MA3227 Numerical Analysis II

Lab Session 1

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General lab rules

- Work through the below questions yourself or in small groups.
- You are highly encouraged to show me your results so I can check them and discuss them with you.

1 Finite differences and sparse LU

Consider Poisson's equation on $(0,1)^2$, i.e. the problem of finding $u : [0,1]^2 \to \mathbb{R}$ such that

$$-\Delta u = f \quad \text{on } (0,1)^2, \qquad u = 0 \quad \text{on } \partial(0,1)^2. \tag{1}$$

Lecture 3 presented a `solve_poisson(f,n)` function which solves this problem using a finite difference discretisation.

1. Demonstrate numerically that $\|u - u_n\|_{2,n} = \mathcal{O}(n^{-2})$ where u denotes the exact solution to (1), u_n is the output of `solve_poisson()`, and

$$\|u\|_{2,n} := \frac{1}{n+1} \sqrt{\sum_{i_1,i_2=1}^{n} u\big(\tfrac{i_1}{n+1}, \tfrac{i_2}{n+1}\big)^2}.$$

   Hint. Poisson's equation can be solved analytically for $f(x_1,x_2) = \sin(\pi x_1)\,\sin(\pi x_2)$.

2. Demonstrate numerically that the runtime of solving the finite-difference linear system using Julia's \ operator is $\mathcal{O}(N^{3/2}) = \mathcal{O}(n^3)$.

   Hint. Have a look at the `@elapsed()` macro (i.e. type `?@elapsed` in the REPL and hit enter to see the documentation).

3. Further ideas if you have the time:
   - Try to rewrite `solve_poisson()` yourself, starting from scratch.
   - Extend your code such that it can solve the variable-coefficient problem

     $$-\nabla \cdot (D(x)\,\nabla u) = f \quad \text{on } (0,1)^2, \qquad u = 0 \quad \text{on } \partial(0,1)^2$$

     where $D : [0,1]^2 \to \mathbb{R}$.
   - Extend your code to three dimensions.