Taller Transformada de fourier (Jerson Maldonado)

1) $F\{e^{-a|t|}\}$; $a \in R^+$ $\quad F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$

$$= \int_{-\infty}^{\infty} e^{-a|t|} \cdot e^{-jwt} dt$$

Ahora definimos el Valor absoluto

$$|t| = \begin{cases} t \text{ si } t \geq 0 \\ -t \text{ si } t < 0 \end{cases}$$

$$= \int_{-\infty}^{0} e^{-a(-t)} e^{-jwt} dt + \int_{0}^{\infty} e^{-a(t)} e^{-jwt} dt$$

$$= \int_{-\infty}^{0} e^{(a-jw)t} dt + \int_{0}^{\infty} e^{(-a-jw)t}$$

$$= \frac{1}{(a-jw)} e^{(a-jw)t} \Big|_{-\infty}^{0} + \frac{1}{(-a-jw)} e^{(-a-jw)t} \Big|_{0}^{\infty}$$

$$= \frac{e^{(a-jw)t}}{(a-jw)} \Big|_{-\infty}^{0} + \frac{e^{(-a-jw)t}}{(-a-jw)} \Big|_{0}^{\infty}$$

$u = (a-jw)t$

$du = (a-jw)\, dt \Rightarrow \frac{du}{(a-jw)} = dt$

$$= \int_{-\infty}^{0} e^{ut} \frac{du}{(a-jw)} = \frac{1}{(a-jw)} \int e^{ut} du = \frac{1}{(a-jw)} e^{ut} \Big|_{-\infty}^{0}$$

$$= \frac{1}{(a-jw)} e^{(a-jw)t} \Big|_{-\infty}^{0}$$

$$\left(\frac{e^{(a-jw)(0)}}{a-jw} - \cancel{\frac{e^{-\infty(a-jw)}}{a-jw}}^{\,0} \right) + \left(\cancel{\frac{e^{\infty(-a-jw)}}{-a-jw}}^{\,0} + \frac{e^{(-a-jw)(0)}}{a+jw} \right)$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$\underbrace{x(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}}$$

b). $F\{\cos(\omega_c t)\}$; $\omega_c \in R$

$$= \int_{-\infty}^{\infty} \cos(\omega_c t) \cdot e^{-j\omega t} \, dt \quad \rightarrow \cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

$$\omega(t) = \frac{1}{2} \int_{-\infty}^{\infty} (e^{j\omega_c t} + e^{-j\omega_c t}) \cdot e^{-j\omega t} \, dt$$

$$\omega(t) = \frac{1}{2} \int_{-\infty}^{\infty} (e^{j\omega_c t} \cdot e^{-j\omega t}) \, dt + \frac{1}{2} \int_{-\infty}^{\infty} (e^{-j\omega_c t} \cdot e^{-j\omega t}) \, dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{j(\omega_c - \omega)(t)} \, dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{j(-\omega_c - \omega)(t)} \, dt$$

Aplicando la Propiedad del Delta de Dirac.

$$\rightarrow \int_{-\infty}^{\infty} e^{-j\alpha t} \, dt = 2\pi \delta(\alpha)$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-j(\omega - \omega_c)(t)} \, dt + \frac{1}{2} \int_{-\infty}^{\infty} e^{-j(\omega + \omega_c)(t)} \, dt \right]$$

Aplicando la propiedad

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_c) + 2\pi\,\delta(\omega+\omega_c)\right]$$

$$X(\omega) = \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

c). $\mathcal{F}\{\operatorname{Sen}(\omega_c t)\}$; $\omega_c \in \mathbb{R}$

$$= \int_{-\infty}^{\infty} \operatorname{Sen}(\omega_c t) \cdot e^{-j\omega t}\, dt \rightarrow \operatorname{Sen}(\omega_c t) = \frac{1}{2j}\left(e^{j\omega_c t} - e^{-j\omega_c t}\right)$$

$$= \frac{1}{2j}\int_{-\infty}^{\infty}\left(e^{j\omega_c t} - e^{-j\omega_c t}\right) \cdot e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty}\left(e^{j\omega_c t} \cdot e^{-j\omega t}\right) dt - \int_{-\infty}^{\infty}\left(e^{-j\omega_c t} \cdot e^{-j\omega t}\right) dt\right]$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty}\left(e^{-j(-\omega_c+\omega)t}\right) dt - \int_{-\infty}^{\infty}\left(e^{-j(\omega_c+\omega)t}\right) dt\right]$$

→ Aplicando la propiedad de delta de Dirac.

$$X(\omega) = \frac{1}{2j}\left[\mathcal{F}\{e^{j\omega_c t}\} - \mathcal{F}\{e^{-j\omega_c t}\}\right]$$

$$\therefore \mathcal{F}\{e^{j\omega_c t}\} = 2\pi\,\delta(\omega-\omega_d)$$

$$X(\omega) = \frac{1}{\cancel{2}j}\left[\cancel{2}\pi\,\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right] \quad \therefore \frac{1}{j} = -j$$

$$X(\omega) = -j\pi\left[\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right]$$

$$X(\omega) = j\pi\left[\delta(\omega+\omega_c) - \delta(\omega-\omega_c)\right]$$

d $\mathcal{F}\left\{ f(t) \cos(\omega_c t) \right\}$; $\omega_c \in \mathbb{R}$; $f(t) \in \mathbb{R}, \mathbb{C}$

$$X(\omega) = \int_{-\infty}^{\infty} f(t) \cos(\omega_c t) \cdot e^{-j\omega t} \cdot dt \qquad \bullet \cos(\omega_c t) = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

$$X(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} f(t) \left(e^{j\omega_c t} + e^{-j\omega_c t} \right) \cdot e^{-j\omega t} \, dt$$

$$X(\omega) = \frac{1}{2} \left[\int_{-\infty}^{\infty} f(t) \, e^{jt(\omega_c - \omega)} \, dt + \int_{-\infty}^{\infty} f(t) \, e^{-jt(\omega_c + \omega)} \, dt \right]$$

$$X(\omega) = \frac{1}{2} \left[\int_{-\infty}^{\infty} f(t) \, e^{-jt(\omega - \omega_c)} \, dt + \int_{-\infty}^{\infty} f(t) \, e^{-jt(\omega + \omega_c)} \, dt \right]$$

$$\omega' = (\omega - \omega_c) \quad ; \quad \omega'' = (\omega + \omega_c)$$

$$X(\omega) = \frac{1}{2} \left[\int_{-\infty}^{\infty} f(t) \, e^{-jt(\omega')} \, dt + \int_{-\infty}^{\infty} f(t) \, e^{-jt(\omega'')} \, dt \right]$$

$$X(\omega) = \frac{1}{2} \left[\mathcal{F}\left\{ f(t) \right\} + \mathcal{F}\left\{ f(t) \right\} \right]$$

$$X(\omega) = \frac{1}{2} \left[F(\omega') + F(\omega'') \right]$$

$$X(\omega) = \frac{1}{2} \left[F(\omega - \omega_c) + F(\omega + \omega_c) \right]$$

$$X(\omega) = \frac{1}{2} \left[F(\omega - \omega_c) + F(\omega + \omega_c) \right]$$

e. $\mathcal{F}\left\{e^{-a|t|^2}\right\}$; $a \in \mathbb{R}^+$

$$X(\omega) \int_{-\infty}^{\infty} e^{-a|t|^2} \cdot e^{-j\omega t}\, dt$$

$$|t| = \begin{cases} t \,;\; t \geq 0 \\ -t \,;\; t < 0 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{0} e^{-a(-t)^2} \cdot e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at^2} \cdot e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{0} e^{-at^2} \cdot e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at^2} \cdot e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at^2 - j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a\left(t^2 + \frac{j\omega t}{a}\right)}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a\left(\left(t + \frac{j\omega}{2a}\right)^2 - \left(\frac{j\omega}{2a}\right)^2\right)}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a\left(t + \frac{j\omega}{2a}\right)^2 - a\left(\frac{j\omega}{2a}\right)^2}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a\left(t + \frac{j\omega}{2a}\right)^2 - \frac{\omega^2}{4a}}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-a\left(t + \frac{j\omega}{2a}\right)^2} \cdot e^{-\frac{\omega^2}{2a}}\, dt$$

$$X(\omega) = e^{-\frac{\omega^2}{2a}} \int_{-\infty}^{\infty} e^{-a\left(t+\frac{j\omega}{2a}\right)^2} dt.$$

$$t + \frac{j\omega}{2a} = v$$

$$dt = dv.$$

Ahora los limites tambien cambian, debida al cambio de Variable.

$$X(\omega) = e^{-\omega^2/2a} \int_{-\infty+\frac{j\omega}{2a}}^{\infty+\frac{j\omega}{2a}} e^{-av^2} dv.$$

Sin embargo, el desplazamiento complejo no interfiere, entonces

$$X(\omega) = e^{-\frac{\omega^2}{2a}} \int_{-\infty}^{\infty} e^{-au^2} du \Rightarrow \text{Integral de Gauss}$$

$$X(\omega) = e^{-\frac{\omega^2}{2a}} \cdot \sqrt{\frac{\pi}{a}}$$

$$X(\omega) = \sqrt{\frac{\pi}{a}} \cdot e^{-\frac{\omega^2}{2a}}$$

F $\mathcal{F}\{A\,\text{rect}_d(t)\}, \; A, d \in \mathbb{R}$,

$$\text{rect}_d(t) = \begin{cases} 1, & \text{si } |t| \leq \frac{d}{2} \\ 0, & \text{en otro caso.} \end{cases} \qquad x(t) = A \quad \text{si } |t| \leq \frac{d}{2}$$

$$x(\omega) = \int_{-\infty}^{\infty} A\,\text{rect}_d(t) \cdot e^{-j\omega t}\, dt,$$ Acotando los limites de integración

$$x(\omega) = A \int_{-d/2}^{d/2} e^{-j\omega t}\, dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-d/2}^{d/2}\right] = A \cdot \frac{1}{-j\omega}\left(e^{-j\omega(d/2)} - e^{-j\omega(-d/2)}\right)$$

Aplicando identidad trigonometrica

$$e^{-j\theta} - e^{j\theta} = -2j\,\text{sen}\,\theta.$$

$$x(\omega) = \frac{A}{-j\omega}(-2j)\,\text{sen}(\omega d/2) = \frac{A \cdot 2\,\text{sen}(\omega d/2)}{\omega}$$

Utilizando la definición de Sinc normalizado (en radianes)

$$\text{sinc}(x) = \frac{\sin(x)}{x}, \qquad x(\omega) = A \cdot d\,\frac{\text{sen}(\omega d/2)}{(\omega d/2)}$$

$$x(\omega) = A \cdot d \cdot \text{sinc}\left(\frac{\omega d}{2}\right)$$

4. a) $\mathcal{F}\left\{e^{-j\omega_1 t}\cos(\omega_c t)\right\}$ $\omega_1, \omega_c \in \mathbb{R}$

Aplicando la identidad trigonometrica: $\cos(\omega_c t) = \frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})$

$$e^{-j\omega t}\cos(\omega_c t) = e^{-j\omega_1 t}\cdot\frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$$

$$= \frac{1}{2}\left(e^{-j(\omega_1-\omega)t} + e^{-j(\omega_1+\omega_d)t}\right)$$

Aplicando la transformada de Fourier: $\mathcal{F}\left\{e^{-j\omega_0 t}\right\} = 2\pi\,\delta(\omega-\omega_0)$

$$\mathcal{F}\left\{e^{-j\omega_1 t}\cos(\omega_c t)\right\} = \frac{1}{2}\left[\mathcal{F}\left\{e^{-j(\omega_1-\omega_c)t}\right\} + \mathcal{F}\left\{e^{-j(\omega_1+\omega_c)t}\right\}\right]$$

$$\mathcal{F}\left\{e^{-j\omega_1 t}\cos(\omega_c t)\right\} = \frac{1}{2}\left[2\pi\,\delta(\omega-(\omega_1-\omega_c)) + 2\pi\,\delta(\omega-(\omega_1+\omega_2))\right]$$

$$\mathcal{F}\left\{e^{-j\omega_1 t}\cos(\omega_c t)\right\} = \pi\left[\delta(\omega-(\omega_1-\omega_c)) + \delta(\omega-(\omega_1+\omega_c))\right]$$

b) $\mathcal{F}\left\{u(t)\cos^2(\omega_c t)\right\}$, $\omega_c \in \mathbb{R}$

$u(t)$ = Función escalon

Aplicando la identidad trigonometrica: $\cos^2(\omega_c t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)$

$$u(t)\cos^2(\omega_c t) = u(t)\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)$$

$$= \frac{1}{2}u(t) + \frac{1}{2}u(t)\cos(2\omega_c t).$$

- Aplicando propiedades lineales de la transformada de fourier.

$$\mathcal{F}\left\{u(t)\cos^2(\omega_c t)\right\} = \frac{1}{2}\mathcal{F}\left\{u(t)\right\} + \frac{1}{2}\mathcal{F}\left\{u(t)\cos(2\omega_c t)\right\}$$

La transformada de fourier de $u(t)$ es

$$\mathcal{F}\left\{u(t)\right\} = \pi\delta(\omega) + \frac{1}{j\omega}$$

Ahora, para $u(t)\cos(2\omega_c t)$; Utilizamos la propiedad de modulación:

$$\mathcal{F}\left\{u(t)\cos(\omega_0 t)\right\} = \frac{1}{2}\left[\mathcal{F}\left\{u(t)e^{j\omega_0 t}\right\} + \mathcal{F}\left\{u(t)e^{-j\omega_0 t}\right\}\right]$$

Además, $\mathcal{F}\left\{u(t)e^{\pm j\omega_0 t}\right\} = \pi\delta(\omega \mp \omega_0) + \frac{1}{j(\omega \mp \omega)}$

Entonces,

$$\mathcal{F}\left\{u(t)\cos(2\omega_c t)\right\} = \frac{1}{2}\left[\pi\delta(\omega - 2\omega_c) + \frac{1}{j(\omega - 2\omega_c)} + \pi\delta(\omega + 2\omega_c) + \frac{1}{j(\omega + 2\omega_c)}\right]$$

$$\mathcal{F}\left\{u(t)\cos^2(\omega_c t)\right\} = \frac{1}{2}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) + \frac{1}{4}\left[\pi\delta(\omega - 2\omega_c) + \frac{1}{j(\omega - 2\omega_c)} + \pi\delta(\omega + 2\omega_c) + \frac{1}{j(\omega + 2\omega_c)}\right]$$

$$\mathcal{F}\left\{u(t)\cos^2(\omega_c t)\right\} = \frac{\pi}{2}\delta(\omega) + \frac{1}{2j\omega} + \frac{\pi}{4}\left[\delta(\omega - 2\omega_c) + \delta(\omega + 2\omega_c)\right] + \frac{1}{4j}\left[\frac{1}{\omega - 2\omega_c} + \frac{1}{\omega + 2\omega_c}\right]$$

c) $\mathcal{F}^{-1}\left\{\frac{7}{\omega^2+6\omega+45} * \frac{10}{(8+j\omega/3)^2}\right\}$

→ Aplicando el teorema de convolución para la transformada de Fourier

$$\mathcal{F}^{-1}\left\{F(\omega) * G(\omega)\right\} = 2\pi . f(t) . g(t)$$

Donde.

$$f(t) = \mathcal{F}^{-1}\left\{F(\omega)\right\}, \quad g(t) = \mathcal{F}^{-1}\left\{G(\omega)\right\}$$

calcular la transformada inversa de la primera función $f(t)$

$$F(\omega) = \frac{7}{\omega^2+6\omega+45}$$

$$\omega^2+6\omega+45 = (\omega^2+6\omega+9)+36 = (\omega+3)^2+6^2$$

$F(\omega) = \frac{7}{(\omega+3)^2+6^2}$; Aplicando la Propiedad par de transformada. (Decaimiento exponencial)

$$F\left\{e^{-a|t|}\right\} = \frac{2a}{a^2+\omega^2} \longrightarrow H(\omega) = \frac{7}{\omega^2+6^2} \quad \cdot a=6$$

$\mathcal{F}^{-1}\left\{\frac{2(6)}{6^2+\omega^2}\right\} = e^{-6|t|}$, Ajustando las constantes

$$H(\omega) = \frac{7}{12} \cdot \frac{12}{\omega^2+6^2} \rightarrow h(t) = \mathcal{F}^{-1}\left\{H(\omega)\right\} = \frac{7}{12} e^{-6|t|}$$

Aplicamos la propiedad de desplazamiento en frecuencia

$H(\omega+3) \rightarrow \omega+3=0$ $\quad$ $f(t) = \mathcal{F}^{-1}\left\{H(\omega+3)\right\} = e^{-j3t} h(t)$

$\omega_0 = -3$

$$\underbrace{f(t) = \frac{7}{12} e^{-j3t} e^{-6|t|}}$$

Calcular la transformada inversa de la segunda función $g(t)$.

$G(\omega) = \frac{10}{(8+j\omega/3)^2}$, Aplicando el par de transformada:

$$\mathcal{F}\{te^{-at}u(t)\} = \frac{1}{(a+j\omega)^2}$$

Reescribiendo $G(\omega)$: $\frac{10}{\left(\frac{1}{3}(24+j\omega)\right)^2} = \frac{10}{\frac{1}{9}(24+j\omega)^2} = \frac{90}{(24+j\omega)}$

Entonces.

$\frac{90}{(24+j\omega)^2}$ coincide con la forma. $\frac{1}{(a+j\omega)^2}$.

$$g(t) = \mathcal{F}^{-1}\left\{\frac{90}{(24+j\omega)^2}\right\} = 90.\mathcal{F}^{-1}\left\{\frac{1}{(24+j\omega)^2}\right\}$$

$$g(t) = 90te^{-24t}u(t)$$

Aplicamos el teorema de convolución: $y(t) = 2\pi.f(t).g(t)$

$$y(t) = \cancel{2}\pi\left(\frac{7}{\cancel{12}_{\cancel{6}\,1}}e^{-j3t}.e^{-6|t|}\right)\left(\cancel{90}^{15}te^{-24t}u(t)\right)$$

$$y(t) = 105\pi e^{-j3t}e^{-6|t|}te^{-24t}u(t)$$

$$\begin{cases} u(t) \rightarrow 0 \text{ para } t<0 \\ |t| \rightarrow t \text{ para } t \geq 0 \end{cases}$$

$$e^{-6|t|}.e^{-24t} = e^{-6t}.e^{-24t} = e^{-30t} \quad (\text{para } t \geq 0)$$

$$y(t) = 105\pi.t.e^{-(30+j3)t}u(t)$$

d) $\mathcal{F}\{3t^3\}$

Aplicando la propiedad de Linealidad.

$\mathcal{F}\{3t^3\} = 3 \cdot \mathcal{F}\{t^3\}$, Aplicamos la Propiedad de Diferenciación en frecuencia.

$\mathcal{F}\{t^n x(t)\} = j^n \frac{d^n}{d\omega^n} x(\omega) \rightarrow n=3$, $x(t)=1$ $\quad X(\omega) = \mathcal{F}\{x(t)=1\}$

$\mathcal{F}\{1\} = 2\pi\,\delta(\omega)$.

$\mathcal{F}\{t^3\} = j^3 \frac{d^3}{d\omega^3}\left[2\pi\,\delta(\omega)\right]$

$j^3 = j^2 \cdot j = (-1)\,j = -j$

$\mathcal{F}\{t^3\} = -j\,2\pi \frac{d^3}{d\omega^3}\delta(\omega)$ $\quad \therefore \frac{d^3}{d\omega^3}\delta(\omega) = \delta^{(3)}(\omega)$

$\mathcal{F}\{3t^3\} = 3\left(-j\,2\pi\,\delta^{(3)}(\omega)\right)$ $\quad \therefore \mathcal{F}\{3t^3\} = -j\,6\,2\pi\,\delta^{(3)}(\omega)$

e) $\frac{B}{T}\sum_{n=-\infty}^{+\infty} F(\omega - n\omega_0)$. si $x(t) =$

$X(\omega) = C \sum_{n=-\infty}^{+\infty} F(\omega - n\omega_0)$ si $x(t) = f(t) \sum_{k=-\infty}^{+\infty} \delta(t - kT)$

Multiplicación de una señal periodica $F(t)$ por un tren de impulsos periodico.

$X(\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} F(\omega - n\omega_0)$.

Constante de escala D

Función de forma base en frecuencia es:

$$F(\omega) = \frac{1}{a^2+\omega^2} + \frac{1}{a+j(\omega)}$$

$F(t) = \mathcal{F}^{-1}\{F(\omega)\}$; Como $F(\omega)$; Aplicando la propiedad de Linealidad ya que $F(\omega)$ es una suma.

$$\mathcal{F}^{-1}\left\{\frac{1}{a^2+\omega^2}\right\} + \mathcal{F}^{-1}\left\{\frac{1}{a+j\omega}\right\}$$

- Aplicamos la Propiedad de par de transformada para el primer termino

$$\mathcal{F}^{-1}\left\{e^{-a|t|}\right\} = \frac{2a}{a^2+\omega^2} \rightarrow \mathcal{F}^{-1}\left\{\frac{1}{a^2+\omega^2}\right\} = \frac{1}{2a}e^{-a|t|}$$

Aplicando la propiedad de Par de transformada para el segundo termino.

$$\mathcal{F}^{-1}\left\{e^{-a|t|}u(t)\right\} = \frac{1}{a+j\omega} \rightarrow \mathcal{F}^{-1}\left\{\frac{1}{a+j\omega}\right\} = e^{-at}u(t)$$

Entonces la función base $F(t)$ es:

$$\mathcal{F}(t) = \frac{1}{2a}e^{-a|t|} + e^{-at}u(t)$$

Construimos la función Final en el dominio del tiempo.

$$x(t) = B.\, F(t)\left(\sum_{n=-\infty}^{+\infty}\delta(t-kT)\right)$$

$$x(t) = B.\left(\frac{1}{2a}e^{-a|t|} + e^{-at}u(t)\right)\sum_{n=-\infty}^{+\infty}\delta(t-kT)$$

$$x(t) = B\sum_{n=-\infty}^{+\infty} F(kT)\cdot\delta(t-kT), \quad F(kT) = \frac{1}{2a}e^{-a|kT|} + e^{-akT}u(kT)$$

Entonces, La expresión final para la señal en el tiempo es:

$$x(t) = B \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2a} e^{-a|kT|} + e^{-akT} u(kT) \right) \delta(t - kT)$$

Modulación AM

$$y(t) = \left(1 + \frac{m(t)}{A_c} \right) c(t).$$

$$c(t) = A_c \operatorname{SEN}(2\pi F_c t).$$

$$y(\omega) = \mathcal{F}\left\{ c(t) + \frac{c(t)\, m(t)}{A_c} \right\}.$$

$$y(\omega) = \mathcal{F}\left\{ c(t) \right\} + \frac{2}{A_c} \mathcal{F}\left\{ c(t)\, m(t) \right\}.$$

- $\mathcal{F}\left\{ c(t) \right\} = C(\omega)$

$$= A_c \int_{-\infty}^{\infty} \operatorname{SEN}(\omega_c t) . e^{-j\omega t}\, dt =$$

$$= A_c \mathcal{F}\left\{ \frac{e^{jmct} - e^{-j\omega_c t}}{2j} \right\}$$

$$= \frac{A_c}{2j} \left[\mathcal{F}\left\{ e^{-j2\pi f c} \right\} - \mathcal{F}\left\{ e^{-j\omega c t} \right\} \right]$$