Hand-in B

We consider two Einstein solids (A and B), each having N oscillators. The total energy of the two solids is q in harmonic oscillator units, and the multiplicity function of the combined system is

$$\Omega(q_A) = \frac{(q_A + N - 1)!}{q_A!(N-1)!} \cdot \frac{(q - q_A + N - 1)!}{(q - q_A)!(N-1)!},$$

where q_A is the number of energy units in Einstein solid A.

Question a:

Show that the average value of the number of energy units in solid A is

$$\overline{q_A} = \frac{\sum_{q_A=0}^{q} q_A \Omega(q_A)}{\sum_{q_A=0}^{q} \Omega(q_A)} = \frac{q}{2}. \tag{1}$$

Argue for both the first and the second equality sign.
Hint for the second equality sign: Write $q_A = \frac{q}{2} + x$, reexpress the sums in terms of x, and use symmetry arguments.

We will now investigate the fluctuations in the energy distribution between the two Einstein solids when $q \gg N \gg 1$. In this case, the multiplicity function Ω for the combined system is given by equation (2.27) in Schroeder, and we can approximate sums by integrals.

Question b:

Show that in this approximation

$$\overline{q_A} = \frac{\int_{-\infty}^{\infty} (\frac{q}{2} + x)\Omega dx}{\int_{-\infty}^{\infty} \Omega dx} = \frac{q}{2}. \tag{2}$$

Argue for both the first and the second equality sign.

Question c:

A useful measure for the fluctuations away from the mean value is the standard deviation σ given by

$$\sigma^2 \equiv \overline{(q_A - \overline{q_A})^2} = \overline{\left(q_A - \frac{q}{2}\right)^2}.$$

Show that

$$\sigma = \frac{q}{2^{3/2}\sqrt{N}}. \tag{3}$$

Question d:

Comment on the size of the standard deviation σ of q_A compared to $\overline{q_A}$.