$H_0: \mu_{\text{method1}} = \mu_{\text{method2}} = \mu_{\text{method3}}$$

H_a: At least one of the means with respect to the **three methods is different**

$$H_a: \mu_{\text{method1}} \neq \mu_{\text{method2}} \neq \mu_{\text{method3}}$$

STEP 2: Select the appropriate test

Check the assumptions of the hypothesis test

This is a problem, concerning three population means. ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

ANOVA test

In an ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.

- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene's test is applied to the response variable.

Shapiro-Wilk's test – Alloy1

We will test the null hypothesis

H_0: The responses follow a normal distribution

Against the alternative hypothesis

H_a: The responses do not follow a normal distribution

The p-value is 1.1945308699072215e-05

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene's test- – Alloy1

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

Shapiro-Wilk's test – Alloy2

We will test the null hypothesis
H_0: The responses follow a normal distribution
Against the alternative hypothesis
H_a: The responses do not follow a normal distribution

The p-value is 0.00040293129942514585

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene’s test- – Alloy2

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

The test for ANOVA fails for two different alloys

- The response does not follow normal distribution
- All the population variances are not equal

We proceed with the test by designing the formula and using the Ordinary Least Squares algorithm

Considering the mean implant hardness response for both the alloys together

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	5.934275e+05	296713.744444	19.89268	7.683892e-08
Residual	87.0	1.297668e+06	14915.724904	NaN	NaN

Table 12: P-value for mean implant hardness response for both alloys together

Considering two alloys together

The p-value is 7.683892e-08

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that mean response of three different methods are same.**

Visualize the plot of mean response among different methods with respect to alloys

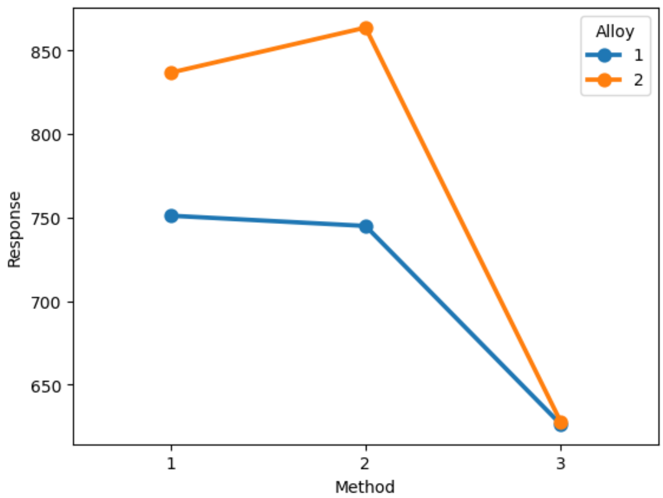

Figure 11: Visual analysis of mean responses for methods based on alloys

STEP 3: Analysis of variation in hardness of implants depending on methods while using ALLOY-1

Considering the mean implant hardness response for methods while treating with alloy-1

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Table 13: P-value for mean implant hardness response for methods while treating with alloy-1

Considering the hardness implant response with respect to ALLOY-1
The p-value is 0.004163
Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that mean implant hardness response for three different methods are same while treating the dental cavities with alloy-1.**

STEP 4: Analysis of variation in hardness of implants depending on methods while using ALLOY-2

Considering the mean implant hardness response for methods while treating with alloy-2

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Table 14: P-value for mean implant hardness response for methods while treating with alloy-2

Considering the hardness implant response with respect to ALLOY-2
The p-value is 0.000005
Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that mean implant hardness response for three different methods are same while treating the dental cavities with alloy-2.**

Thus the hardness of implants vary with methods irrespective of the alloys used to treat the dental cavities

STEP 5: In case the implant hardness differs, identifying for which pairs it differs

Multiple Comparison test (Tukey HSD)

In order to identify for which method mean response is different from other groups, the null hypothesis is

$$H_0: \mu_{\text{method1}} = \mu_{\text{method2}} \text{ and } \mu_{\text{method1}} = \mu_{\text{method3}} \text{ and } \mu_{\text{method2}} = \mu_{\text{method3}}$$

Against the alternative hypothesis

$$H_a: \mu_{\text{method1}} \neq \mu_{\text{method2}} \text{ and } \mu_{\text{method1}} \neq \mu_{\text{method3}} \text{ and } \mu_{\text{method2}} \neq \mu_{\text{method3}}$$

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True

Table 15: P-value for comparing means of methods while treating with alloy-1

While using ALLOY-1 for treating the dental cavities

- The response of hardness of dental implants **differs between method 1 and method 3**
- The response of hardness of dental implants also **differs between method 2 and method 3**
- The response of hardness of dental implants is not so different between method 1 and method 2

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

Table 16: P-value for comparing means of methods while treating with alloy-2

While using ALLOY-2 for treating the dental cavities

- The response of hardness of dental implants **differs between method 1 and method 3**
- The response of hardness of dental implants also **differs between method 2 and method 3**
- The response of hardness of dental implants is not so different between method 1 and method 2

Thus the hardness of implants with method3 is different from method 1 and method2 irrespective of using any of the alloys

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

RUBRIC - Create Interaction Plot - Inferences from the plot Note: Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys."

STEP 1: Null and Alternate Hypotheses

Given that two different alloys say "Alloy-1" and "Alloy-2" are used by 5 different dentists with 3 different methods to treat dental cavities through implants, the interaction effect between the dentists and methods on the hardness of dental implants can be hypothesized for both the alloys as follows:

The null and alternate hypothesis can be formulated as:

H_0: There is **no interaction effect between the dentist and the method**

H_a: There is **an interaction effect between the dentist and the method**

STEP 2: Analysis of interaction effect between dentist and the method while using ALLOY-1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Table 17: P-value for interaction effect between dentist and the method while treating with alloy-1

Considering the interaction effect between dentist and the method with respect to ALLOY-1

The p-value is 0.006793

- Since p-value of the test is less than the 5% significance level, we reject the null hypothesis
- So there is an interaction effect between the dentist and the methods used to treat the dental cavities while treating with alloy-1

The point plot in Figure 12 also shows that there is interactions between the dentist and the methods while using alloy-1 for treating the dental cavities

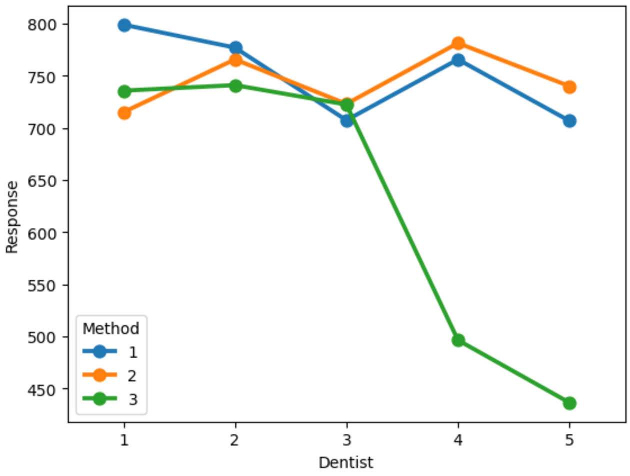

Figure 12: Point plot depicting interaction between dentist and method

STEP 3: Analysis of interaction effect between dentist and the method while using ALLOY-2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Table 18: P-value for interaction effect between dentist and the method while treating with alloy-2

Considering the interaction effect between dentist and the method with respect to ALLOY-2

The p-value is 0.093234

- Since p-value of the test is greater than the 5% significance level, we cannot reject the null hypothesis
- So there is no interaction effect between the dentist and the methods used to treat the dental cavities while treating with alloy-2

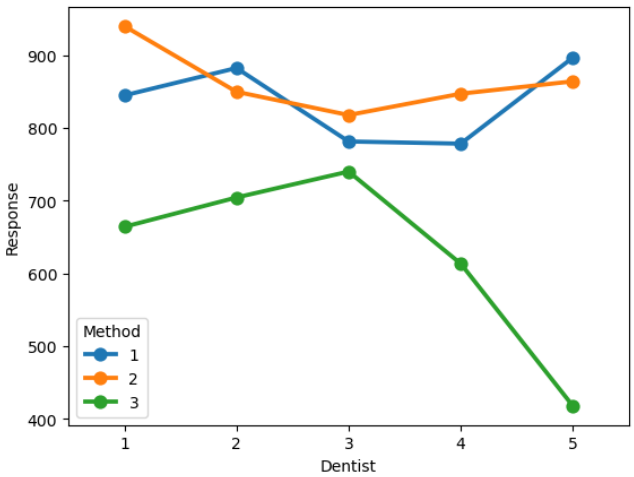

Figure 13: Point plot depicting no interaction between dentist and method

The point plot in Figure 13 also shows that there is no interactions between the dentist and the methods while using alloy-2 for treating the dental cavities

STEP 4: Create Interaction Plot

The analysis of the effect of both the ‘Dentist’ and ‘Method’ on the ‘Response’ variable is done using interaction plots

ALLOY-1

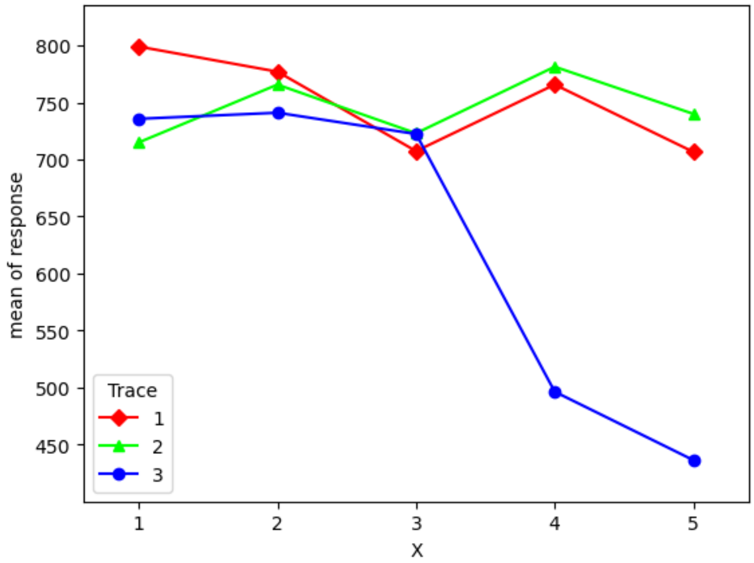

Figure 14: Interaction plot between dentist and method – alloy1

There is some sort of an interaction between the dentist and the methods used to treat the dental cavities while treating with alloy-1

ALLOY-2

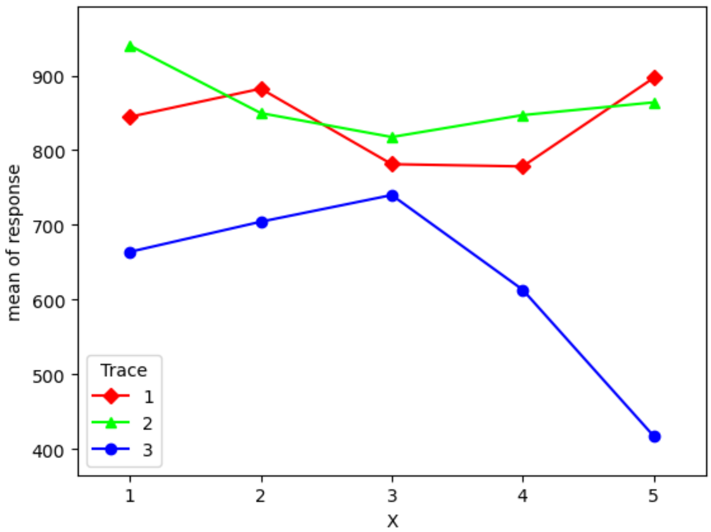

Figure 15: Interaction plot between dentist and method – alloy2

There is no interaction between the dentist and the methods used to treat the dental cavities while treating with alloy-2

4.4 How does the hardness of implants vary depending on dentists and methods together?

RUBRIC - State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - Identify which dentists and methods combinations are different, and which interaction levels are different. Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test.

SOLUTION:

EXPLORATORY DATA ANALYSIS

Count plot of different types of dentists

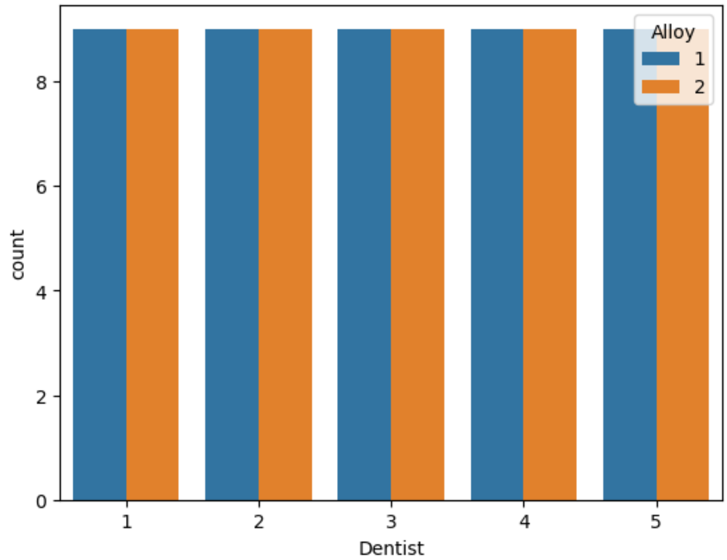

Figure 16: Count plot for different types of dentists with hue ='Alloy'

- Based on the information of the data, there are Five different Dentists: 1, 2, 3, 4 and 5
- In the sample of 90 people, each 9 were treated by any one of the five dentists using alloy-1 and each 9 were treated by any one of the five dentists using alloy-2

<u>Count plot of different types of methods</u>

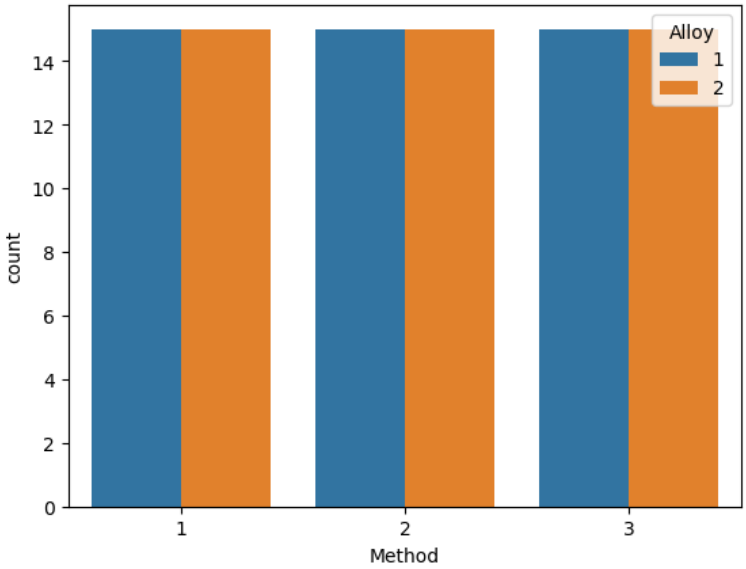

Figure 17: Count plot for different types of methods with hue ='Alloy'

- Based on the information of the data, there are Three different Methods: 1, 2, and 3
- In the sample of 90 people, each 15 were treated using any one of the three methods with alloy-1 and each 15 were treated using any one of the three methods with alloy-2

<u>Count plot of different types of alloys wrt dentists</u>

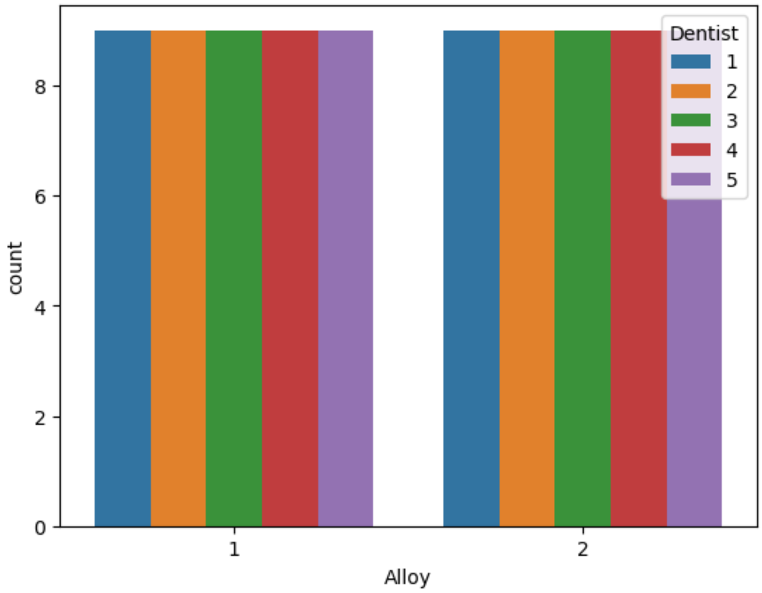

Figure 18: Count plot for different types of alloys with hue ='Dentist'

- Based on the information of the data, there are Two different Alloys: 1, and 2
- In the sample of 90 people, each 45 were implanted using alloy-1 by any one of the 5 dentists and each 45 were implanted using alloy-2 by any one of the 5 dentists

Count plot of different types of alloys wrt methods

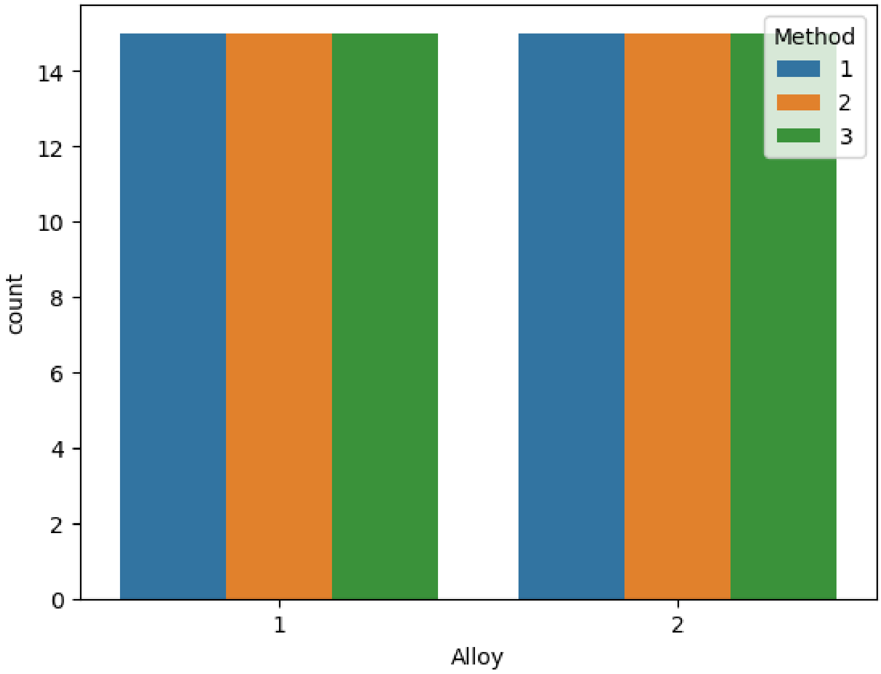

Figure 19: Count plot for different types of alloys with hue ='Method'

- Based on the information of the data, there are Two different Alloys: 1, and 2
- In the sample of 90 people, each 45 were implanted using alloy-1 by any one of the 3 methods and each 45 were implanted using alloy-2 by any one of the 3 methods

BIVARIATE ANALYSIS

Dentist vs Response (with alloy1)

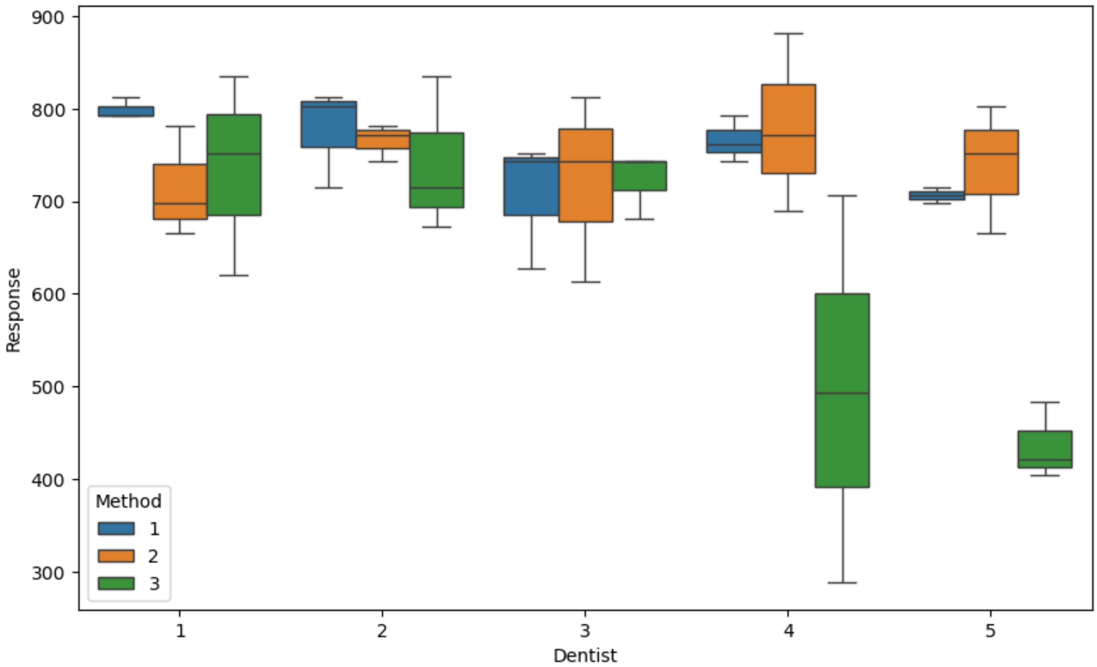

Figure 20: Box plot for Dentist vs Response with hue ='Method' (alloy1)

Dentist vs Response (with alloy2)

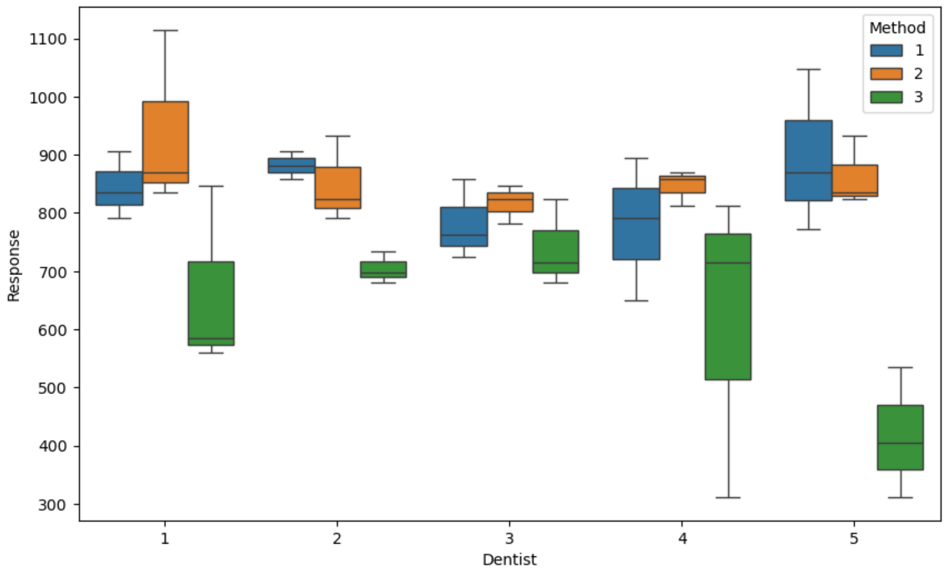

Figure 21: Box plot for Dentist vs Response with hue ='Method' (alloy2)

Method vs Response (with alloy1)

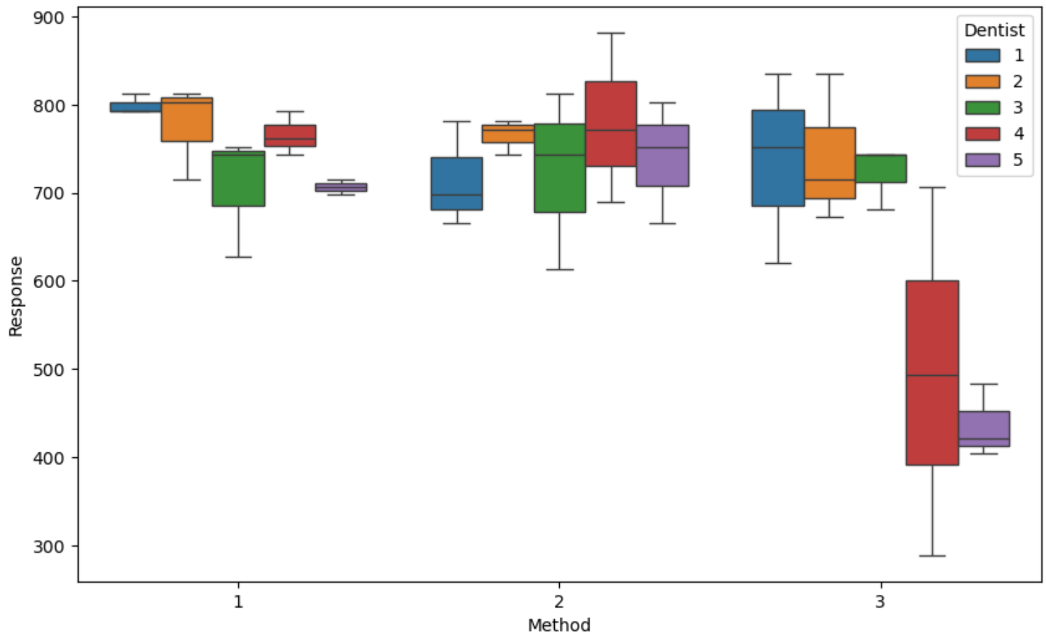

Figure 22: Box plot for Method vs Response with hue ='Dentist' (alloy1)

Method vs Response (with alloy2)

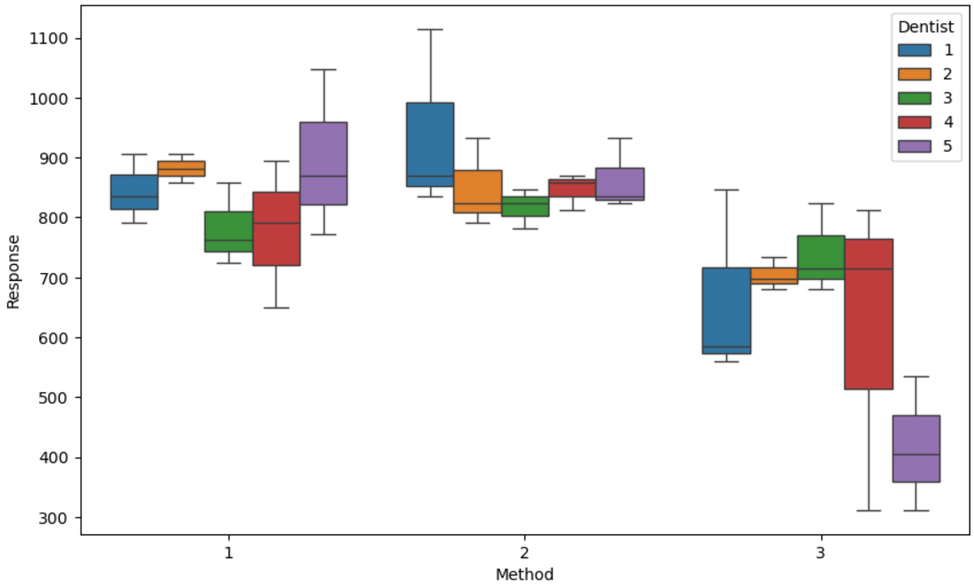

Figure 23: Box plot for Method vs Response with hue ='Dentist' (alloy2)

STEP 1: Null and Alternate Hypotheses

Given that two different alloys say "Alloy-1" and "Alloy-2" are used by 5 different dentists implementing 3 different methods to treat dental cavities through implants, the dental implant hardness can be hypothesized for both the alloys as follows:

The null and alternate hypothesis can be formulated as:

H_0: The mean responses to each **dentist along with the respective method** is equal.

$$\begin{aligned} H_0: \mu_{\text{dentist1\&method1}} &= \mu_{\text{dentist1\&method2}} = \mu_{\text{dentist1\&method3}} = \mu_{\text{dentist2\&method1}} \\ &= \mu_{\text{dentist2\&method2}} = \mu_{\text{dentist2\&method3}} = \mu_{\text{dentist3\&method1}} \\ &= \mu_{\text{dentist3\&method2}} = \mu_{\text{dentist3\&method3}} = \mu_{\text{dentist4\&method1}} \\ &= \mu_{\text{dentist4\&method2}} = \mu_{\text{dentist4\&method3}} = \mu_{\text{dentist5\&method1}} \\ &= \mu_{\text{dentist5\&method2}} = \mu_{\text{dentist5\&method3}} \end{aligned}$$

H_a: At least one of the mean responses with respect to the **5 dentists and 3 methods is different**.

$$\begin{aligned} H_a: \mu_{\text{dentist1\&method1}} &\neq \mu_{\text{dentist1\&method2}} \neq \mu_{\text{dentist1\&method3}} \neq \mu_{\text{dentist2\&method1}} \\ &\neq \mu_{\text{dentist2\&method2}} \neq \mu_{\text{dentist2\&method3}} \neq \mu_{\text{dentist3\&method1}} \\ &\neq \mu_{\text{dentist3\&method2}} \neq \mu_{\text{dentist3\&method3}} \neq \mu_{\text{dentist4\&method1}} \\ &\neq \mu_{\text{dentist4\&method2}} \neq \mu_{\text{dentist4\&method3}} \neq \mu_{\text{dentist5\&method1}} \\ &\neq \mu_{\text{dentist5\&method2}} \neq \mu_{\text{dentist5\&method3}} \end{aligned}$$

STEP 2: Select the appropriate test

Check the assumptions of the hypothesis test

This is a problem, concerning three population means. ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

ANOVA test

In an ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.

- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene's test is applied to the response variable.

Shapiro-Wilk's test – Alloy1

We will test the null hypothesis

H_0: The responses follow a normal distribution

Against the alternative hypothesis

H_a: The responses do not follow a normal distribution

The p-value is 1.1945308699072215e-05

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene's test- – Alloy1

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

Shapiro-Wilk's test – Alloy2

We will test the null hypothesis
H_0: The responses follow a normal distribution

Against the alternative hypothesis
H_a: The responses do not follow a normal distribution

The p-value is 0.00040293129942514585

Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that the response follows the normal distribution**.

Levene's test- – Alloy2

We will test the null hypothesis

H_0: All the population variances are equal

Against the alternative hypothesis

H_a: At least one variance is different from the rest

The p-value is NaN

Since p-value of the test is NaN, **we reject the null hypothesis that all the population variances are equal**

The test for ANOVA fails for two different alloys

- The response does not follow normal distribution
- All the population variances are not equal

We proceed with the test by designing the formula and using the Ordinary Least Squares algorithm

STEP 3: Analysis of variation in hardness of implants depending on dentists and methods together while using ALLOY-1

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	2.591255	0.051875
C(Method)	2.0	148472.177778	74236.088889	7.212522	0.002211
Residual	38.0	391121.377778	10292.667836	NaN	NaN

Table 19: P-value for variation in hardness implants depending on dentists and methods while treating with alloy-1

Considering the hardness implant response depending on dentists and methods together with respect to ALLOY-1

The p-value is 0.051875 with respect to DENTIST

- Since p-value of the test is greater than the 5% significance level, **we fail to reject the null hypothesis that mean implant hardness response for five different dentists are same while treating the dental cavities with alloy-1.**
- So there is **no significant variation in hardness of implants depending on dentists** while treating with alloy-1

Considering the hardness implant response depending on dentists and methods together with respect to ALLOY-1

The p-value is 0.002211 with respect to METHOD

- Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that mean implant hardness response for 3 different metho are same while treating the dental cavities with alloy-1.**
- So there is **significant variation in hardness of implants depending on methods** while treating with alloy-1

STEP 4: Analysis of variation in hardness of implants depending on dentists and methods together while using ALLOY-2

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	0.926215	0.458933
C(Method)	2.0	499640.400000	249820.200000	16.295479	0.000008
Residual	38.0	582564.488889	15330.644444	NaN	NaN

Table 20: P-value for variation in hardness implants depending on dentists and methods while treating with alloy-2

Considering the hardness implant response depending on dentists and methods together with respect to ALLOY-2

The p-value is 0.458933 with respect to DENTIST

- Since p-value of the test is greater than the 5% significance level, **we fail to reject the null hypothesis that mean implant hardness response for five different dentists are same while treating the dental cavities with alloy-2.**
- So there is **no significant variation in hardness of implants depending on dentists** while treating with alloy-2

The p-value is 0.000008 with respect to METHOD

- Since p-value of the test is less than the 5% significance level, **we reject the null hypothesis that mean implant hardness response for 3 different methods are same while treating the dental cavities with alloy-2.**
- So there is **significant variation in hardness of implants depending on methods** while treating with alloy-2

STEP 5: Identify which dentists and methods combinations are different, and which interaction levels are different.

<u>Multiple Comparison test (Tukey HSD) - Dentist combinations –(Alloy1)</u>

In order to identify for which method mean response is different from other groups, the null hypothesis is

H_0: The mean response for any pair of dentist combinations is the same while treating with alloy1

Against the alternative hypothesis
H_a: The mean response for at least one pair of dentist combinations is different while treating with alloy1

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	11.3333	0.9996	-145.0423	167.709	False
1	3	-32.3333	0.9757	-188.709	124.0423	False
1	4	-68.7778	0.7189	-225.1535	87.5979	False
1	5	-122.2222	0.1889	-278.5979	34.1535	False
2	3	-43.6667	0.9298	-200.0423	112.709	False
2	4	-80.1111	0.5916	-236.4868	76.2646	False
2	5	-133.5556	0.1258	-289.9312	22.8201	False
3	4	-36.4444	0.9626	-192.8201	119.9312	False
3	5	-89.8889	0.4805	-246.2646	66.4868	False
4	5	-53.4444	0.8643	-209.8201	102.9312	False

Table 21: P-value for comparing means of dentists combinations while treating with alloy-1

Thus the mean hardness of implants remains the same for any pair of dentist combinations while treating with alloy-1

<u>Multiple Comparison test (Tukey HSD) - Dentist combinations –(Alloy2)</u>

In order to identify for which method mean response is different from other groups, the null hypothesis is

H_0: The mean response for any pair of dentist combinations is the same while treating with alloy2

Against the alternative hypothesis
H_a: The mean response for at least one pair of dentist combinations is different while treating with alloy2

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-4.1111	1.0	-225.5687	217.3465	False
1	3	-36.5556	0.9895	-258.0131	184.902	False
1	4	-70.0	0.8941	-291.4576	151.4576	False
1	5	-90.1111	0.7724	-311.5687	131.3465	False
2	3	-32.4444	0.9933	-253.902	189.0131	False
2	4	-65.8889	0.9132	-287.3465	155.5687	False
2	5	-86.0	0.8008	-307.4576	135.4576	False
3	4	-33.4444	0.9925	-254.902	188.0131	False
3	5	-53.5556	0.9574	-275.0131	167.902	False
4	5	-20.1111	0.999	-241.5687	201.3465	False

Table 22: P-value for comparing means of dentists combinations while treating with alloy-2

Thus the mean hardness of implants remains the same for any pair of dentist combinations while treating with alloy-2

Multiple Comparison test (Tukey HSD) - Method combinations –(Alloy1 & Alloy2)

In order to identify for which method mean response is different from other groups, the null hypothesis is

$$H_0: \mu_{\text{method1}} = \mu_{\text{method2}} \text{ and } \mu_{\text{method1}} = \mu_{\text{method3}} \text{ and } \mu_{\text{method2}} = \mu_{\text{method3}}$$

Against the alternative hypothesis

$$H_a: \mu_{\text{method1}} \neq \mu_{\text{method2}} \text{ and } \mu_{\text{method1}} \neq \mu_{\text{method3}} \text{ and } \mu_{\text{method2}} \neq \mu_{\text{method3}}$$

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True

Table 23: P-value for comparing means of methods while treating with alloy-1

While using ALLOY-1 for treating the dental cavities

- The response of hardness of dental implants **differs between method 1 and method 3**
- The response of hardness of dental implants also **differs between method 2 and method 3**
- The response of hardness of dental implants is not so different between method 1 and method 2

```
   Multiple Comparison of Means - Tukey HSD, FWER=0.05
========================================================
group1 group2 meandiff p-adj    lower     upper   reject
--------------------------------------------------------
     1      2     27.0 0.8212  -82.4546  136.4546  False
     1      3   -208.8 0.0001 -318.2546  -99.3454   True
     2      3   -235.8    0.0 -345.2546 -126.3454   True
--------------------------------------------------------
```

Table 24: P-value for comparing means of methods while treating with alloy-2

While using ALLOY-2 for treating the dental cavities

- The response of hardness of dental implants **differs between method 1 and method 3**
- The response of hardness of dental implants also **differs between method 2 and method 3**
- The response of hardness of dental implants is not so different between method 1 and method 2

Thus the hardness of implants with method3 is different from method 1 and method2 irrespective of using any of the alloys

Creating a new column 'class' representing the dentists and methods combinations – ALLOY1

	Dentist	Method	Alloy	Temp	Response	class
79	5	2	1	1600	665	D5M2
80	5	2	1	1700	752	D5M2
84	5	3	1	1500	421	D5M3
85	5	3	1	1600	483	D5M3
86	5	3	1	1700	405	D5M3

Table 25: Dataframe with alloy1 and class representing dentist and method combinations

Multiple Comparison test (Tukey HSD) - Dentist and Method combinations –(Alloy1)

In order to identify for which dentist and method combinations, the mean response is different from other groups, the null hypothesis is

H_0: The mean response for any pair of dentist and method combinations are the same while treating with alloy1

Against the alternative hypothesis
H_a: The mean response for at least one pair of dentist and method combinations is different while treating with alloy1

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
D1M1	D1M2	-84.0	0.9933	-332.8283	164.8283	False
D1M1	D1M3	-63.3333	0.9996	-312.1617	185.495	False
D1M1	D2M1	-22.0	1.0	-270.8283	226.8283	False
D1M1	D2M2	-33.3333	1.0	-282.1617	215.495	False
D1M1	D2M3	-58.0	0.9999	-306.8283	190.8283	False
D1M1	D3M1	-91.6667	0.9853	-340.495	157.1617	False
D1M1	D3M2	-76.0	0.9975	-324.8283	172.8283	False
D1M1	D3M3	-76.6667	0.9972	-325.495	172.1617	False
D1M1	D4M1	-33.3333	1.0	-282.1617	215.495	False
D1M1	D4M2	-17.6667	1.0	-266.495	231.1617	False
D1M1	D4M3	-302.6667	0.007	-551.495	-53.8383	True
D1M1	D5M1	-92.3333	0.9844	-341.1617	156.495	False
D1M1	D5M2	-59.0	0.9998	-307.8283	189.8283	False
D1M1	D5M3	-362.6667	0.0007	-611.495	-113.8383	True
D1M2	D1M3	20.6667	1.0	-228.1617	269.495	False
D1M2	D2M1	62.0	0.9997	-186.8283	310.8283	False
D1M2	D2M2	50.6667	1.0	-198.1617	299.495	False
D1M2	D2M3	26.0	1.0	-222.8283	274.8283	False
D1M2	D3M1	-7.6667	1.0	-256.495	241.1617	False
D1M2	D3M2	8.0	1.0	-240.8283	256.8283	False
D1M2	D3M3	7.3333	1.0	-241.495	256.1617	False
D1M2	D4M1	50.6667	1.0	-198.1617	299.495	False
D1M2	D4M2	66.3333	0.9994	-182.495	315.1617	False
D1M2	D4M3	-218.6667	0.1324	-467.495	30.1617	False
D1M2	D5M1	-8.3333	1.0	-257.1617	240.495	False
D1M2	D5M2	25.0	1.0	-223.8283	273.8283	False
D1M2	D5M3	-278.6667	0.0173	-527.495	-29.8383	True
D1M3	D2M1	41.3333	1.0	-207.495	290.1617	False
D1M3	D2M2	30.0	1.0	-218.8283	278.8283	False
D1M3	D2M3	5.3333	1.0	-243.495	254.1617	False
D1M3	D3M1	-28.3333	1.0	-277.1617	220.495	False
D1M3	D3M2	-12.6667	1.0	-261.495	236.1617	False
D1M3	D3M3	-13.3333	1.0	-262.1617	235.495	False
D1M3	D4M1	30.0	1.0	-218.8283	278.8283	False
D1M3	D4M2	45.6667	1.0	-203.1617	294.495	False
D1M3	D4M3	-239.3333	0.0688	-488.1617	9.495	False
D1M3	D5M1	-29.0	1.0	-277.8283	219.8283	False
D1M3	D5M2	4.3333	1.0	-244.495	253.1617	False
D1M3	D5M3	-299.3333	0.0079	-548.1617	-50.505	True
D2M1	D2M2	-11.3333	1.0	-260.1617	237.495	False
D2M1	D2M3	-36.0	1.0	-284.8283	212.8283	False
D2M1	D3M1	-69.6667	0.999	-318.495	179.1617	False
D2M1	D3M2	-54.0	0.9999	-302.8283	194.8283	False
D2M1	D3M3	-54.6667	0.9999	-303.495	194.1617	False
D2M1	D4M1	-11.3333	1.0	-260.1617	237.495	False
D2M1	D4M2	4.3333	1.0	-244.495	253.1617	False
D2M1	D4M3	-280.6667	0.016	-529.495	-31.8383	True
D2M1	D5M1	-70.3333	0.9989	-319.1617	178.495	False
D2M1	D5M2	-37.0	1.0	-285.8283	211.8283	False
D2M1	D5M3	-340.6667	0.0016	-589.495	-91.8383	True
D2M2	D2M3	-24.6667	1.0	-273.495	224.1617	False
D2M2	D3M1	-58.3333	0.9999	-307.1617	190.495	False
D2M2	D3M2	-42.6667	1.0	-291.495	206.1617	False
D2M2	D3M3	-43.3333	1.0	-292.1617	205.495	False
D2M2	D4M1	0.0	1.0	-248.8283	248.8283	False
D2M2	D4M2	15.6667	1.0	-233.1617	264.495	False
D2M2	D4M3	-269.3333	0.0243	-518.1617	-20.505	True
D2M2	D5M1	-59.0	0.9998	-307.8283	189.8283	False
D2M2	D5M2	-25.6667	1.0	-274.495	223.1617	False
D2M2	D5M3	-329.3333	0.0025	-578.1617	-80.505	True

D2M3	D3M1	-33.6667	1.0	-282.495	215.1617	False
D2M3	D3M2	-18.0	1.0	-266.8283	230.8283	False
D2M3	D3M3	-18.6667	1.0	-267.495	230.1617	False
D2M3	D4M1	24.6667	1.0	-224.1617	273.495	False
D2M3	D4M2	40.3333	1.0	-208.495	289.1617	False
D2M3	D4M3	-244.6667	0.0576	-493.495	4.1617	False
D2M3	D5M1	-34.3333	1.0	-283.1617	214.495	False
D2M3	D5M2	-1.0	1.0	-249.8283	247.8283	False
D2M3	D5M3	-304.6667	0.0065	-553.495	-55.8383	True
D3M1	D3M2	15.6667	1.0	-233.1617	264.495	False
D3M1	D3M3	15.0	1.0	-233.8283	263.8283	False
D3M1	D4M1	58.3333	0.9999	-190.495	307.1617	False
D3M1	D4M2	74.0	0.9981	-174.8283	322.8283	False
D3M1	D4M3	-211.0	0.166	-459.8283	37.8283	False
D3M1	D5M1	-0.6667	1.0	-249.495	248.1617	False
D3M1	D5M2	32.6667	1.0	-216.1617	281.495	False
D3M1	D5M3	-271.0	0.0229	-519.8283	-22.1717	True
D3M2	D3M3	-0.6667	1.0	-249.495	248.1617	False
D3M2	D4M1	42.6667	1.0	-206.1617	291.495	False
D3M2	D4M2	58.3333	0.9999	-190.495	307.1617	False
D3M2	D4M3	-226.6667	0.1035	-475.495	22.1617	False
D3M2	D5M1	-16.3333	1.0	-265.1617	232.495	False
D3M2	D5M2	17.0	1.0	-231.8283	265.8283	False
D3M2	D5M3	-286.6667	0.0128	-535.495	-37.8383	True
D3M3	D4M1	43.3333	1.0	-205.495	292.1617	False
D3M3	D4M2	59.0	0.9998	-189.8283	307.8283	False
D3M3	D4M3	-226.0	0.1057	-474.8283	22.8283	False
D3M3	D5M1	-15.6667	1.0	-264.495	233.1617	False
D3M3	D5M2	17.6667	1.0	-231.1617	266.495	False
D3M3	D5M3	-286.0	0.0131	-534.8283	-37.1717	True
D4M1	D4M2	15.6667	1.0	-233.1617	264.495	False
D4M1	D4M3	-269.3333	0.0243	-518.1617	-20.505	True
D4M1	D5M1	-59.0	0.9998	-307.8283	189.8283	False
D4M1	D5M2	-25.6667	1.0	-274.495	223.1617	False
D4M1	D5M3	-329.3333	0.0025	-578.1617	-80.505	True
D4M2	D4M3	-285.0	0.0137	-533.8283	-36.1717	True
D4M2	D5M1	-74.6667	0.9979	-323.495	174.1617	False
D4M2	D5M2	-41.3333	1.0	-290.1617	207.495	False
D4M2	D5M3	-345.0	0.0013	-593.8283	-96.1717	True
D4M3	D5M1	210.3333	0.1692	-38.495	459.1617	False
D4M3	D5M2	243.6667	0.0596	-5.1617	492.495	False
D4M3	D5M3	-60.0	0.9998	-308.8283	188.8283	False
D5M1	D5M2	33.3333	1.0	-215.495	282.1617	False
D5M1	D5M3	-270.3333	0.0234	-519.1617	-21.505	True
D5M2	D5M3	-303.6667	0.0067	-552.495	-54.8383	True

Table 26: P-value for comparing means of dentist and method combinations while treating with alloy-1

While using ALLOY-1 for treating the dental cavities

The response of hardness of dental implants **differs for the following combinations of dentists and methods (Inference from Table 24)**

- **Dentist 1 with Method 1 and Dentist 4 with Method 3**
- **Dentist 1 with Method 1 and Dentist 5 with Method 3**

The response of hardness of dental implants **differs for the following combinations of dentists and methods (Inference from Table 24)**

- **Dentist 1 with Method 2 and Dentist 5 with Method 3**
- **Dentist 1 with Method 3 and Dentist 5 with Method 3**
- **Dentist 2 with Method 1 and Dentist 4 with Method 3**
- **Dentist 2 with Method 1 and Dentist 5 with Method 3**
- **Dentist 2 with Method 2 and Dentist 4 with Method 3**
- **Dentist 2 with Method 2 and Dentist 5 with Method 3**
- **Dentist 2 with Method 3 and Dentist 5 with Method 3**
- **Dentist 3 with Method 1 and Dentist 5 with Method 3**
- **Dentist 3 with Method 2 and Dentist 5 with Method 3**
- **Dentist 3 with Method 3 and Dentist 5 with Method 3**
- **Dentist 4 with Method 1 and Dentist 4 with Method 3**
- **Dentist 4 with Method 1 and Dentist 5 with Method 3**
- **Dentist 4 with Method 2 and Dentist 4 with Method 3**
- **Dentist 4 with Method 2 and Dentist 5 with Method 3**
- **Dentist 5 with Method 1 and Dentist 5 with Method 3**
- **Dentist 5 with Method 2 and Dentist 5 with Method 3**

Creating a new column 'class' representing the dentists and methods combinations – ALLOY2

	Dentist	Method	Alloy	Temp	Response	class
82	5	2	2	1600	933	D5M2
83	5	2	2	1700	835	D5M2
87	5	3	2	1500	536	D5M3
88	5	3	2	1600	405	D5M3
89	5	3	2	1700	312	D5M3

Table 27: Dataframe with alloy2 and class representing dentist and method combinations

Multiple Comparison test (Tukey HSD) - Dentist and Method combinations –(Alloy2)

In order to identify for which dentist and method combinations, the mean response is different from other groups, the null hypothesis is

H_0: The mean response for any pair of dentist and method combinations are the same while treating with alloy2

Against the alternative hypothesis
H_a: The mean response for at least one pair of dentist and method combinations is different while treating with alloy2

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
D1M1	D1M2	95.3333	0.999	-245.5625	436.2292	False
D1M1	D1M3	-180.6667	0.8085	-521.5625	160.2292	False
D1M1	D2M1	37.6667	1.0	-303.2292	378.5625	False
D1M1	D2M2	5.0	1.0	-335.8958	345.8958	False
D1M1	D2M3	-140.3333	0.9635	-481.2292	200.5625	False
D1M1	D3M1	-63.3333	1.0	-404.2292	277.5625	False
D1M1	D3M2	-27.0	1.0	-367.8958	313.8958	False
D1M1	D3M3	-104.6667	0.9973	-445.5625	236.2292	False
D1M1	D4M1	-66.3333	1.0	-407.2292	274.5625	False
D1M1	D4M2	2.3333	1.0	-338.5625	343.2292	False
D1M1	D4M3	-231.3333	0.4686	-572.2292	109.5625	False
D1M1	D5M1	52.0	1.0	-288.8958	392.8958	False
D1M1	D5M2	19.3333	1.0	-321.5625	360.2292	False
D1M1	D5M3	-427.0	0.0049	-767.8958	-86.1042	True
D1M2	D1M3	-276.0	0.2169	-616.8958	64.8958	False
D1M2	D2M1	-57.6667	1.0	-398.5625	283.2292	False
D1M2	D2M2	-90.3333	0.9994	-431.2292	250.5625	False
D1M2	D2M3	-235.6667	0.4396	-576.5625	105.2292	False
D1M2	D3M1	-158.6667	0.912	-499.5625	182.2292	False
D1M2	D3M2	-122.3333	0.9884	-463.2292	218.5625	False
D1M2	D3M3	-200.0	0.6868	-540.8958	140.8958	False
D1M2	D4M1	-161.6667	0.9005	-502.5625	179.2292	False
D1M2	D4M2	-93.0	0.9992	-433.8958	247.8958	False
D1M2	D4M3	-326.6667	0.0709	-667.5625	14.2292	False
D1M2	D5M1	-43.3333	1.0	-384.2292	297.5625	False
D1M2	D5M2	-76.0	0.9999	-416.8958	264.8958	False
D1M2	D5M3	-522.3333	0.0003	-863.2292	-181.4375	True
D1M3	D2M1	218.3333	0.5587	-122.5625	559.2292	False
D1M3	D2M2	185.6667	0.7793	-155.2292	526.5625	False
D1M3	D2M3	40.3333	1.0	-300.5625	381.2292	False
D1M3	D3M1	117.3333	0.992	-223.5625	458.2292	False
D1M3	D3M2	153.6667	0.9291	-187.2292	494.5625	False
D1M3	D3M3	76.0	0.9999	-264.8958	416.8958	False
D1M3	D4M1	114.3333	0.9937	-226.5625	455.2292	False
D1M3	D4M2	183.0	0.7951	-157.8958	523.8958	False
D1M3	D4M3	-50.6667	1.0	-391.5625	290.2292	False
D1M3	D5M1	232.6667	0.4596	-108.2292	573.5625	False
D1M3	D5M2	200.0	0.6868	-140.8958	540.8958	False
D1M3	D5M3	-246.3333	0.3717	-587.2292	94.5625	False
D2M1	D2M2	-32.6667	1.0	-373.5625	308.2292	False
D2M1	D2M3	-178.0	0.8234	-518.8958	162.8958	False
D2M1	D3M1	-101.0	0.9981	-441.8958	239.8958	False
D2M1	D3M2	-64.6667	1.0	-405.5625	276.2292	False
D2M1	D3M3	-142.3333	0.9594	-483.2292	198.5625	False
D2M1	D4M1	-104.0	0.9975	-444.8958	236.8958	False
D2M1	D4M2	-35.3333	1.0	-376.2292	305.5625	False
D2M1	D4M3	-269.0	0.2485	-609.8958	71.8958	False
D2M1	D5M1	14.3333	1.0	-326.5625	355.2292	False
D2M1	D5M2	-18.3333	1.0	-359.2292	322.5625	False
D2M1	D5M3	-464.6667	0.0017	-805.5625	-123.7708	True
D2M2	D2M3	-145.3333	0.9525	-486.2292	195.5625	False
D2M2	D3M1	-68.3333	1.0	-409.2292	272.5625	False
D2M2	D3M2	-32.0	1.0	-372.8958	308.8958	False
D2M2	D3M3	-109.6667	0.9958	-450.5625	231.2292	False
D2M2	D4M1	-71.3333	1.0	-412.2292	269.5625	False
D2M2	D4M2	-2.6667	1.0	-343.5625	338.2292	False
D2M2	D4M3	-236.3333	0.4352	-577.2292	104.5625	False
D2M2	D5M1	47.0	1.0	-293.8958	387.8958	False
D2M2	D5M2	14.3333	1.0	-326.5625	355.2292	False
D2M2	D5M3	-432.0	0.0043	-772.8958	-91.1042	True

D2M3	D3M1	77.0	0.9999	-263.8958	417.8958	False
D2M3	D3M2	113.3333	0.9942	-227.5625	454.2292	False
D2M3	D3M3	35.6667	1.0	-305.2292	376.5625	False
D2M3	D4M1	74.0	0.9999	-266.8958	414.8958	False
D2M3	D4M2	142.6667	0.9586	-198.2292	483.5625	False
D2M3	D4M3	-91.0	0.9994	-431.8958	249.8958	False
D2M3	D5M1	192.3333	0.7376	-148.5625	533.2292	False
D2M3	D5M2	159.6667	0.9083	-181.2292	500.5625	False
D2M3	D5M3	-286.6667	0.1746	-627.5625	54.2292	False
D3M1	D3M2	36.3333	1.0	-304.5625	377.2292	False
D3M1	D3M3	-41.3333	1.0	-382.2292	299.5625	False
D3M1	D4M1	-3.0	1.0	-343.8958	337.8958	False
D3M1	D4M2	65.6667	1.0	-275.2292	406.5625	False
D3M1	D4M3	-168.0	0.8735	-508.8958	172.8958	False
D3M1	D5M1	115.3333	0.9932	-225.5625	456.2292	False
D3M1	D5M2	82.6667	0.9998	-258.2292	423.5625	False
D3M1	D5M3	-363.6667	0.0279	-704.5625	-22.7708	True
D3M2	D3M3	-77.6667	0.9999	-418.5625	263.2292	False
D3M2	D4M1	-39.3333	1.0	-380.2292	301.5625	False
D3M2	D4M2	29.3333	1.0	-311.5625	370.2292	False
D3M2	D4M3	-204.3333	0.657	-545.2292	136.5625	False
D3M2	D5M1	79.0	0.9999	-261.8958	419.8958	False
D3M2	D5M2	46.3333	1.0	-294.5625	387.2292	False
D3M2	D5M3	-400.0	0.0105	-740.8958	-59.1042	True
D3M3	D4M1	38.3333	1.0	-302.5625	379.2292	False
D3M3	D4M2	107.0	0.9967	-233.8958	447.8958	False
D3M3	D4M3	-126.6667	0.9842	-467.5625	214.2292	False
D3M3	D5M1	156.6667	0.9191	-184.2292	497.5625	False
D3M3	D5M2	124.0	0.9869	-216.8958	464.8958	False
D3M3	D5M3	-322.3333	0.0786	-663.2292	18.5625	False
D4M1	D4M2	68.6667	1.0	-272.2292	409.5625	False
D4M1	D4M3	-165.0	0.8868	-505.8958	175.8958	False
D4M1	D5M1	118.3333	0.9914	-222.5625	459.2292	False
D4M1	D5M2	85.6667	0.9997	-255.2292	426.5625	False
D4M1	D5M3	-360.6667	0.0302	-701.5625	-19.7708	True
D4M2	D4M3	-233.6667	0.4529	-574.5625	107.2292	False
D4M2	D5M1	49.6667	1.0	-291.2292	390.5625	False
D4M2	D5M2	17.0	1.0	-323.8958	357.8958	False
D4M2	D5M3	-429.3333	0.0046	-770.2292	-88.4375	True
D4M3	D5M1	283.3333	0.1871	-57.5625	624.2292	False
D4M3	D5M2	250.6667	0.3458	-90.2292	591.5625	False
D4M3	D5M3	-195.6667	0.7158	-536.5625	145.2292	False
D5M1	D5M2	-32.6667	1.0	-373.5625	308.2292	False
D5M1	D5M3	-479.0	0.0011	-819.8958	-138.1042	True
D5M2	D5M3	-446.3333	0.0028	-787.2292	-105.4375	True

Table 28: P-value for comparing means of dentist and method combinations while treating with alloy-2

While using ALLOY-2 for treating the dental cavities

The response of hardness of dental implants **differs for the following combinations of dentists and methods (Inference from Table 26)**

- **Dentist 1 with Method 1 and Dentist 5 with Method 3**
- **Dentist 1 with Method 2 and Dentist 5 with Method 3**

The response of hardness of dental implants **differs for the following combinations of dentists and methods (Inference from Table 26)**

- **Dentist 2 with Method 1 and Dentist 5 with Method 3**
- **Dentist 2 with Method 2 and Dentist 5 with Method 3**
- **Dentist 3 with Method 1 and Dentist 5 with Method 3**
- **Dentist 3 with Method 2 and Dentist 5 with Method 3**
- **Dentist 4 with Method 1 and Dentist 5 with Method 3**
- **Dentist 4 with Method 2 and Dentist 5 with Method 3**
- **Dentist 5 with Method 1 and Dentist 5 with Method 3**
- **Dentist 5 with Method 2 and Dentist 5 with Method 3**

STEP 6: Interaction levels between Dentist and Method, Alloy and Method, Dentist and Alloy.

<u>Interaction levels between Dentist and Method- Alloy1</u>

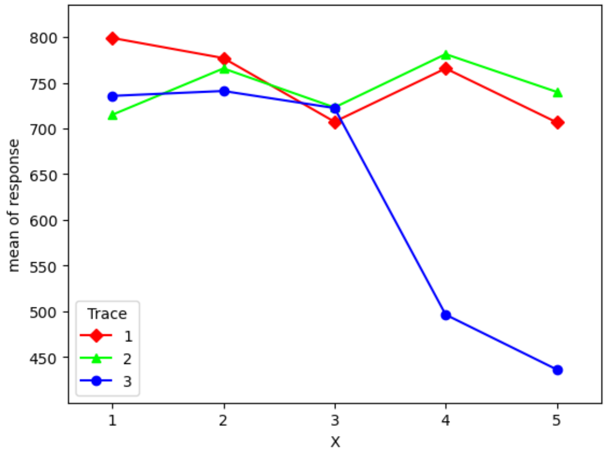

Figure 24: Interaction plot between dentist and method (alloy1)

<u>Interaction levels between Dentist and Method- Alloy2</u>

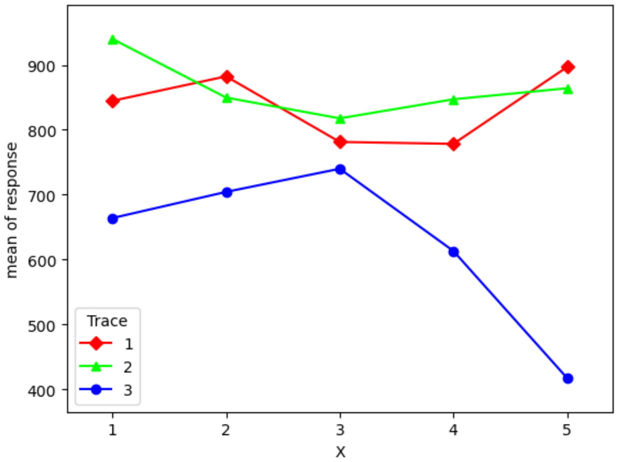

Figure 25: Interaction plot between dentist and method (alloy2)

Interaction levels between Alloy and Method

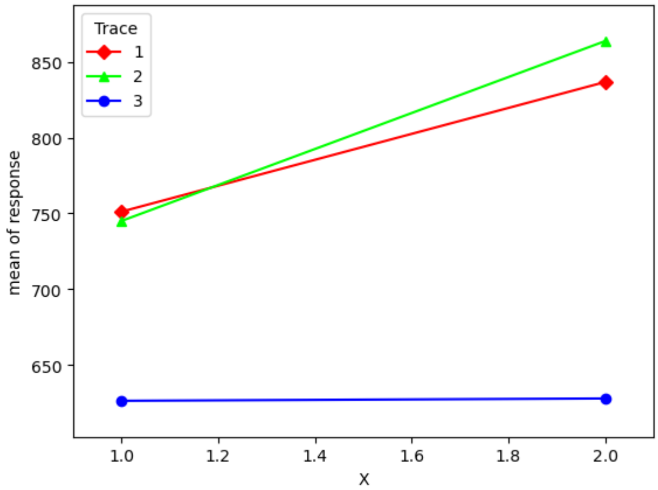

Figure 26: Interaction plot between alloy and method

Interaction levels between Dentist and Alloy

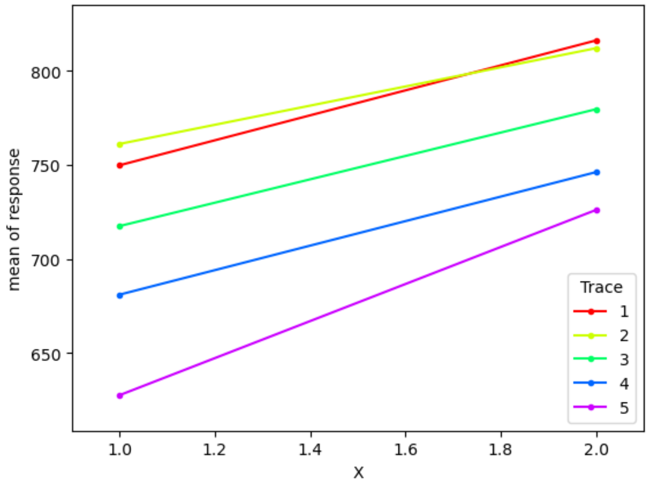

Figure 27: Interaction plot between Dentist and Alloy

- There is some sort of interaction between dentist and method while using alloy-1
- There is no interaction between dentist and method while using alloy-2
- There is some sort of interaction between alloy-1 and methods 1& 2
- There is no interaction between alloy-2 and any of the three methods
- There is some sort of interaction between alloy-2 and dentists 1& 2
- There is no interaction between alloy-1 and any of the five dentists