Work Summary

Thoughts on the flaws expressed in the previous report.

Main Work

1. The potential flaws

1 The potential flaws

Following from the previous report, I want to show that $\hat{U}_{ge}$ and $\hat{U}_{eg}$ are also minized when $\hat{U}_{gg}$ is optimized to be unitary. Let the full unitary be

$$\hat{U} = \begin{bmatrix} \hat{U}_{gg} & \hat{U}_{eg} \\ \hat{U}_{ge} & \hat{U}_{ee} \end{bmatrix}.$$

Since all time slices are driven by unitary operators, the evolution $\hat{U}$ should also be unitary. Therefore $\hat{U}^\dagger \hat{U} = \hat{I}$. Expanding the expression gives

$$\hat{U}^\dagger_{gg}\hat{U}_{gg} + \hat{U}^\dagger_{eg}\hat{U}_{eg} = \hat{U}^\dagger_{ee}\hat{U}_{ee} + \hat{U}^\dagger_{ge}\hat{U}_{ge} = \hat{I}$$

and

$$\hat{U}^\dagger_{gg}\hat{U}_{ge} + \hat{U}^\dagger_{eg}\hat{U}_{ee} = \left(\hat{U}^\dagger_{gg}\hat{U}_{ge} + \hat{U}^\dagger_{eg}\hat{U}_{ee}\right)^\dagger = \hat{0}.$$

When optimized according to the definition of f_1 in the previous report, ideally

$$\hat{U}^\dagger_{gg}\hat{U}_{gg} = \hat{I} - \hat{\delta}, \text{ where } \hat{\delta} \ll \hat{I}.$$

Then, $\hat{U}^\dagger_{eg}\hat{U}_{eg} = \hat{\delta}$ and therefore the entries of $\hat{U}_{eg}$ are much smaller than that of $\hat{U}_{gg}$. Looking at the second equation obtained above then implies that the entries of $\hat{U}_{ge}$ are much smaller than $\hat{U}_{ee}$ in order to satisfy $\hat{U}^\dagger_{gg}\hat{U}_{ge} = -\hat{U}^\dagger_{eg}\hat{U}_{ee}$. If this is true, then the fidelity f_1 or f_2 works without the need of constraining $\hat{U}_{ge} = \hat{0}$ or $\hat{U}_{ee}\hat{U}_{ee} = \hat{I}$.