MA5233 Computational Mathematics

Lecture 18: Adaptive Quadrature

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Adaptive Quadrature

Adaptive quadrature and ODE solvers

Both composite quadrules and ODE solvers involve an interval width / step size Δ such that error decreases but work increases for $\Delta \to \infty$.

Algorithms formulated so far require us to choose Δ and then compute a result with a certain error $e(\Delta)$.

In most applications, what we actually want is the opposite: we want to specify an error tolerance ε, and algorithm should determine the Δ that is needed to meet this tolerance.

Such algorithms exist, and in addition to being more convenient they can also be more efficient.

We will focus on quadrature in this lecture for convenience, but all ideas apply equally to ODE solvers.

Adaptive Quadrature

Basic idea

- Use two quadrules of different orders to estimate $\int_a^b f(x)\,dx$.
- If results are within tolerance ε of each other, than the approximation is probably good enough.
- If not, split $\int_a^b f(x)\,dx = \int_a^m f(x)\,dx + \int_m^b f(x)\,dx$ with $m = \frac{a+b}{2}$ and use recursion to compute each integral up to tolerance $\frac{\varepsilon}{2}$.

Refinements

- We can reduce the number of function evaluations if quadrules are *embedded*, i.e. if the quadpoints of the coarse rule are a subset of the quadpoints of the fine rule.
- Instead of halving the integration interval, it is also possible to increase the orders of the quadrules, but this is less common in practice (and would be more difficult for ODE solvers).

Implementation

See `18_adaptive_quadrature.jl`.