B9-06 Optical Quantum Effects*

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We entangle the polarization of photon pairs and show core properties of quantum entanglement. Through type I spontaneous parametric down-conversion happening within β-Barium Borate crystals, we create photon pairs with entangled polarization states. The polarization states are detected by a combination of half-wave plates and polarizing beam splitters. The entanglement of photons is shown clearly by measuring coincidence rates and performing remote state preparations. The ratio between coincidence rate and single detector counts reaches 3%, which is a remarkable improvement compared to other experiments alike.

I. INTRODUCTION

Quantum entanglement is one of the core physical phenomena in quantum physics. It is the basis of many potentially revolutionary technologies such as quantum computing algorithms [1] and quantum cryptography [2]. However, when the concept of quantum entanglement was first raised, it confused generations of world-class scientists. The raise of Einstein-Podolsky-Rosen paradox [3] on local hidden variable theory is a piece of evidence highlighting its counter-intuitive nature. This paradox was only resolved after the proposal of Bell's inequality [4] in 1964. Thus, it is important to develop undergraduate teaching labs to enhance students' understandings through hands-on experience. Unlike entanglements in other physical systems such as atoms or superconducting circuits, demonstrations of photon entanglement are more suitable for teaching labs because they are less demanding on the laboratory environment and the required equipment is more accessible. Besides, efforts on improving the experimental loopholes of proving Bell's inequality have been underway even until 2015 [5–7]. Therefore, these teaching labs can serve as a first introduction to cutting-edge research topics.

Here, we describe labs to demonstrate spontaneous parametric down-conversion (SPDC), photon polarization entanglement, and remote state preparation. All labs are based on the SPDC within β-Barium Borate (BBO) crystals, a type of nonlinear crystal. By comparing the theoretical prediction with measured data, properties of quantum entanglement are clearly shown in the end. The coincidence rate can be up to 10,000 counts per second, and the ratio of coincidence rate to single detector counts is at 3% while similar experiments only had around 1% in the past.

* Raw data and codes are uploaded to Github, and can be available upon request.

II. THEORY

A. Spontaneous Parametric Down-conversion

Spontaneous parametric down-conversion (SPDC) happens when the pump beam passes through a birefringent nonlinear crystal. Figure 1 demonstrates the process with a single crystal. When a single photon passes through the crystal, a signal and an idler photon are created. The signal and idler photons' possible paths can be represented by light cones. The creation of the signal and idler photons satisfies the energy and momentum conservation laws as shown in Equation 1. Here, subscripts p, i, and s represented pump beam photons, idler photons, and signal photons respectively.

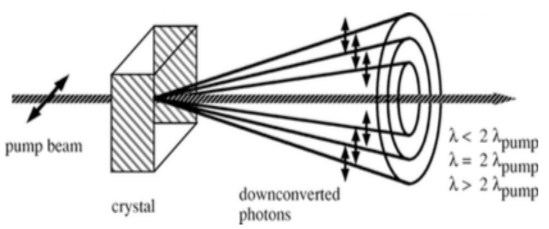


FIG. 1: Type I SPDC with single crystal. Reprinted from Ref [8].

$$\begin{aligned} \hbar\omega_p &= \hbar\omega_i + \hbar\omega_s, \\ \hbar\vec{k}_p &= \hbar\vec{k}_i + \hbar\vec{k}_s. \end{aligned} \tag{1}$$

Equation 2 gives the dispersion relation of the crystal, where n is the index of refraction and $|\Psi\rangle$ is the polarization state of the incoming photon.

$$k = \frac{n\left(\omega, |\Psi\rangle\right)\omega}{c}. \tag{2}$$

Only signal and idler photon frequencies satisfying these relations can exist. In our case, type I β-Barium Borate (BBO) crystals are used. This type of nonlinear crystal has the property that only pump beam photons with polarization perpendicular to the crystal's optical axis split into signal photons and idler photons, and pump beam photons polarized parallel to the optical axis are not affected. The signal and idler photons are polarized parallel to the optical axis. Moreover, although there is a spectrum of light cones as shown in Figure 1, the crystal is cut such that the light cones are centered at the light cone with $\lambda_i = \lambda_s = 2\lambda_p$, and the angles that the outgoing signal and idler photons make with the incoming pump beam photons are $\theta_i = \theta_s \approx 3°$.

A single crystal can only produce signal and idler photons in one direction. To create entangled states, a double crystal is needed. As shown in Figure 2, the double crystal consists of two single crystals with their optical axis perpendicular to each other, normally one horizontal and the other vertical. When the pump beam passes through the crystal, the horizontally polarized component is only affected by the crystal with a vertical optical axis, and vice versa. Thus, when a pump beam photon polarized along 45° goes through the double crystal, the process is described with Equation 3, assuming the first and second crystals' axis are in vertical and horizontal directions respectively.

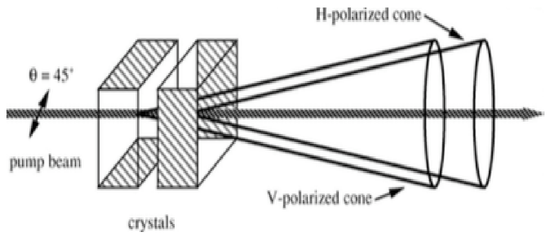


FIG. 2: Type I SPDC with double crystal. When pump beam is polarized at $45°$ with the crystals' axis, the outgoing photons are in entangled states. Reprinted from Ref [8].

$$\frac{|H\rangle_p + |V\rangle_p}{\sqrt{2}} \xrightarrow{BBO_1} \frac{|VV\rangle_{s,i} + |V\rangle_p}{\sqrt{2}} \xrightarrow{BBO_2} \frac{|VV\rangle_{s,i} + e^{i\psi}|HH\rangle_{s,i}}{\sqrt{2}}. \tag{3}$$

As seen in Equation 3, the resulting state of the idler and signal photons is a superposition of the $|HH\rangle$ and $|VV\rangle$ states with a relative phase ψ. ψ is introduced by both the dependence of the dispersion relation on wavelength and polarization and the thickness of the crystals. Evident in Figure 2, the two light cones produced by the two single crystals are concentric but are some distance apart, which makes it possible to distinguish which crystal each light cone comes from. This distinguishability reduces the extent to which they are entangled. When ψ is 0 or π, the outgoing state is one of the Bell's states $\Phi^+ = \frac{|HH\rangle+|VV\rangle}{\sqrt{2}}$ or $\Phi^- = \frac{|HH\rangle-|VV\rangle}{\sqrt{2}}$. Bell states yield optimum results for testing entanglement properties and non-locality. Therefore, an additional single BBO crystal or a quarter-wave plate can be installed to compensate for the phase shift [9].

B. Photon Polarization

In this experiment, photons are measured as a function of their polarizations. A polarizing beam splitter (PBS) only lets through horizontally polarized photons and reflects vertically polarized photons. Place a half-wave plate (HWP) before the PBS. Assume the HWP's axis and the photon's polarization are at angles α and θ with the horizontal direction respectively. The light intensity after the PBS is given by Equation 4. Here, I_0 is the intensity before the HWP.

$$I = I_0 \cos^2(2\alpha - \theta). \tag{4}$$

Therefore, fitting the intensity as a cosine-squared function of the half-wave plate angle α reveals the photon's polarization angle θ before entering this system. Note that this assumes the incoming light is linearly polarized.

C. Remote State Preparation

Assume the incoming light is in state $|\Psi\rangle = \frac{|VV\rangle_{s,i}+e^{i\psi}|HH\rangle_{s,i}}{\sqrt{2}}$ and the two detectors each belongs to Alice and Bob. If Alice receives the signal photon, the signal photon is measured when it gets through the polarizing beam splitter. Then, Alice's measurement prepares the Bob's idler photon in polarization states depending on the angle of Alice's HWP. The matrix forms of HWP operators [10] and measurements are shown in Equation 5. Here, HWP angle α is with respect to the horizontal direction. Single photon Hilbert space is expressed in the basis of $\{|H\rangle, |V\rangle\}$, and double photon Hilbert space is expressed in $\{|HH\rangle, |HV\rangle, |VH\rangle, |VV\rangle\}$.

$$\begin{aligned} \langle PBS| &= \langle H| = \begin{pmatrix} 1 & 0 \end{pmatrix}, \\ \hat{A}_{HWP}(\alpha) &= \begin{pmatrix} -\cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}. \end{aligned} \tag{5}$$

When the HWP angle $\alpha = 77.5°$, the remotely prepared state is given by Equation 6 by performing partial inner product. The resulting idler photon state is elliptically polarized except when ψ equals 0 or π.

$$\begin{aligned} |\Psi\rangle_i &= \left(\langle PBS|\hat{A}_{HWP}(\alpha)\right)_s \otimes \mathbb{1}_i|\Psi\rangle \\ &= \frac{|V\rangle_i + e^{i\psi}|H\rangle_i}{\sqrt{2}} \end{aligned} \tag{6}$$

III. SETUP

A schematic diagram of the experiment setup is shown in Figure 3.

A. Optics

In this experiment, there are two laser sources, one for the pump beam and the other for the back-alignment laser. The pump beam laser source has is labelled to have 100mW power at wavelength 405nm, but measurement shows it only has 82.9mW. The back-alignment laser is a standard 1mW power laser at 635nm. However, after placing a high pass wavelength filter in front of the pump beam that only transmits light of wavelength larger than 780nm, the spectrum of the pump beam shows a component higher than 780nm. This is problematic because it mixes with the signal and idler photons and is received by the detectors. Therefore, a lens tube that includes a low-pass wavelength filter should be fixed in front of the pump beam laser source. Moreover, a polarizer P is added in front of the pump beam laser source to ensure the polarization of the pump beam. Because of the limit of optical table space, a few mirror are used.

Along the path, there are half-wave plates and quarter-wave plates. Since pump beam photon wavelength is at 405nm and down-converted photon wavelength is at 810nm, it is important to use components centered at the correct wavelength. We used zero-order half-wave plates WPH10E-405 for HWP0 and WPH10E-808 for HWP1 and HWP2. At the two ends, photons are coupled into multimode fibers, which can connect to either lasers or photon counters. Each fiber coupler assembly, including a high-pass wavelength filter and a lens with 11mm focal length, are installed within a lens tube, as shown in Figure 10d.

B. Electronics

The detectors pass the photons through fibers into photon counters, which is connected to a frequency meter, a coincidence unit, and a oscilloscope. Pictures of the above equipment are shown in Figure 10 in the appendix.

The coincidence unit is set to have an allowed time interval of 10ns for coincidence events. The major part of the photon counter is the Photon Counting Mod-

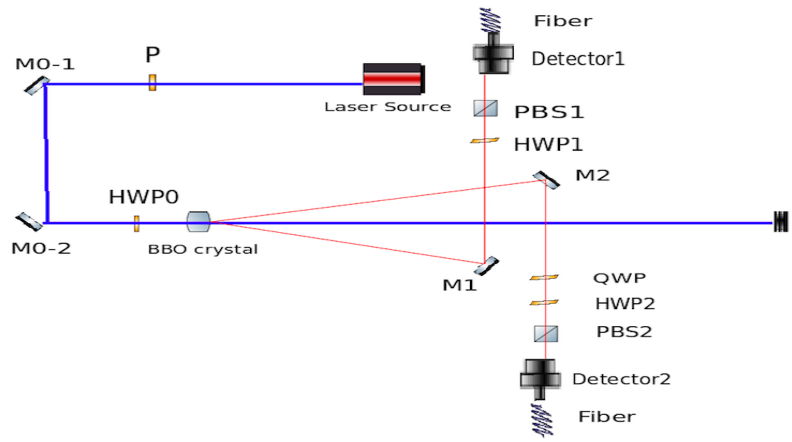


FIG. 3: The optics diagram of the full setup on the optical table. Here P stands for polarizer, M for mirrors, HWP for half-wave plate, QWP for quarter-wave plate, and PBS for polarizing beam splitter. The blue laser lines represents the pump beam at 405nm, and the red laser lines represent the down-converted photons at 810nm.

ule (PCM). Here, a high efficiency is needed. The total efficiency of capturing photons is the product of all individual components. Specifically, the coincidence rate should be at least $\sim$ 100 counts per second to perform reliable analysis. For our PCM, the detection efficiency at around 800nm is 45%. Be aware that the setup cannot take in too many photons. If too many photons are received, the PCM could get damaged because of overheat. Therefore, cover electronic screens, and do not expose the counter to strong light.

IV. ALIGNMENT

The alignment process is the most crucial step of this experiment. Since the signal and idler photons created by SPDC are in the infra-red range with very low intensity, coupling these photons into the fiber is the hardest and most time-consuming part. Thus, it is necessary to have a back-alignment laser source to aid with aligning. The following includes some critical points for the alignment process. See Appendix A for a detailed description.

A. Rough alignment

a. Pump beam alignment Use an iris to set the height of the pump beam path. Keep this height fixed for the all subsequent experiments. It is important to align pump beam path to be along a row or column of holes on the optical table. Also make sure the crystal is assembled on a mirror mount for the purpose of tilt axis adjustment.

b. Back-alignment The mirrors M1 and M2 are inserted on the expected paths of the signal and idler photons, given that the photons make 3° angles with the pump beam. Make sure at least one of the detectors is on a rail mount for later fine adjustments. Connect the detectors to the back-alignment laser. Since the filter only passes through photons with wavelength higher than 780nm and the back-alignment laser is at 635nm, the filter should be taken off when back-alignment laser is connected. A crucial step is to adjust the distance from the lens to the fiber such that the back-alignment laser beam is collimated in the length range of the experiment. The distance we measured was 9.85mm. This distance should be fixed for both lens tubes throughout the experiment. Aim the back-alignment laser at the center of the crystal.

B. Fine Alignment

a. Detector 1 Connect detector 1 to the photon counter. Maximise the single detector counts by adjusting mirror M1, half-wave plate HWP0, detector 1, and the tilt axis of the crystal.

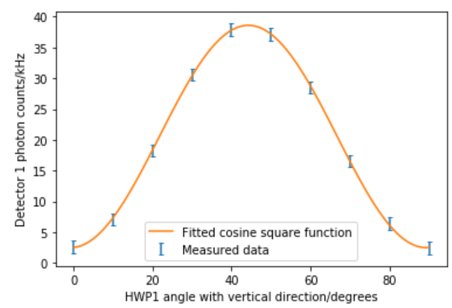
b. Detector 2 Connect detector 2 to the photon counter. With detector 1's position fixed, adjust mirror M2 and detector 2 only to maximise the coincidence counts between detector 1 and 2.

After completing the previous steps, the coincidence rate should be around 3% of the single detector counts. If the coincidence rate is significantly smaller than that fraction, it is likely the rough alignment steps need to be redone. Ideally, the detectors should be placed at the horizontal diameter of the SPDC light cones. To verify this, turn mirrors M1 and M2 and adjust the detector 1 and 2 in opposite directions to see if the coincidence rates can remain at the same level.

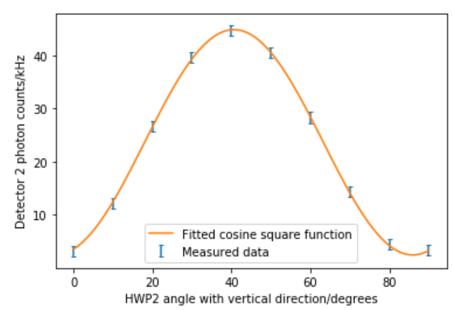
V. RESULTS

A. SPDC with single crystal

Insert the single BBO crystal in the setup with its axis pointing approximately in the vertical direction. As stated in Section II A, all signal and idler photons created by the SPDC process should be in the vertical direction. Insert the half-wave plates and polarizing beam splitters as seen in Figure 3. Rotate HWP1 and HWP2, and fit the single detector counts to the cosine-squared relation derived. As seen from Figure 4. The function fits the measured data very well.



(a) Detector 1



(b) Detector 2

FIG. 4: Single detector counts versus half-wave plate angles for detector 1 and 2 when single crystal is inserted.

The fitted cosine-squared function is

shown in Equation 7.

$$I(\alpha) = I_0 \cos^2(\omega\alpha + \psi) + I_1. \tag{7}$$

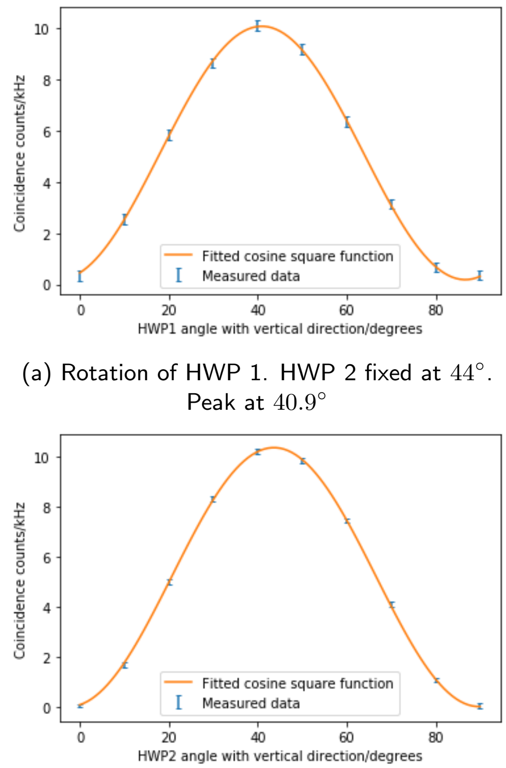
Here, I_1 is a bias in the photon counts from background photons, scattered pump beam photons, and dark counts. As derived in Equation 4, $I \propto \cos^2 2\alpha \propto \cos 4\alpha$, which agrees with the approximately 90° period as shown in Figure 4.

B. SPDC with double crystal

To verify the polarization states of signal and idler photons after the double crystal, set pump beam polarization to horizontal. The outgoing signal and idler photons should both be vertically polarized. Rotate the half-wave plates and record the single detector counts. We also record the coincidence counts when the other detector's single counts is maximised. Plot the measured coincidence counts and fitted functions in Figure 5. Calculate the peak positions from the fitted functions.

Perform similar measurements when the pump beam polarization is set to vertical. In this case, the signal and idler photons should be polarized in the horizontal direction. The graphs are plotted in Figure 6.

Ideally, the peaks should be reached at 0° and 45° for both half-wave plates when pump beam polarization is set to vertical and horizontal respectively. However, in practice, the double crystal does not have its axis precisely pointing at horizontal and vertical directions. Also, the phase shift introduced by the double crystal can cause the idler photon to be elliptically polarized instead of linearly polarized as shown in Equation 6. The components such as half-wave plates are not ideal as well, and finding the peak angles calibrates the half-wave plates' readings for the rest of the experiment.



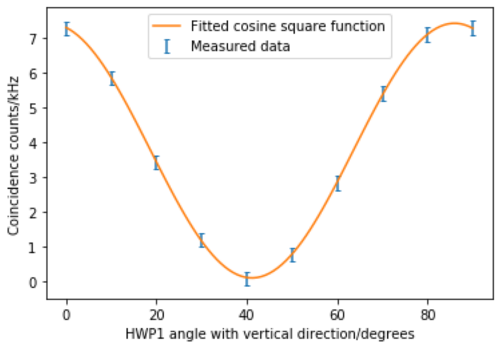
(a) Rotation of HWP 1. HWP 2 fixed at 44°. Peak at 40.9°

(b) Rotation of HWP 2. HWP 1 fixed at 41°. Peak at 43.7°

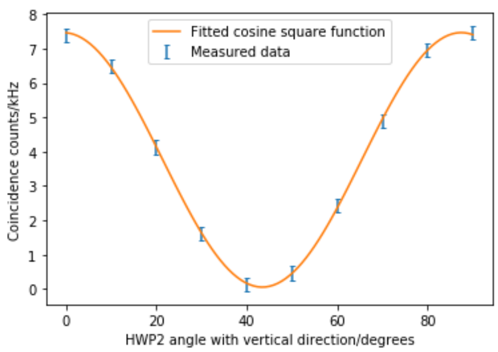
FIG. 5: Coincidence counts versus half-wave plate angles. Double crystal is inserted and pump beam polarization set to horizontal.

C. Maximally entangled state

In previous sections, the pump beam is polarized in either horizontal or vertical direction, which means only one of the double crystal are producing signal and idler photons. Thus, the photons created are



(a) Rotation of HWP 1. HWP 2 fixed at $87°$. Peak at $86.0°$



(b) Rotation of HWP 2. HWP 1 fixed at $88°$. Peak at $87.4°$

FIG. 6: Coincidence counts versus half-wave plate angles. Double crystal is inserted and pump beam polarization set to vertical.

still in separable superposition states. When HWP0 is set to a certain intermediate angle, the pump beam can be adjusted to be diagonally polarized. At this angle, the signal and idler photons should be in the state of $\frac{|VV\rangle_{s,i}+e^{i\psi}|HH\rangle_{s,i}}{\sqrt{2}}$. Therefore, we measured the coincidence rate of $|VV\rangle$ and $|HH\rangle$ when rotating HWP0 to find the optimum angle at which they are at equal magnitude. Here, adopt the peak angles measured from the previous section for different polarization states. The result is shown in Figure 7. At HWP0 angles $17.1°$ and $58.5°$, $|VV\rangle$ and $|HH\rangle$ have the same coincidence counts. The entangled states should differ from each other by a $e^{\pi i}$ phase shift in one of the $\{|VV\rangle, |HH\rangle\}$ basis. The pump beam should also be polarized at two mutually orthogonal diagonal states at these two angles respectively. Moreover, the coincidence counts of $|VH\rangle$ is shown. As expected, the amplitude of $|VH\rangle$ is negligible compared to that of $|VV\rangle$ and $|HH\rangle$.

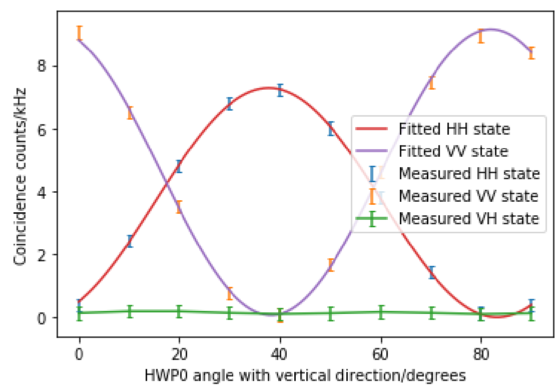


FIG. 7: Coincidence rates for different polarization states as labelled. The intersections of $|VV\rangle$ and $|HH\rangle$ states' counts are at HWP0 angles $17.1°$ and $58.5°$.

D. Remote state preparation

Insert the quarter-wave plate QWP as shown in Figure 3. The quarter wave plate can convert elliptically polarized light into linearly polarized light. Set HWP1 angles to be $0°$, $22.5°$, and $45°$. Then, rotate QWP to find an angle at which the visibility of the coincidence rate is maximised for each HWP1 angle respectively. The visibility is defined

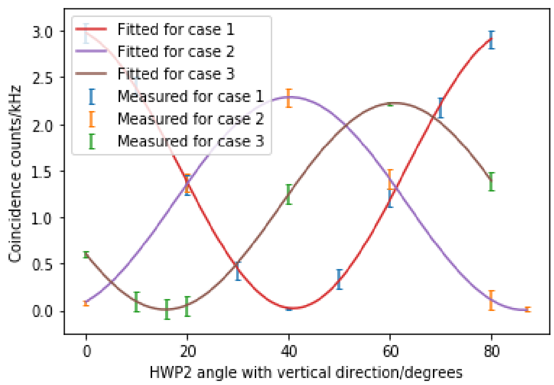


FIG. 8: Coincidence rates as a function of HWP2 angles. The three cases are defined by combination of HWP1 and QWP angles $(\theta_{HWP1}, \theta_{QWP})$. Case 1, 2, and 3 are $(0°, 292°)$, $(22.5°, 246°)$, $(45°, 23°)$.

as the ratio between the maximum and minimum of the coincidence rate when rotating HWP2. The data and fits are shown in Figure 8. Assume Alice has detector 1 and Bob has detector 2. For various HWP1 positions there exist a QWP2 position that converts the photons received by Bob to linearly polarized. Thus, Alice can remotely prepare the photons Bob receives through a rotation of measurement basis.

VI. NEXT STEPS

Many potential next steps are within reach.

a. Remote state preparation One can measure more sets of $(\theta_{HWP1}, \theta_{QWP})$ that achieve linear polarization and construct the theoretical model to calculate the phase shift ψ described in Equation 3.

b. Phase precompensation Instead of using a quarter-wave plate after the crystal, one can do phase precompensation by inserting a single crystal in front of the crystal [9].

c. Bell's inequality This step can be easily achieved with the current setup. Measure and calculate the CHSH form of Bell's inequality to prove non-locality.

d. Further experiments There are some experiment that can be demonstrated using the same setup with some modifications. For example, demonstration of Hong-Ou-Mandel effect [11] and single photon interference experiment through Mach-Zehnder interferometer.

VII. CONCLUSION

This experiment demonstrates spontaneous parametric down-conversion, photon entanglement, and remote state preparation using single and double β-Barium Borate (BBO) crystals. Overall, the most critical step is alignment. Aligning the weak and invisible signal and idler photons into the detectors is demanding in experimental techniques. The aid of back-alignment laser is crucial to the completion of alignment.

The measurements done for single and double crystals are fitted to cosine-squared functions. The fits are excellent within the error bounds, and the fitted parameters are consistent with the theoretical prediction for photon polarization after the SPDC process. The detected coincidence rate is around 3° of the single detector counts, which is 3 times as high as similar experiments performed in other universities. From the measurement results, when the half-wave plate HWP0 is rotated at 17.1° and 58.5°, the setup produces entangled states that differs with the Bell's state only by a phase shift. In the end, a quarter-wave plate is inserted to convert the elliptically polarized states to linearly polarized state, and thus show remote state preparation.

All three labs can be developed into advanced teaching labs for undergraduate students in the future, and detailed step-by-step operations are described in the lab manual in Appendix A.

VIII. ACKNOWLEDGEMENTS

I worked on this project with one colleague. My colleague worked with me on the alignment and measurements with single crystal, and I completed the parts with double crystals myself. Moreover, lab technician Richard Gardner helped me greatly on familiarizing with the equipment. The whole project is completed under Professor Alex Lvovsky's supervision, and I deeply appreciate his suggestions on the experimental designs.

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Appendix A: Lab Manual

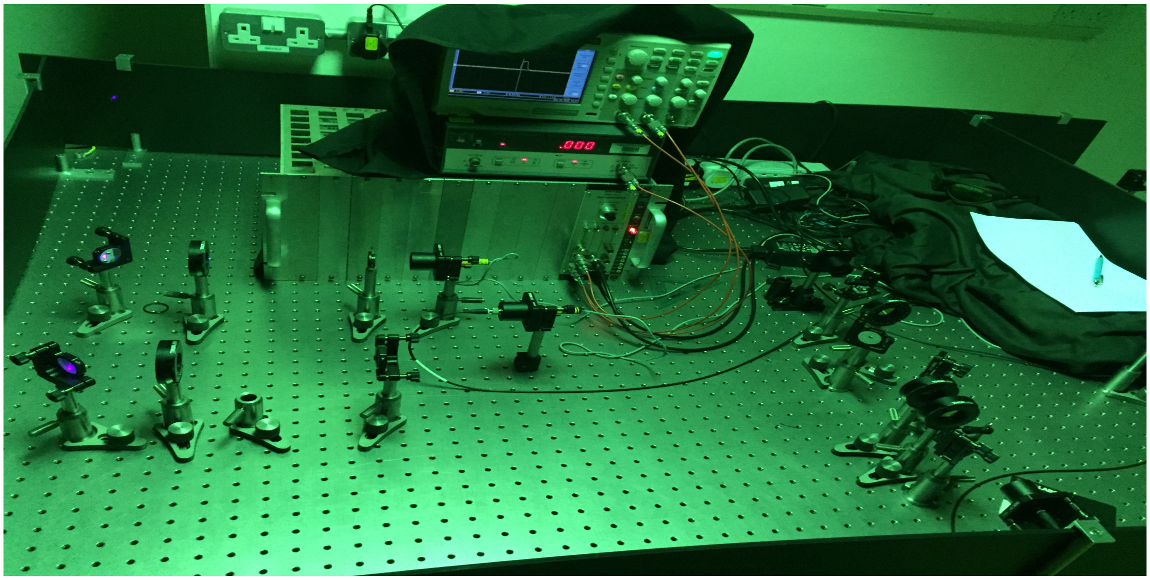


FIG. 9: A picture of our experimental setup. For a schematic diagram, refer to Figure 3.

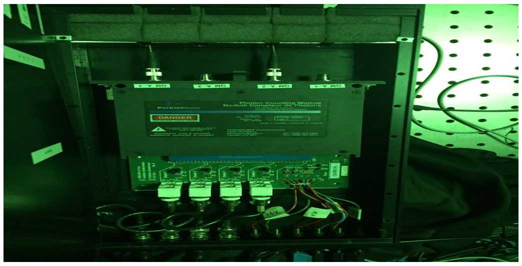
Three experiments are described in this manual: spontaneous parametric down-conversion (SPDC), photon polarization entanglement, and remote state preparation. A picture of the actual experimental apparatus can be seen in Figure 9.

Experiment apparatus

Most of the experiment apparatus is decribed in shown in Section III. To add up on the adjustment of lens distance in the lens tubes. In order to converge the parallel input light to the fiber coupler, the distance should be first set to around 11mm, its focal length. Then, connect the back-alignment laser to the back of the tube head. The optimum distance is achieved when the back-alignment laser beam are approximately collimated in the length range of the experiment. For our setup, the optimum value is measured to be 9.85mm. Keep this fixed for both lens head for all experiments.

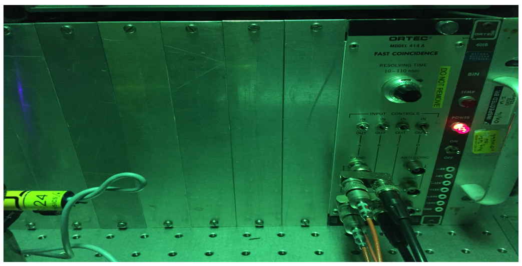
The frequency meter we have has several modes. For gate mode it has choices of 0.1s, 1s, and 10s, and for range mode, it has choices of 10MHz and 200MHz. The default modes are 1s and 200MHz. However, in our case, to obtain measurements that are not only stable but also sensitive to changes, the modes should be set to 1s and 10MHz.

Pictures of the photon counter, frequency meter, coincidence unit, and lens tube are shown in Figure 10.

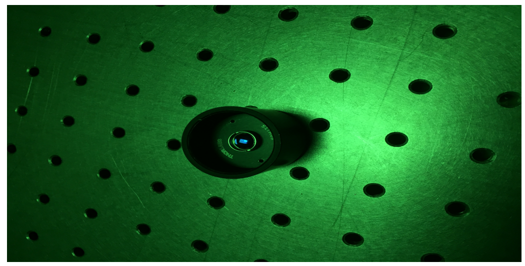


(a) Photon counter

(b) Frequency meter



(c) Coincidence unit



(d) Lens tube without filters

FIG. 10: Measurement and readout apparatus.

a. Pump beam alignment

Referring to Fig. 3, the pump beam is reflected by mirrors M0-1 and M0-2 because our optical table isn't long enough. Therefore, M0-1 and M0-2 are used to adjust the pump beam's horizontal and vertical angles. The mirror mounts each has a horizontal and a vertical screw that can adjust the angle of the mirror surface. The pump beam should travel horizontally and right above a row of holes on the optical table. The alignment procedure goes as follows, starting with only the pump beam

- Insert mirror M0-1. Orient the laser source such that the beam approximately hits at the center of M0-1.
- Insert mirror M0-2. Adjust the horizontal and vertical screws such that the pump beam shoots approximately at the center of M0-2.
- Adjust the horizontal and vertical orientations of the pump beam after reflected from M0-2. The adjustments are done with the help of an iris. The iris is placed on a mount, and set the height of the iris to a desired position. This height should be fixed throughout all steps.
  - Vertical orientation: put the iris close to M0-2, adjust the vertical screw of M0-1 to match the heights of the pump beam and iris. Then, move the iris

further away, adjust the vertical screw of M0-2 to match the heights.

  - Horizontal orientation: move the iris close to M0-2, and align the iris with one of the holes on the line of holes. Adjust the horizontal screw of M0-1 to match the horizontal positions of the pump beam and iris. Then, move the iris further away while keeping the iris on the line of holes. Adjust the horizontal screw of M0-2 to match the horizontal positions again.

  - Repeat the above two steps until the alignment is optimum.

- Insert polarizer P and half-wave plate HWP0. Fix the axis of P to maximise the light intensity passing through P. Look at the reflections of pump beam on P and HWP0 on the laser source and M0-2 respectively. Adjust the angles to make the reflected beam match with the position of the source to ensure the surfaces of P and HWP0 are roughly perpendicular to the pump beam.

- Insert a single BBO crystal. Let the pump beam go through the center of the crystal and make sure the crystal's axis is in vertical direction. The crystal should be placed in a mirror mount instead of a rotational mount for flexibility. For rough alignments, similarly match the reflection with incoming pump beam by adjusting the screws.

b. Back-alignment

Back-alignment process directs the red back-alignment laser into the two detectors through the fibers, which should shoot on the center of the BBO crystal after adjustments.

- The angles the signal and idler beams make with the pump beam are roughly 3° as described in Section II A. Calculate the expected beam paths for the signal and idler beams. Measure 50cm away from the crystal along the pump beam direction. Calculate the distance of the signal and idler photons to this point, which should be $50cm \times \tan 3^\circ$. Place mirror M1 at that position.

- Place detector 1 along the same column of holes as M1.

- Make sure the lens tube doesn't have a high-pass filter in it. Connect detector 1 to back-alignment laser.

- Move the iris close to detector 1 and on a column of holes. Adjust the horizontal and vertical positions of detector 1 by aligning it with the iris.

- Move the iris further away while still on the same column of holes. Adjust the horizontal and vertical screws to align the laser beam with the iris.

- Adjust the position of mirror M1 to make sure the laser beam from detector 1 hits the center of the mirror.
- Adjust the horizontal and vertical screws of mirror M1 such that the reflected back-alignment laser can hit the center of the crystal.
- Repeat the above steps for M2 and detector 2 on the other path.

c. Fine alignment

The difficulty in fine alignment is imposed by the sensitivity of the photon counts. The goal should be to have the coincidence counts reaching around 3% of the single photon counts. If the final goal is to prove Bell's inequality, the absolute coincidence counts should be at least above 100 counts per second to do statistical analysis later on.

- Put the high-pass filters into the lens tubes for detector 1. Connect detector 1 to the photon counter.
- Make sure the room's light is turned off. Open pump beam and photon counter. Turn the coincidence unit's mode to single photon counting on detector 1.
- If the preliminary alignments are well-done, there should be some readings on the frequency meter already. Rotate HWP0 to look for sinusoidal changes in the number of photons received. If the sinusoidal behavior is observed, then the photons observed are generated by spontaneous down conversion. Otherwise, redo the preliminary alignment processes above.
- Fix HWP to angle where the photon count is maximised. Define the angle as θ_0. Since the single crystal is installed now, the photon counts will only be sensitive to one of the screws. When the crystal axis is in vertical direction, the horizontal screw is more sensitive, and the vertical screw should only have relatively small effects. Adjust the horizontal screw to maximise the counts.
- First adjust the vertical screw of M1, then the vertical screw of detector 1 to maximise the counts. Repeat for a few times. Then, repeat for the horizontal screws of M1 and detector 1.
- Sometimes physically moving the mount of detector 1 can improve the situation, but this operation should only be performed when absolutely necessary. The counts are very sensitive to changes, so doing so might result in losing all the counts obtained.
- Fix the positions of detector 1 and M1. They should be kept the same throughout the rest of the experiments.

- Similarly insert M2 and detector 2 for the other arm. The HWP0 angle maximized for detector 1's photon counts should also be the optimum angle for detector 2. An estimation of how well the alignments are can be done by observing the horizontal screw of the crystal. If the detectors are catching the down-converted photon pairs, the maximum of detector 1 and 2's counts should be achieved at around the same position of crystal's horizontal screw.

- Turn the coincidence unit's mode to coincidence between detector 1 and 2. Instead of maximising single detector counts, adjust the horizontal and vertical screws of M2 and detector 2 to maximise the coincidence rate.

- Fix the positions of detector 2 and M2. They should be kept the same throughout the rest of the experiments.

- Insert PBS1, PBS2, HWP1, and HWP2 on the expected paths for the signal and idler beams.

1. Lab 1: spontaneous parametric down-conversion (SPDC)

Spontaneous parametric down-conversion lab aims to demonstrate the predicted non-linear optics phenomenon produced by the single BBO crystal by measuring the polarization of the down-converted photons.

- Turn the coincidence unit to single photon counts of detector 1.

- Rotate HWP1's axis through a range of 90 degrees, which is the period of the sinusoidal function. Record the values of single detector counts at each point. Input the data and fit to a sine/cosine function to find the angle at which the maximum is reached. Define this angle to be θ_1.

- Repeat for the other arm. Find the angle of HWP2's axis, and define this angle to be θ_2.

- By checking θ_1 and θ_2, calculated what is the polarization of the incoming photons. As a result, verify the effect the BBO crystal has on the polarization of the signal and idler beam.

The angles θ_1 and θ_2 found are 41° and 44° respectively. Here, be aware that if all components are ideal, the angles should be both at 45°. However, the crystal isn't guaranteed to be strictly vertical, and the half-wave plates and polarizing beam-splitters aren't exactly centered at 810nm, which is the wavelength of the signal and idler beam passing through. Therefore, it is useful to find which angles of the half-wave plates' axis give maximised single detector counts.

2. Lab 2: photon entanglement

In lab 2, photon entanglement is explored. Here, the use of double crystal enables the photons to be in superposition of horizontally and vertically polarized states. When HWP1 and HWP2's axis are set at θ_1 and θ_2, the detectors are measuring vertically polarized photons. Therefore, when HWP1 and HWP2's axis are set at $\theta_1 + 45°$ and $\theta_2 + 45°$, the detectors are measuring horizontally polarized photons. Similarly, the signal beam and idler beam are at states $|VV\rangle$ and $|HH\rangle$ when HWP0 is set at θ_0 and $\theta_0 + 45°$ respectively.

- Install the double crystal to replace the single crystal. The double crystal consists of two single crystals such that the two crystals' axis are perpendicular to each other. Orient the double crystals so their axis are in the horizontal and vertical direction respectively.

- Rotate HWP1 and HWP2 to be at θ_1 and θ_2 respectively. Switch the coincidence unit to coincidence counts. Adjust the horizontal screw of the double crystal to maximise the coincidence counts. Verify that the vertical screw is not sensitive in this configuration. Record the maximum coincidence counts to be at N_{HH}.

- Rotate HWP1 and HWP2 to be at $\theta_1 + 45°$ and $\theta_2 + 45°$ respectively. Rotate HWP0 to be at $\theta_0 + 45°$. Adjust the vertical screw of the crystal to maximise the coincidence counts. Record the maximum counts to be N_{VV}. N_{HH} and N_{VV} should be roughly equal.

- Set HWP0 to angle θ_0.
  - Set HWP1 to angle θ_1, then rotate the angle of HWP2 through a range of 90°. Record multiple points' value of detector 2's single detector counts and coincidence counts.
  - Similarly set HWP2 to angle θ_2. Perform the operations on HWP1 and measure detector 1's single detector counts and coincidence counts.

- Rotate HWP0 through a range of 90 degrees. Measure the coincidence counts of $|HH\rangle$, $|VV\rangle$, and $|HV\rangle$ of the two detectors.

The last step should provide two angles at which $|HH\rangle$ and $|VV\rangle$ have the same coincidence counts. At these angles, the photon pairs are only a phase shift different from the Bell's states.

3. Lab 3: remote state preparation

- According to the results from Appendix A 2, find the angle of HWP0 at which $|HH\rangle$ and $|VV\rangle$ coincidence counts are about the same. Fix HWP0 at this angle.
- Insert QWP in front of HWP2 as shown in Fig. 3.
- Set the angle of HWP1 to be $0°$.
  - Find the angle of QWP's axis such that the visibility of coincidence counts is maximised with respect to rotation in HWP2's axis. Here, visibility is defined to be the relative difference between the maximum and minimum value of the coincidence counts when the HWP2's axis is rotated through a period, i.e. $90°$.
  - Fix QWP at this angle. Measure the coincidence counts as a function of HWP2's angle.
- Repeat the previous step for HWP1 angles at $15°$, $22.5°$, $45°$.

By plotting coincidence rates as a function of HWP1 angles, observe the photon state, and thus conclude remote state preparation has been done.