Work Summary

Thoughts on variational method under symmetric spin configurations.

Main Work

1. Variational under symmetric spin configurations

1 Variational under symmetric spin configurations

As shown in the previous report, when simulating the dynamics, symmetry properties of the Hamiltonian in its spin up and spin down components can be used to reduce the controls to the ones with the same symmetry. For example, see the following table for a list of controls that exhibit the symmetry. In simulating dynamics, it is often observed that the performance of GRAPE with these controls applied is normally better than the ones with independent controls.

Controls		
Control terms	Names	Nicknames
$\left\{\hat{a}_1 + \hat{a}_1^\dagger, i\left(\hat{a}_1 - \hat{a}_1^\dagger\right), \sigma_{1,x}, \sigma_{1,y}\right\}$ and $\left\{\hat{a}_2 + \hat{a}_2^\dagger, i\left(\hat{a}_2 - \hat{a}_2^\dagger\right), \sigma_{2,x}, \sigma_{2,y}\right\}$	Independent controls	independent
$\left\{\hat{a}_1 + \hat{a}_1^\dagger + \hat{a}_2 + \hat{a}_2^\dagger, i\left(\hat{a}_2 - \hat{a}_2^\dagger\right) + i\left(\hat{a}_1 - \hat{a}_1^\dagger\right), \sigma_{1,x} + \sigma_{2,x}, \sigma_{1,y} + \sigma_{2,y}\right\}$	Identical controls	identical
$\left\{\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger, i\left(\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_1\hat{a}_2^\dagger\right)\right\}$	Beam splitter	bs
$\left\{\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_1\hat{a}_2^\dagger\right\}$	Beam splitter with symmtry	symbs
$\left\{\hat{a}_1^\dagger\hat{a}_2^\dagger + \hat{a}_1\hat{a}_2, i\left(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2\right)\right\}$	Mode squeezer	sq
$\left\{\hat{a}_1^\dagger\hat{a}_2^\dagger + \hat{a}_1\hat{a}_2, i\left(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2\right)\right\}$	Mode squeezer with symmtry	symsq

Therefore, it is desirable to used these properties in variational method as well. However, for variational method, the ground state's symmetry properties can not be determined beforehand.

Let the Hilbert space be expressed as $\hat{H}_\uparrow \otimes \hat{H}_\downarrow$. The Hamiltonian is exchange symmetric, i.e.

$$\hat{H} = \sum_{i,j} \hat{H}_i \otimes \hat{H}_j = \hat{P}\hat{H}\hat{P} = \sum_{i,j} \hat{H}_j \otimes \hat{H}_i.$$

Then, applying the conventional derivation, if $|v\rangle$ is an eigenstate of Hamiltonian $\hat{H}$,

$$\hat{P}\left(\hat{H}|v\rangle\right) = \hat{P}\left(\lambda|v\rangle\right) = \lambda\hat{P}|v\rangle$$

and

$$\hat{P}\left(\hat{H}|v\rangle\right) = \left(\hat{P}\hat{H}\hat{P}\right)\hat{P}|v\rangle = \hat{H}\left(\hat{P}|v\rangle\right).$$

Thus, any eigenstate of $\hat{H}$ should also be an eigenstate of the exchange operator $\hat{P}$, which only has eigenvalues of ± 1. Therefore, the ground state should either be symmetric or anti-symmetric under exchange of spin-up and spin-down states.

If using the symmtric version of the controls, then the unitary

$$U = e^{-i\left(\sum_i \epsilon_i \hat{A}_i\right)\tau}$$

is also exchange symmetric. The final states evolved from the initial state through the unitary has the same symmetry as the initial state. Thus, for example, I can try the symmetric initial state

$$|\uparrow\downarrow, \uparrow\downarrow, -, -\rangle$$

and the anti-symmetric initial state

$$\frac{1}{\sqrt{2}}\left(|\uparrow\uparrow, \downarrow\downarrow, -, -\rangle - |\downarrow\downarrow, \uparrow\uparrow, -, -\rangle\right)$$

to cover both possible cases, and use the lower energy one as the ground state.