18.338 Eigenvalues of Random Matrices

Fall 2019 Syllabus

Mon. Wed. 2:30-4:00pm in 4-261

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Course website: http://web.mit.edu/18.338/www/

Description

This course covers mathematical, computational, and applied areas of random matrix theory. We want and expect students in mathematics, science, and engineering/finance applications.

Prerequisites

No particular prerequisites are needed though a proficiency in linear algebra and basic probability theory will be assumed. A familiary with numerical computing languages such as Julia, MATLAB, or Mathematica may be useful .... our primary focus will be Julia (http://julialang.org) and some Mathematica.

Content

- **Highlights of random matrix theory**: A crash course on the main ideas
- **Classical random matrix theory**: β-Gaussian ensembles ($\beta = 1, 2, 4$ corresponding to real, complex and quaternions).
- **Infinite random matrix theory**
  - Wigner's semicircle law, Marcenko-Pastur Law, Wachter's Law, Circular Law
  - using combinatorial techniques to derive the limiting distributions of classical random matrix ensembles
  - Tracy-Widom Distribution, Fredholm Determinants and Eigenvalue spacings
  - Stochastic operators
  - The Riemann-Hilbert problem and equilibrium measure. Interpretation of limiting distribution as the equilibrium measure. Applications to physics.
  - Universality and beyond universality
- **Finite random matrix theory**
  - Orthogonal polynomials, Jacobi matrix, quadrature and all that: Hermite, Laguerre and Jacobi polynomials
  - Jack polynomials, zonal polynomials and Hypergeometric functions. Combinatorial aspects.
  - Level density for finite random matrices. Correlation functions. Fredholm Determinants
- **Free probability**: The concept of freeness and partial freeness. Free cumulants and non-crossing partitions. The R-transform. Fluctuations.
- **Random growth models, Determinantal Point Process and its Application in Machine Learning.**
- **Understanding Deep Learning with Random Matrix Theory**: landscape of nonconvex functions, random polynomials, spin-glass models.

Homework

Homework assignments will be geared towards both math and computation (your choice!). There will also be readings of my notes and handouts. Students are required to comment on the notes. (This will help future students!) Students can write Julia code and/or prove new theorems depending on one's interest!

Course Project

You will be asked to come up with a project on a random matrix problem that is of interest to you. Also students who are highly into projects with permission can trade off projects for homework. A list of project suggestions will be handed out shortly.

Grading

The course has no official TA.

Textbook

There is no one book that covers any significant portion of this syllabus. Instead, we will use extracts from a book that is currently being written. There will also be course readers. However, we recommend looking at

- *An introduction to Random Matrices* by Greg Anderson, Ofer Zeitouni and Alice Guionnet
- *Topics in Random Matrix Theory* by Terry Tao
- *Log-Gases and Random Matrices* by Peter Forrester is a comprehensive book for finite random matrix theory
- *Oxford Handbook of Random Matrix Theory* edited by Gernot Akemann, Jinho Baik, and Philippe Di Francesco which contains a number of specialized articles
- the original book by Mehta (*Random Matrices*) is still worth looking at for Hermite and Circular Ensembles
- Murihead's *Aspects of Multivariate Statistical Theory* remains a favorite case for real Laguerre and Jacobi ensembles.

*If you have a disability accommodation letter from SDS, please speak with the Mathematics disabilities accommodation coordinator Galina Lastovkina in the MAS (galina@math.mit.edu) to make arrangements.