

Grid Generator, Prerequisites

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Problem Statement

Given a digital elevation model (of Earth's surface), we need a domain of point coordinates in three dimensions suitable for solving partial differential equations on.

Requirements

Inputs

Data to be supplied by the user are:

- a digital elevation model covering the desired area,
- longitude and latitude intervals,
- a desired depth.

Outputs

Data to be output by the program are three tab-delimited .txt files containing the separate (x, y, z) -coordinates of a computational grid spanning the longitude and latitude limits and down to the desired depth.

2D Transfinite interpolation

$$\begin{aligned} x(\xi, \eta) = & (1 - \xi)x(0, \eta) + \xi x(1, \eta) + (1 - \eta)x(\xi, 0) + \eta x(\xi, 1) - (1 - \eta)(1 - \xi)x(0, 0) \\ & - \xi(1 - \eta)x(1, 0) - (1 - \xi)\eta x(0, 1) - \eta\xi x(1, 1) \end{aligned}$$

$$\begin{aligned} y(\xi, \eta) = & (1 - \xi)y(0, \eta) + \xi y(1, \eta) + (1 - \eta)y(\xi, 0) + \eta y(\xi, 1) - (1 - \eta)(1 - \xi)y(0, 0) \\ & - \xi(1 - \eta)y(1, 0) - (1 - \xi)\eta y(0, 1) - \eta\xi y(1, 1) \end{aligned}$$

3D Transfinite Interpolation

As in [1], the formula for 3D transfinite interpolation is as follows:

$$\begin{aligned}
U(\xi, \eta, \zeta) &= (1 - \xi)X(0, \eta, \zeta) + \xi X(1, \eta, \zeta), \\
V(\xi, \eta, \zeta) &= (1 - \eta)X(\xi, 0, \zeta) + \eta X(\xi, 1, \zeta), \\
W(\xi, \eta, \zeta) &= (1 - \zeta)X(\xi, \eta, 0) + \zeta X(\xi, \eta, 1), \\
UW(\xi, \eta, \zeta) &= (1 - \xi)(1 - \zeta)X(0, \eta, 0) + \zeta(1 - \xi)X(0, \eta, 1) \\
&\quad + \xi(1 - \zeta)X(1, \eta, 0) + \xi\zeta X(1, \eta, 1), \\
UV(\xi, \eta, \zeta) &= (1 - \xi)(1 - \eta)X(0, 0, \zeta) + \eta(1 - \xi)X(0, 1, \zeta) \\
&\quad + \xi(1 - \eta)X(1, 0, \zeta) + \xi\eta X(1, 1, \zeta), \\
VW(\xi, \eta, \zeta) &= (1 - \eta)(1 - \zeta)X(\xi, 0, 0) + \zeta(1 - \eta)X(\xi, 0, 1) \\
&\quad + \eta(1 - \zeta)X(\xi, 1, 0) + \eta\zeta X(\xi, 1, 1), \\
UVW(\xi, \eta, \zeta) &= (1 - \xi)(1 - \eta)(1 - \zeta)X(0, 0, 0) + (1 - \xi)(1 - \eta)\zeta X(0, 0, 1) \\
&\quad + (1 - \xi)\eta(1 - \zeta)X(0, 1, 0) + \xi(1 - \eta)(1 - \zeta)X(1, 0, 0) \\
&\quad + (1 - \xi)\eta\zeta X(0, 1, 1) + \xi(1 - \eta)\zeta X(1, 0, 1) \\
&\quad + (1 - \zeta)\xi\eta X(1, 1, 0) + \xi\eta\zeta X(1, 1, 1).
\end{aligned}$$

Putting these together gives the complete formula:

$$\begin{aligned}
X(\xi, \eta, \zeta) &= U(\xi, \eta, \zeta) + V(\xi, \eta, \zeta) + W(\xi, \eta, \zeta) \\
&\quad - UW(\xi, \eta, \zeta) - UV(\xi, \eta, \zeta) - VW(\xi, \eta, \zeta) \\
&\quad + U VW(\xi, \eta, \zeta).
\end{aligned}$$

References

- [1] Smith, Robert E (1998) Transfinite Interpolation Generation Systems. In Nigel P., et. al. *Handbook of Grid Generation*, CRC Press.