Derivation of the phase compensation

A simple derivation and simulation done for the phase compensation used for remote state preparation

Derivation

Assume initial state of pump beam after the double crystal to be $|\Psi_i\rangle$.

$$|\Psi_i\rangle = \frac{|HH\rangle + e^{i\psi}|VV\rangle}{\sqrt{2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i\psi} \end{pmatrix} \tag{1}$$

Here, ψ is the relative phase created by the double crystal. Ψ_i can be represented by a 4×1 matrix.

The optical components acting on this entangled state can be represented by the following operators.

$$\hat{A}_{PBS} = \begin{pmatrix} 1 & 0 \end{pmatrix} = \langle H| \tag{2}$$

$$\hat{A}_H(\alpha) = \begin{pmatrix} -cos2\alpha & -sin2\alpha \\ -sin2\alpha & cos2\alpha \end{pmatrix} \tag{3}$$

$$\hat{A}_Q(\alpha) = \begin{pmatrix} sin^2\alpha + icos^2\alpha & (i-1)sin\alpha cos\alpha \\ (i-1)sin\alpha cos\alpha & isin^2\alpha + cos^2\alpha \end{pmatrix} \tag{4}$$

Here, H is for half-wave plates, and Q is for quarter-wave plates. Assume the measured angles to be θ_{H_1}, θ_{H_2}, and θ_{Q_2} respectively for each component. Then the final measured coincidence rate should be as follows.

$$coincidence = Amp * (\hat{A}_{PBS}\hat{A}_H(\theta_{H_1}) \bigotimes \hat{A}_{PBS}\hat{A}_H(\theta_{H_2})\hat{A}_Q(\theta_{Q_2}))|\Psi_i\rangle \tag{5}$$

Amp represents the amplitude. Therefore, by fitting measured data of coincidence versus θ_{H_2}, the phase can be calculated.

Question

To test on my theory, I tried to print the function. I chose $\theta_{H_1} = 22.5$, $\theta_{Q_2} = 23$, and $Amp = 1$. Then I printed out the calculated values for coincidence versus θ_{H_2} for different phase values. The resulting graph is shown in Fig 1. Then, I plotted out my measured data in Fig 2. This set of data only contains a few data points because I didn't have time to do more. However, the trend doesn't look similar to the simulated function.

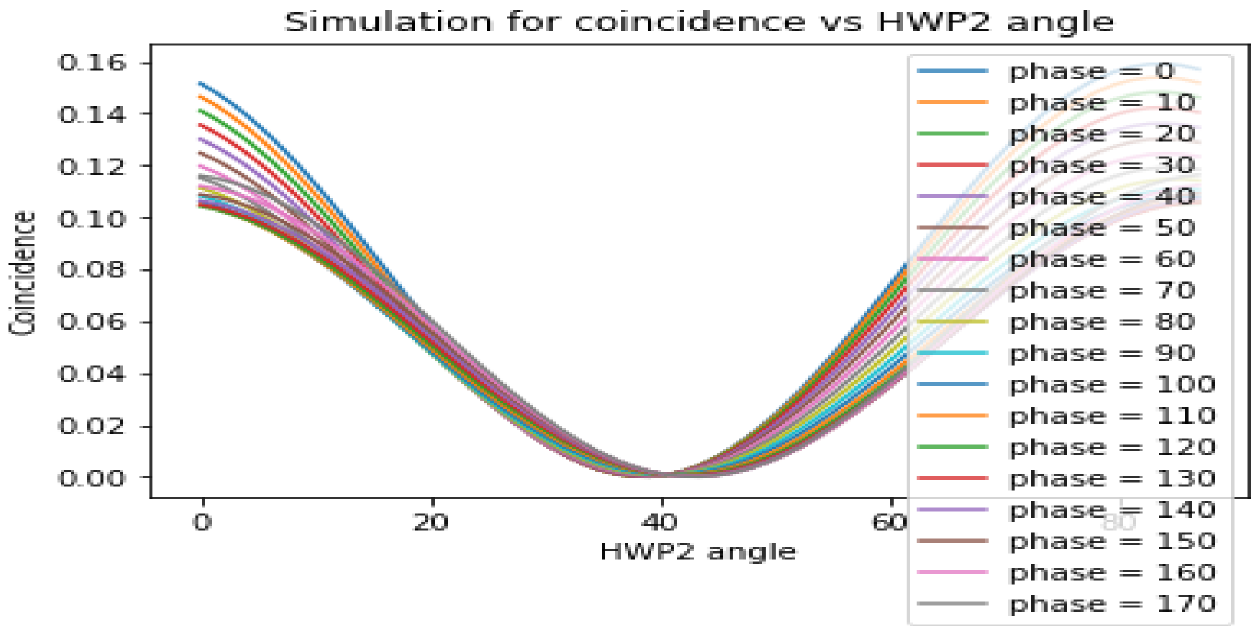

Figure 1: Simulation data

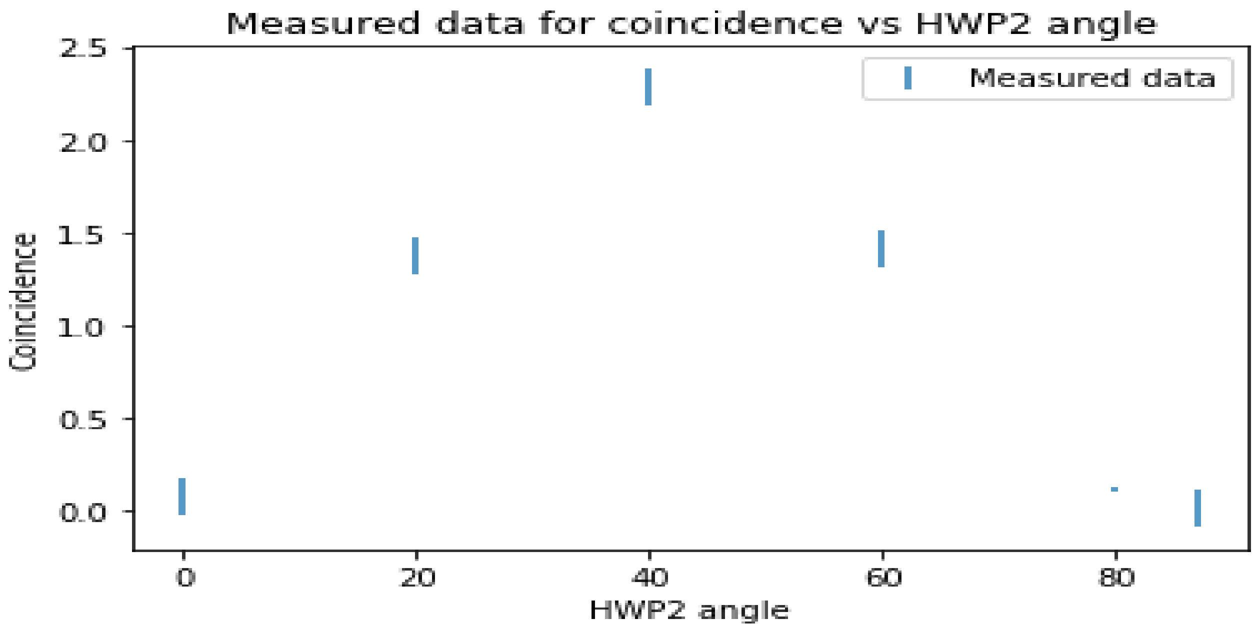

Figure 2: Measured data