Grid Generator, Prerequisites

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Problem Statement

Given a digital elevation model (of Earth's surface), we need a domain of point coordinates in three dimensions suitable for solving partial differential equations on.

Requirements

Inputs

Data to be supplied by the user are:

- a digital elevation model (m above mean sea level) covering the desired area e.g. a GTOPO30 tile,

- longitude and latitude intervals (in degrees easting and northing),

- a desired depth (m).

Outputs

Data to be output by the program are three tab-delimited .txt files containing the separate (x, y, z)-coordinates of a computational grid spanning the longitude and latitude limits and down to the desired depth.

Architecture

Mathematics

2D Transfinite interpolation

$$x(\xi,\eta) = (1-\xi)x(0,\eta) + \xi x(1,\eta) + (1-\eta)x(\xi,0) + \eta x(\xi,1) - (1-\eta)(1-\xi)x(0,0)$$
$$- \xi(1-\eta)x(1,0) - (1-\xi)\eta x(0,1) - \eta\xi x(1,1),$$

$$y(\xi,\eta) = (1-\xi)y(0,\eta) + \xi y(1,\eta) + (1-\eta)y(\xi,0) + \eta y(\xi,1) - (1-\eta)(1-\xi)y(0,0)$$
$$- \xi(1-\eta)y(1,0) - (1-\xi)\eta y(0,1) - \eta\xi y(1,1).$$

3D Transfinite Interpolation

As in [1], the formula for 3D transfinite interpolation is as follows:

$$U(\xi,\eta,\zeta) = (1-\xi)X(0,\eta,\zeta) + \xi X(1,\eta,\zeta),$$
$$V(\xi,\eta,\zeta) = (1-\eta)X(\xi,0,\zeta) + \eta X(\xi,1,\zeta),$$
$$W(\xi,\eta,\zeta) = (1-\zeta)X(\xi,\eta,0) + \zeta X(\xi,\eta,1),$$
$$UW(\xi,\eta,\zeta) = (1-\xi)(1-\zeta)X(0,\eta,0) + \zeta(1-\xi)X(0,\eta,1)$$
$$+ \xi(1-\zeta)X(1,\eta,0) + \xi\zeta X(1,\eta,1),$$
$$UV(\xi,\eta,\zeta) = (1-\xi)(1-\eta)X(0,0,\zeta) + \eta(1-\xi)X(0,1,\zeta)$$
$$+ \xi(1-\eta)X(1,0,\zeta) + \xi\eta X(1,1,\zeta),$$
$$VW(\xi,\eta,\zeta) = (1-\eta)(1-\zeta)X(\xi,0,0) + \zeta(1-\eta)X(\xi,0,1)$$
$$+ \eta(1-\zeta)X(\xi,1,0) + \eta\zeta X(\xi,1,1),$$
$$UVW(\xi,\eta,\zeta) = (1-\xi)(1-\eta)(1-\zeta)X(0,0,0) + (1-\xi)(1-\eta)\zeta X(0,0,1)$$
$$+ (1-\xi)\eta(1-\zeta)X(0,1,0) + \xi(1-\eta)(1-\zeta)X(1,0,0)$$
$$+ (1-\xi)\eta\zeta X(0,1,1) + \xi(1-\eta)\zeta X(1,0,1)$$
$$+ (1-\zeta)\xi\eta X(1,1,0) + \xi\eta\zeta X(1,1,1).$$

Putting these together gives the complete formula:

$$X(\xi,\eta,\zeta) = U(\xi,\eta,\zeta) + V(\xi,\eta,\zeta) + W(\xi,\eta,\zeta)$$
$$- UW(\xi,\eta,\zeta) - UV(\xi,\eta,\zeta) - VW(\xi,\eta,\zeta)$$
$$+ UVW(\xi,\eta,\zeta).$$

References

[1] Smith, Robert E (1998) Transfinite Interpolation Generation Systems. In Nigel P., et. al. *Handbook of Grid Generation*, CRC Press.