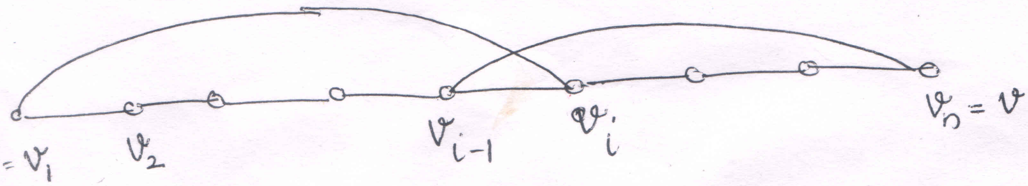
Dirac's Theorem :-

A sufficient condition for a simple graph G to have a Hamiltonian circuit is that the degree of every vertex in G be at least $n/2$, where n is the number of vertices in G.

Proof :-

We have the condition that $d(v) \geq n/2 \ \forall v \in V(G)$.
Hence for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) \geq \frac{n}{2} + \frac{n}{2} = n \longrightarrow ①$

We have to prove that G is Hamiltonian.
If possible suppose that G is not Hamiltonian. Add edges to G (without adding vertices) and get a supergraph G^* of G such that G^* is a maximal simple graph that satisfies the condition of the theorem but G^* is non-Hamiltonian. Such a graph G^* must exist since G is non-Hamiltonian while the complete graph on $V(G)$ is Hamiltonian. Hence, for any pair u and v of nonadjacent vertices of G^*, $G^* + uv$ must contain a Hamiltonian cycle C. This cycle C would certainly contain the edge $e = uv$. (otherwise C will be a Hamiltonian cycle in G^*). Then $C - e$ is a Hamilton path $u = v_1 v_2 v_3 \ldots v_n = v$ of G^*



Now, if v_i is adjacent to u, then v_{i-1} is nonadjacent to v_i; otherwise, $v_1 v_2 \cdots v_{i-1} v_n v_{n-1} v_{n-2} \cdots v_{i+1} v_i v_1$ would be a Hamiltonian cycle in G^*. Hence, for each vertex adjacent to u, there is a vertex of $V - \{v\}$ nonadjacent to v. But then,

$$d_{G^*}(v) \leq n-1 - d_{G^*}(u)$$

This gives that $d_{G^*}(u) + d_{G^*}(v) \leq n-1$, and therefore, $d_G(u) + d_G(v) \leq n-1$ which is a contradiction to 1.

So we conclude that ~~H~~ G is Hamiltonian.

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