Holt-Winters

Abigail Asare

Forecasting Exercise 5

Learning Objectives

- Holt-Winters with trend and no seasonality
- Holt-Winters with trend and seasonality

Holt-Winters Components

Double exponential smoothing is also known as Holt's Linear trend model. Here we assume that trend can change over time.

- Here we assume that trend can change over time
- It is suitable for time series with trend but without seasonality.

$$\hat{x}_{t+h|t} = g_t + b_t h \ldots\ldots\ldots\ldots (1)$$

- g_t is the level estimate at time t
- b_t is the slope of the trend h
- h is the trend estimate at time t which we assumed as additive.

Updating Equations

NB: The level and Trend are updated as more data become available through a pair of updating equations:

$$g_t = \alpha y_t + (1-\alpha)(g_{t-1} + b_{t-1}) \ldots\ldots\ldots\ldots\ldots (2)$$

$$b_t = \beta(g_t - g_{t-1}) + (1-\beta) b_{t-1} \ldots\ldots\ldots\ldots\ldots (3)$$

- g_t is the smoothed level, simply weighted average of the actual values at time t and the level in the previous adjusting for trend.
- b_t estimate of trend at time t
- α is the smoothing parameter of the data.
- β is the trend smoothing parameter.
- α and β determine the rate of learning, higher values faster learning and more weight to recent information.
- $0 < \alpha/\beta < 1$

Starting Values

- What are the starting values for g_t and b_t. Very important
- g_t and b_t

$$\hat{x}_{t+h|t} = g_t + b_t h$$

- β_0 start value for g_t
- β_1 start value for b_t

h-step Forecast

- For h-step forecasts:

$$x_{T+h} \leftarrow \hat{x}_{h|T} = g_T + b_T h$$

- Forecast Error

$$\hat{u}_{h|T} = x_{T+h} - \hat{x}_{h|T}$$

Holt-Winters with Additive Seasonal Fluctuations

- It is suitable for time series with trend but with additive seasonality.

$$\hat{x}_{t+h|t} = g_t + b_t h + s_{t+h-m} \quad \text{.....(4)}$$

- Updating equations:

$$g_t = \alpha(x_t - s_{t-m}) + (1-\alpha)(g_{t-1} + b_{t-1}) \quad \text{.....(5)}$$

$$b_t = \beta(g_t - g_{t-1}) + (1-\beta)b_{t-1} \quad \text{.....(6)}$$

$$s_t = \gamma(x_t - g_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m} \quad \text{.....(7)}$$

- m which indicates seasonal indices used for forecasting is from the final year of the sample.

Holt-Winters with Additive Seasonal Fluctuations

- m is the length of the seasonality.
- We assume seasonality with m to be the number of the seasons in the first half of the sample.
- α is the smoothing constant, β is the slope parameter and γ is the parameter for adjusting seasonal fluctuations.
- We need m start values for the seasonal factors.
- We also need start values for g_t and b_t.

Holt-Winters with Multiplicative Seasonal Fluctuations

- It is suitable for time series with trend but with multiplicative seasonality.

$$\hat{x}_{t+h|t} = (g_t + b_t h)s_{t+h-m} \quad \text{.....(8)}$$

- Updating equations:

$$g_t = \alpha \frac{x_t}{s_{t-m}} + (1-\alpha)(g_{t-1} + b_{t-1}) \quad \text{.....(9)}$$

$$b_t = \beta(g_t - g_{t-1}) + (1-\beta)b_{t-1} \quad \text{.....(10)}$$

$$s_t = \gamma \frac{x_t}{(g_{t-1} + b_{t-1})} + (1-\gamma)s_{t-m} \quad \text{.....(11)}$$

- In multiplicative seasonality, values in different seasons differ by percentage amount