CSCE 221 Homework 4 Cover Page

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I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.
On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.

Your Name Date Nov 10, 2014

Homework 4

due November 7, 2014

Write clearly and give full explanations to solutions for all the problems. Show all steps of your work.

Reading assignment.

- Binary Search Trees, Chap. 10
- Heap and Priority Queue, Chap. 8
- Hash Tables, Maps and Skip Lists, Chap. 9

Problems.

1. (10 points) R-10.17 p. 493
   For the following statements about red-black trees, provide a justification for each true statement and a counterexample for each false one.

   (a) A subtree of a red-black tree is itself a red-black tree.
   False. Because the root of a red-black tree is black, if the root of the sub-tree is red, then it cannot be a red black tree.

   (b) The sibling of an external node is either external or it is red.
   True. A red-black tree keeps its leafs the same black depth. If the sibling of an external node is a black node, the number of black nodes from the root to the sibling's child is bigger than the external node. Therefore the sibling is either external or red.

   (c) There is a unique (2,4) tree associated with a given red-black tree.
   False. If red-black tree has one root with value of 2 and one right leaf with value of 4: 2 could be the root of the (2,4) tree and 4 is the right children of the root; or both 2 and 4 could be the root of (2,4) tree.

   (d) There is a unique red-black tree associated with a given (2,4) tree.
   False. If a (2,4) tree has one root with two values (2,4): 2 could be the root of the red-black tree and 4 is the right children of the root; or 4 could be the root of the red-black tree and 2 is the left child of the root.

2. (10 points) R-10.19 p. 493
   Consider a tree T storing 100,000 entries. What is the worst-case height of T in the following cases?

(a) T is an AVL tree.

$$h = 2\log_2(100000 + 1) = 33$$

(b) T is a (2,4) tree.

$$h = \log_2(100000 + 1) = 17$$

(c) T is a red-black tree.

$$h = 2\log_2(100000 + 1) = 33$$

(d) T is a binary search tree.

$$h = 100000 - 1 = 99999$$

3. (10 points) R-9.7 p. 417

Draw the 11-entry hash table that results from using the has function, h(k) = (3k + 5) mod 11, to hash the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, and 5, assuming collisions are handled by chaining.

0 -> 13
1 -> 39 -> 94
2
3
4
5 -> 11 -> 88 -> 44
6
7
8 -> 23 -> 12
9 -> 5 -> 16
10 -> 20

4. (10 points) R-9.8 p. 417

What is the result of the previous exercise, assuming collisions are handled by linear probing?

Index	0	1	2	3	4	5	6	7	8	9	10
Key	13	94	39	16	5	44	88	11	12	23	20

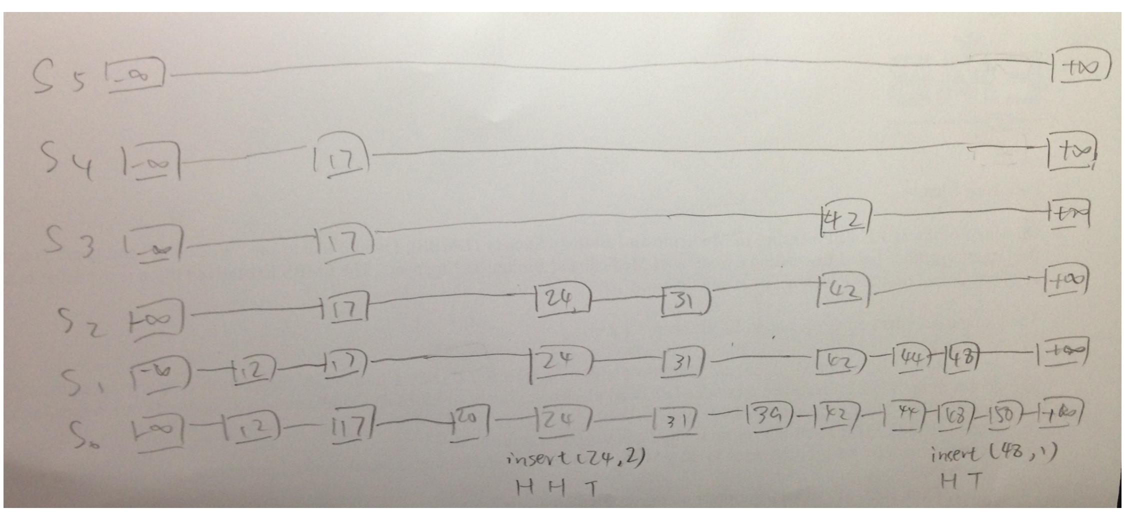
5. (10 points) R-9.10 p. 417

What is the result of Exercise R-9.7, when collisions are handled by double hashing using the secondary hash function hs(k) = 7 − (k mod 7)?

Index	0	1	2	3	4	5	6	7	8	9	10
Key	13	94	23	88	39	44	11	5	12	16	20

6. (10 points) R-9.16 p. 418

Draw an example skip list that results from performing the following series of operations on the skip list shown in Figure 9.12: erase(38), insert(48,x), insert(24,y), erase(55). Record your coin flips, as well.



7. (10 points) R-8.2 p. 361
How long would it take to remove ⌈log n⌉ smallest elements from a heap that contains n entries using the removeMin() operation?

removeMin() function removes the root of the tree, then swaps the root and the last node. It takes O(1) to swap and takes O($\log_2 n$) to walk-down the last node.
$O(\log_2 n)\times \log_2 n = O(log^2 n)$

8. (10 points) R-8.7 p. 361
An airport is developing a computer simulation of air-traffic control that handles events such as landings and takeoffs. Each event has a time-stamp that denotes the time when the event occurs. The simulation program needs to efficiently perform the following two fundamental operations:
• Insert an event with a given time-stamp (that is, add a future event)
• Extract the event with smallest time-stamp (that is, determine the next event to process)
Which data structure should be used for the above operations? Why?

Heap. We create a heap based on time-stamps. Insert into a heap will put the smallest element as the root of the tree and put the largest element as an external node. It takes O(1) to find the min of the minimum time-stamps and takes O(log n).

9. (10 points) R-8.15 p. 362
Let T be a complete binary tree such that node v stores the key-entry pairs (f(v),0), where f(v) is the level number of v. Is tree T a heap? Why or why not?

T is a heap. Two reasons: firstly, each of the children's value is greater than the parent's value; secondly T is a complete binary tree.

10. (10 points) R-12.14 p. 588
Draw the frequency array and Huffman tree for the following string: "dogs do not spot hot pots or cats".

Char	space	a	c	d	g	h	n	o	p	r	s	t
Cnt	7	1	1	2	1	1	1	7	2	1	4	5

Char	a	c	g	h	n	r	d	p	s	t	o	space
Cnt	1	1	1	1	1	1	2	2	4	5	7	7