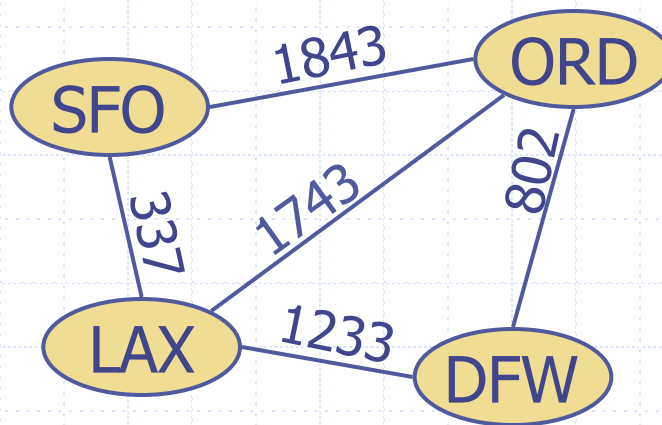
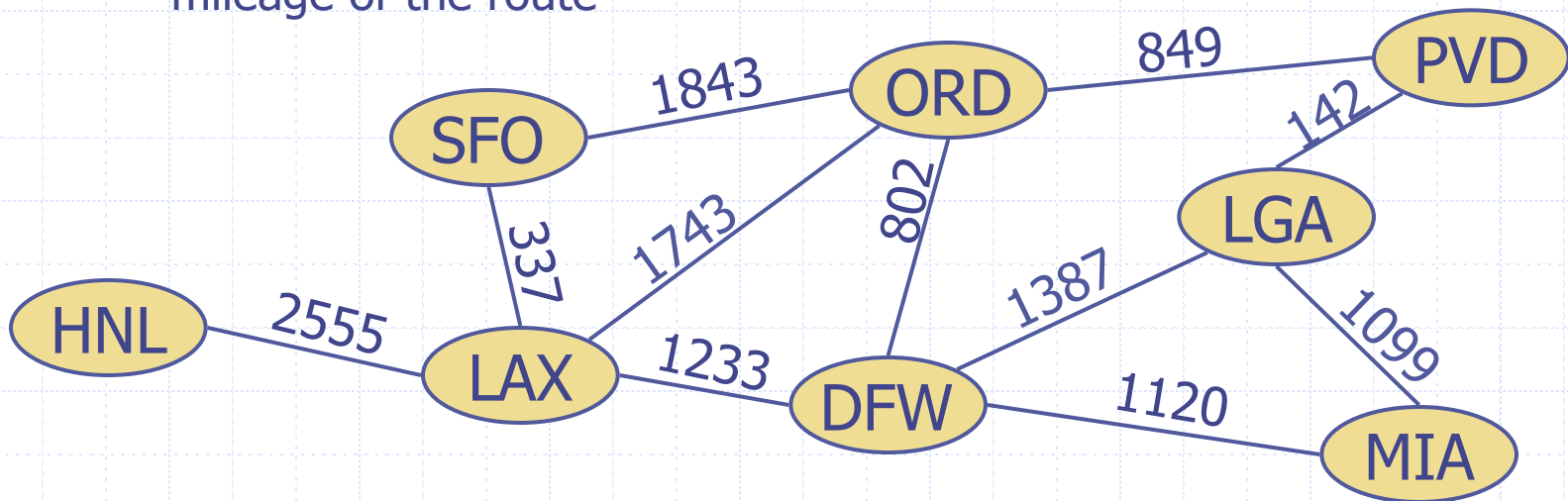


Graphs



Graphs

- A graph is a pair (V, E) , where
 - V is a set of nodes, called **vertices**
 - E is a collection of pairs of vertices, called **edges**
 - Vertices and edges are positions and store elements
- Example:
 - A vertex represents an airport and stores the three-letter airport code
 - An edge represents a flight route between two airports and stores the mileage of the route



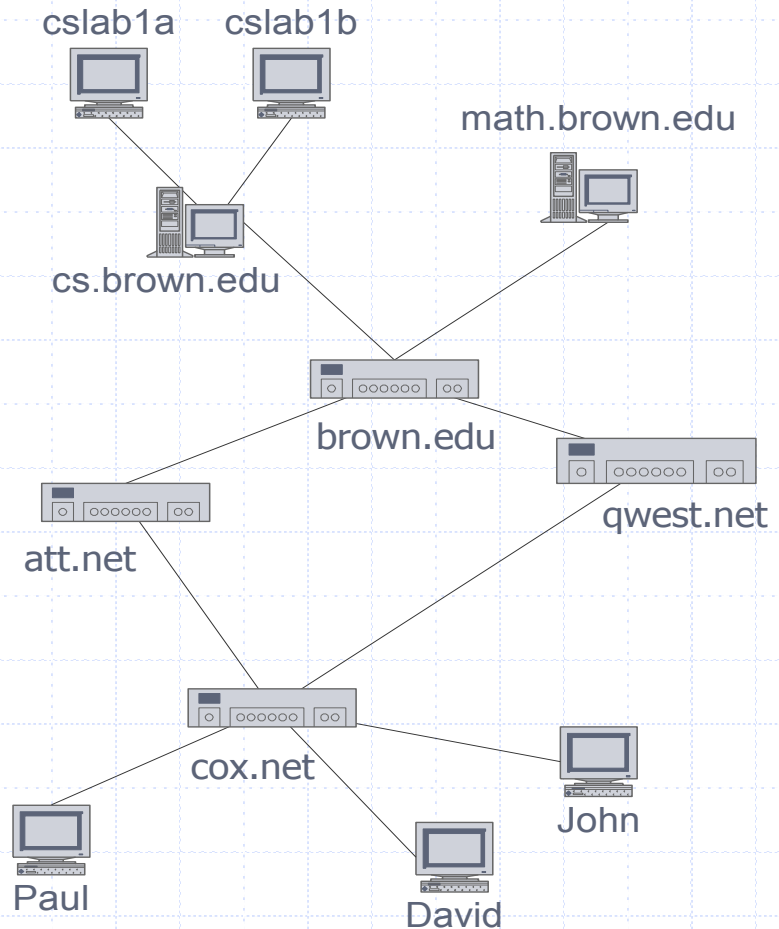
Edge Types

- ❑ Directed edge
 - ordered pair of vertices (u,v)
 - first vertex u is the origin
 - second vertex v is the destination
 - e.g., a flight
- ❑ Undirected edge
 - unordered pair of vertices (u,v)
 - e.g., a flight route
- ❑ Directed graph
 - all the edges are directed
 - e.g., route network
- ❑ Undirected graph
 - all the edges are undirected
 - e.g., flight network



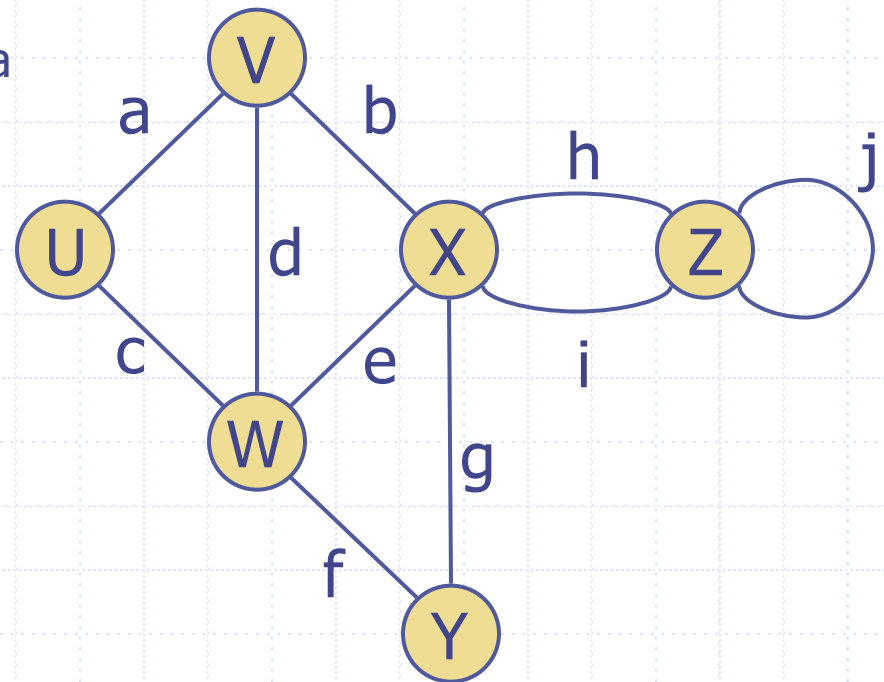
Applications

- ❑ Electronic circuits
 - Printed circuit board
 - Integrated circuit
- ❑ Transportation networks
 - Highway network
 - Flight network
- ❑ Computer networks
 - Local area network
 - Internet
 - Web
- ❑ Databases
 - Entity-relationship diagram



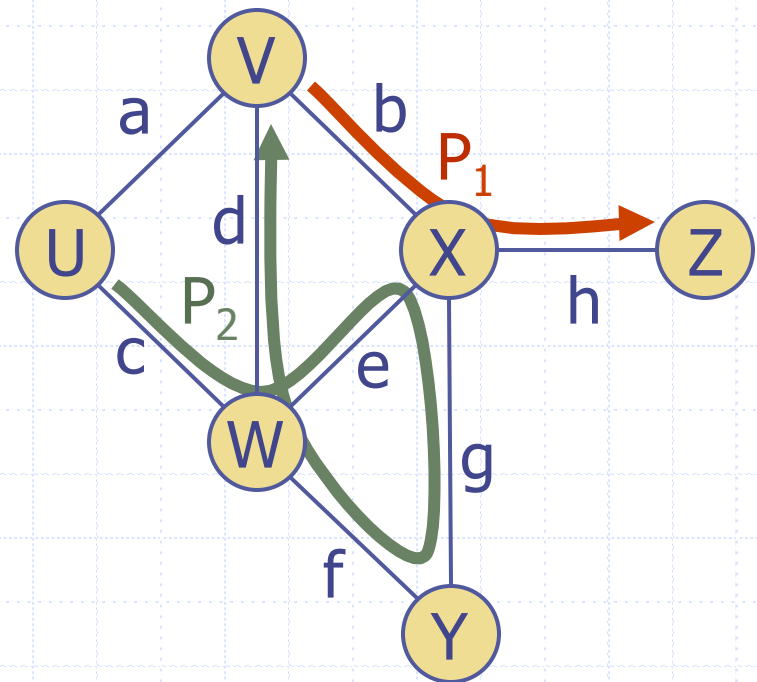
Terminology

- ❑ End vertices (or endpoints) of an edge
 - U and V are the endpoints of a
- ❑ Edges incident on a vertex
 - a, d, and b are incident on V
- ❑ Adjacent vertices
 - U and V are adjacent
- ❑ Degree of a vertex
 - X has degree 5
- ❑ Parallel edges
 - h and i are parallel edges
- ❑ Self-loop
 - j is a self-loop



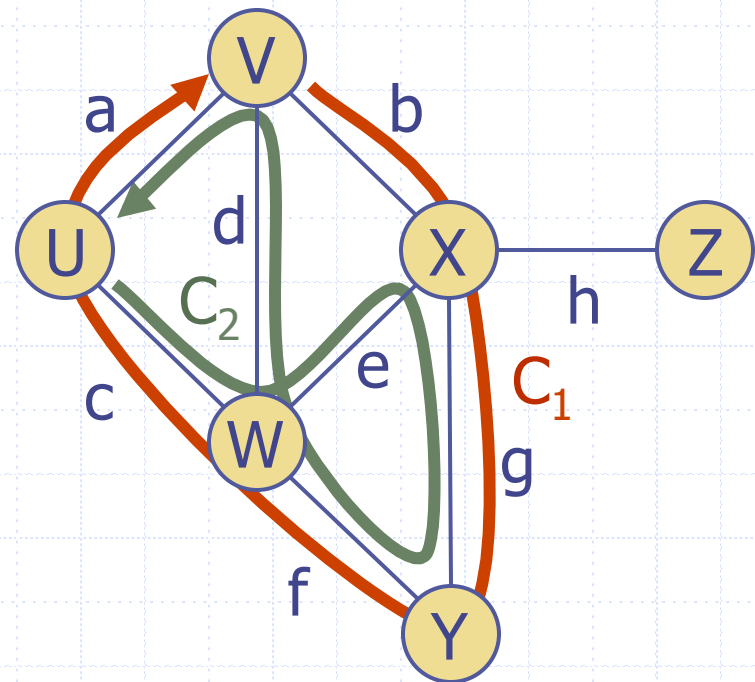
Terminology (cont.)

- Path
 - sequence of alternating vertices and edges
 - begins with a vertex
 - ends with a vertex
 - each edge is preceded and followed by its endpoints
- Simple path
 - path such that all its vertices and edges are distinct
- Examples
 - $P_1 = (V, b, X, h, Z)$ is a simple path
 - $P_2 = (U, c, W, e, X, g, Y, f, W, d, V)$ is a path that is not simple



Terminology (cont.)

- Cycle
 - circular sequence of alternating vertices and edges
 - each edge is preceded and followed by its endpoints
- Simple cycle
 - cycle such that all its vertices and edges are distinct
- Examples
 - $C_1 = (V, b, X, g, Y, f, W, c, U, a, \hookleftarrow)$ is a simple cycle
 - $C_2 = (U, c, W, e, X, g, Y, f, W, d, V, a, \hookleftarrow)$ is a cycle that is not simple



Properties

Property 1

$$\sum_v \deg(v) = 2m$$

Proof: each edge is counted twice

Property 2

In an undirected graph with no self-loops and no multiple edges

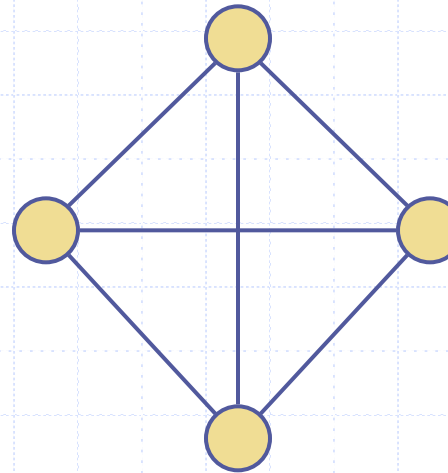
$$m \leq n(n-1)/2$$

Proof: each vertex has degree at most $(n-1)$

What is the bound for a directed graph?

Notation

n	number of vertices
m	number of edges
$\deg(v)$	degree of vertex v



Example

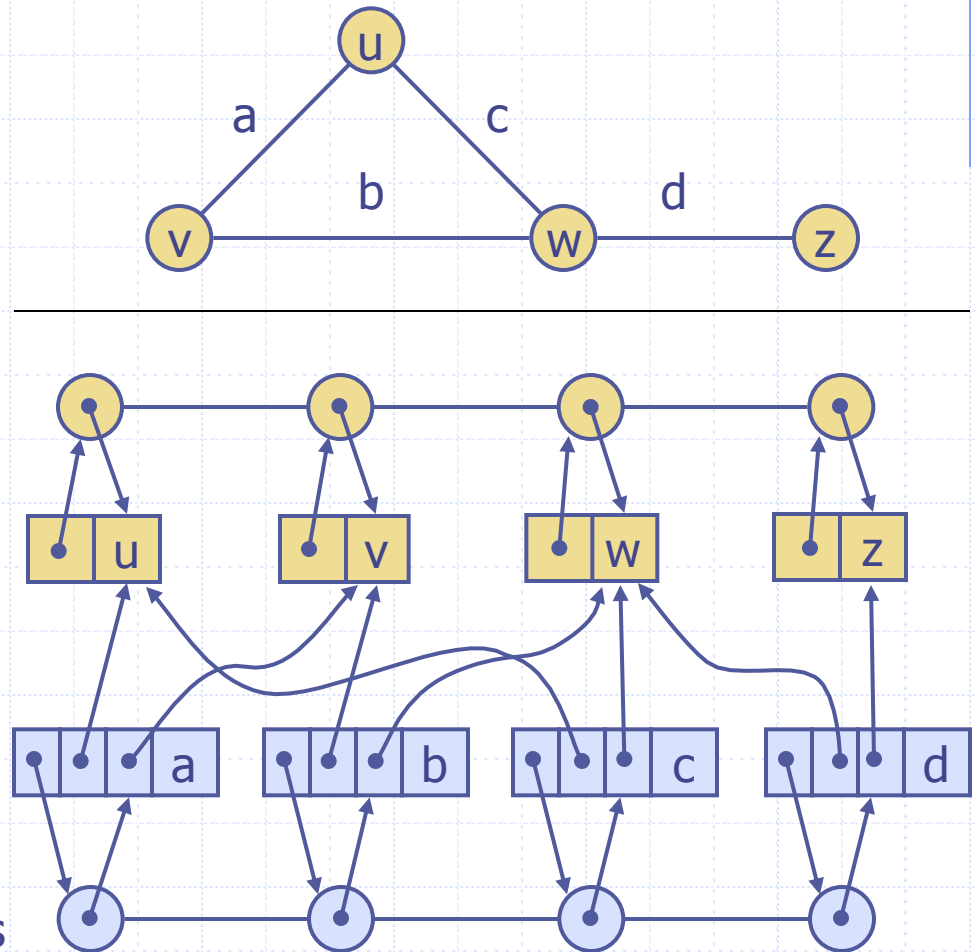
- $n = 4$
- $m = 6$
- $\deg(v) = 3$

Main Methods of the Graph ADT

- Vertices and edges
 - are positions
 - store elements
- Accessor methods
 - `e.endVertices()`: a list of the two endvertices of `e`
 - `e.opposite(v)`: the vertex opposite of `v` on `e`
 - `u.isAdjacentTo(v)`: true iff `u` and `v` are adjacent
 - `*v`: reference to element associated with vertex `v`
 - `*e`: reference to element associated with edge `e`
- Update methods
 - `insertVertex(o)`: insert a vertex storing element `o`
 - `insertEdge(v, w, o)`: insert an edge `(v,w)` storing element `o`
 - `eraseVertex(v)`: remove vertex `v` (and its incident edges)
 - `eraseEdge(e)`: remove edge `e`
- Iterable collection methods
 - `incidentEdges(v)`: list of edges incident to `v`
 - `vertices()`: list of all vertices in the graph
 - `edges()`: list of all edges in the graph

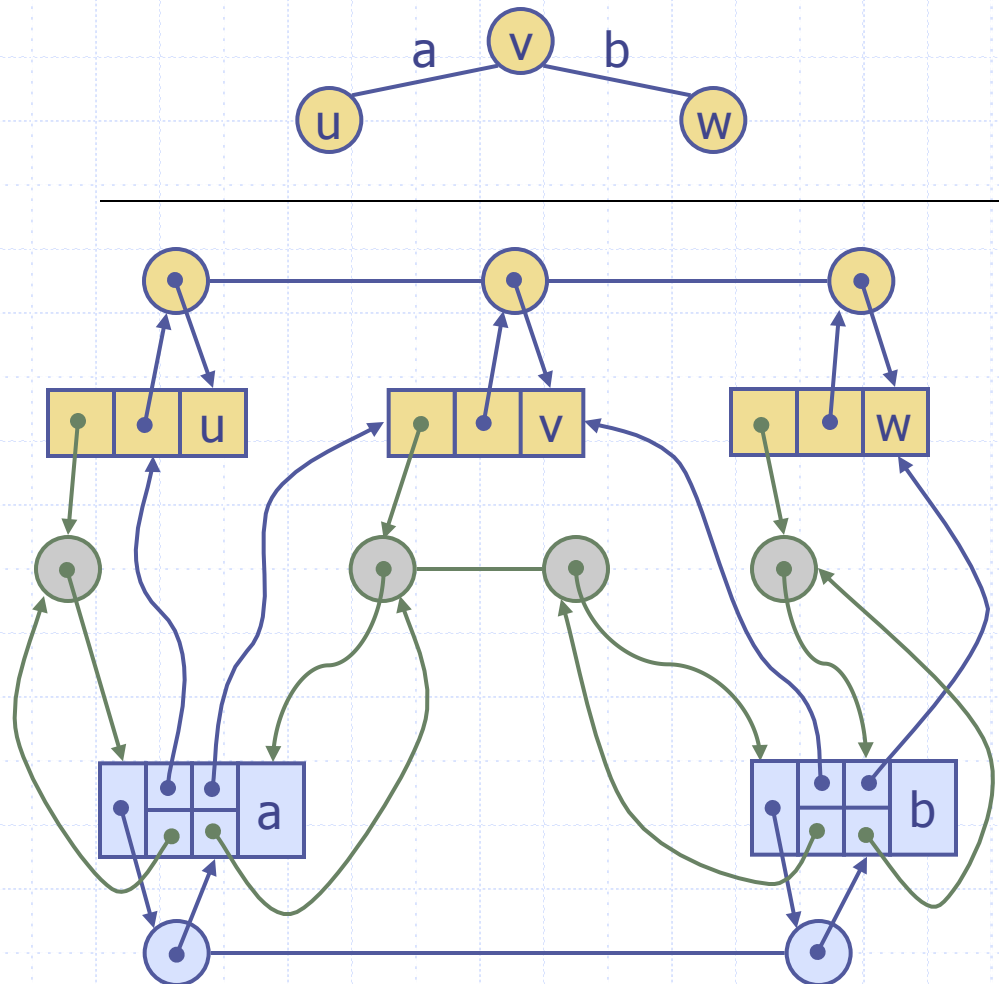
Edge List Structure

- Vertex object
 - element
 - reference to position in vertex sequence
- Edge object
 - element
 - origin vertex object
 - destination vertex object
 - reference to position in edge sequence
- Vertex sequence
 - sequence of vertex objects
- Edge sequence
 - sequence of edge objects



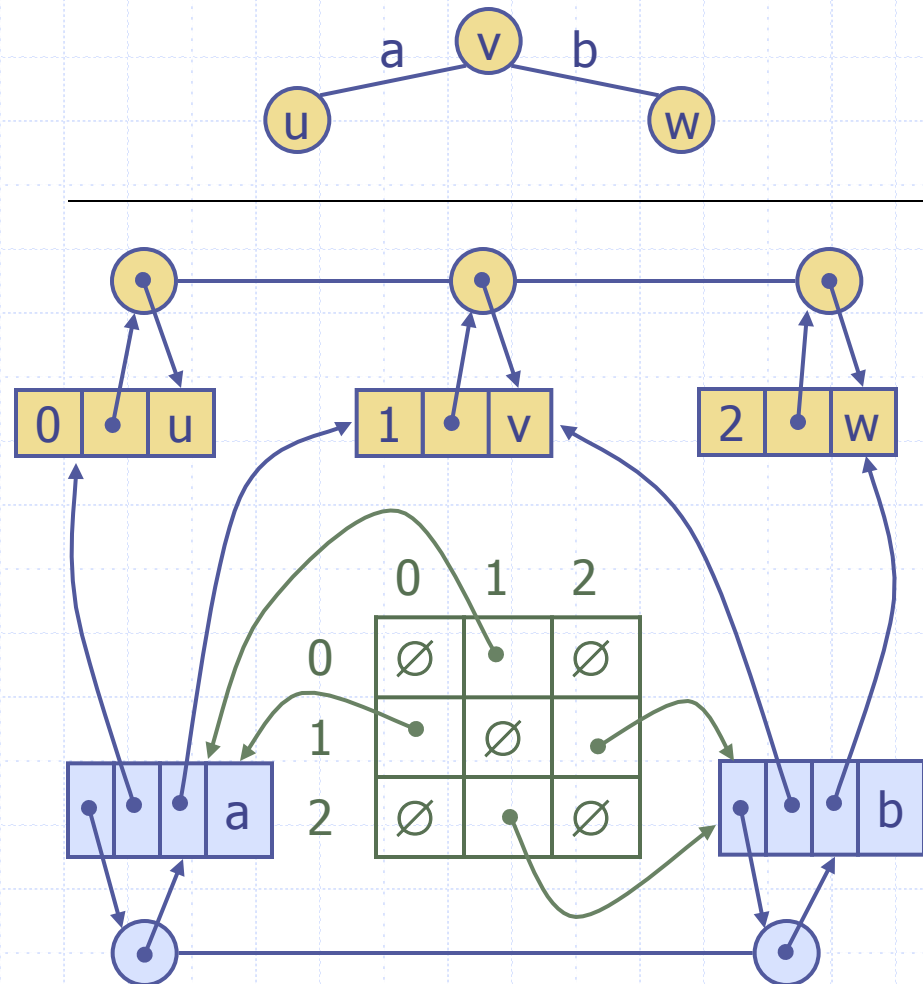
Adjacency List Structure

- Edge list structure
- Incidence sequence for each vertex
 - sequence of references to edge objects of incident edges
- Augmented edge objects
 - references to associated positions in incidence sequences of end vertices



Adjacency Matrix Structure

- ❑ Edge list structure
- ❑ Augmented vertex objects
 - Integer key (index) associated with vertex
- ❑ 2D-array adjacency array
 - Reference to edge object for adjacent vertices
 - Null for non nonadjacent vertices
- ❑ The “old fashioned” version just has 0 for no edge and 1 for edge



Performance

<ul style="list-style-type: none"> ▪ n vertices, m edges ▪ no parallel edges ▪ no self-loops 	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
$v.\text{incidentEdges}()$	m	$\text{deg}(v)$	n
$u.\text{isAdjacentTo}(v)$	m	$\min(\text{deg}(v), \text{deg}(w))$	1
$\text{insertVertex}(o)$	1	1	n^2
$\text{insertEdge}(v, w, o)$	1	1	1
$\text{eraseVertex}(v)$	m	$\text{deg}(v)$	n^2
$\text{eraseEdge}(e)$	1	1	1