

# **Actuarial Pricing & Capital Modeling Study**

Whole Life Insurance Portfolio (Issue Age 40)

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## Executive Summary

This study develops a deterministic pricing, reserving, and capital adequacy framework for a portfolio of 2,000 whole life insurance policies issued at age 40 with a \$1,000 death benefit payable at the end of the year of death. The objective is to evaluate premium adequacy, aggregate underwriting risk, reserve sufficiency, and required capital buffers under baseline actuarial assumptions.

All calculations are performed using life-table mortality and a 6% effective annual interest rate, with no allowance for expenses, lapses, or profit margins.

The equivalence principle is applied to determine net premium levels, and aggregate portfolio risk is analyzed using the Central Limit Theorem to approximate the distribution of total claim present values.

Key findings include:

- Net Single Premium: \$154.93
- Net Annual Premium: \$10.38
- Risk-Adjusted Single Premium (2.5% loss threshold): \$161.42
- Risk-Adjusted Annual Premium: \$10.81
- 20-Year Aggregate Reserve (Prospective Method): \$423,075
- Required Per-Policy Capital for 98% Solvency Confidence: \$838.62

Deterministic equivalence pricing results in zero expected loss per policy; however, variability at the portfolio level creates underwriting risk. This variability requires additional premium margins and capital buffers to maintain solvency targets. Although risk per policy decreases as the portfolio grows, aggregate volatility remains significant when high confidence levels are required.

Additionally, the study outlines how Monte Carlo simulation could extend this framework by incorporating stochastic mortality improvements and interest rate variability, enabling dynamic solvency testing and stress analysis. Overall, the results highlight the interaction between pricing assumptions, reserve development, and capital requirements, illustrating how actuarial valuation techniques translate into portfolio-level financial risk management decisions.

## Model Framework and Assumptions

### Product Design

- Whole life insurance
- Issue age: 40
- \$ 1,000 death benefit
- Benefit paid at end of year of death
- Level annual premium

### Economic Assumptions

- Effective annual interest rate: 6%
- Deterministic interest

### Mortality Assumptions

- Life table mortality
- Independent lives
- No mortality improvement

### Portfolio Structure

- 2,000 identical independent policies
- No expense or lapses
- No profit margin in base pricing

### Random Variable Definitions

- $X = 1000v^{T+1}$
- $T$  = future lifetime of a life aged 40
- $v = (1.06)^{-1}$
- $S = \sum_{i=1}^{2000} X_i$

## Net Single Premium

Under the equivalence principle, the expected present value of future benefits equals the expected present value of future premiums. For a \$1,000 whole life insurance policy issued at age 40:

$$P = 1000A_{40}$$

where  $A_{40}$  represents the present value of a whole life insurance of 1 payable at the end of the year of death.

$$A_{40} = \sum_{t=0}^{\omega-40} v^{t+1} {}_t p_{40} q_{40+t}$$

$$\text{Net Single Premium} \approx \$154.93$$

This amount represents the present value of expected future death benefits under the stated assumptions.

## Net Annual Premium

Under annual premium payments made at the beginning of each year while the insured is alive, the equivalence principle becomes:

$$1000A_{40} = P \times \ddot{a}_{40}$$

where  $\ddot{a}_{40}$  denotes the present value of a whole life annuity-due of 1 issued at age 40.

$$\ddot{a}_{40} = \frac{1 - A_{40}}{d}$$

where  $d = \frac{i}{1+i}$ , the annuity factor is computed under the same mortality and 6% interest assumptions.

Solving for P yields:

$$\text{Net Annual Premium} \approx \$10.38$$

This premium produces zero expected present value loss under deterministic assumptions.

## Aggregate Portfolio Risk Analysis

While deterministic pricing ensures zero expected loss per policy, variability in mortality introduces underwriting risk at the portfolio level. To evaluate this risk, the aggregate present value of claims for a portfolio of 2,000 independent policies is modeled.

Let  $X = 1000v^{T+1}$  denote the present value of the death benefit for a single policy. Then:

$$\mu_X = E[X] = 1000A_{40}$$

The second moment is given by:

$$E[X^2] = {}^2A_{40}(1000)^2$$

where

$${}^2A_{40} = \sum_{t=0}^{\omega-40} v^{2(t+1)} {}_t p_{40} q_{40+t}$$

$$\sigma_X^2 = E[X^2] - E[X]^2 \approx 21929.41$$

Aggregate present value:

$$S = \sum_{i=1}^{2000} X_i$$

Assuming independence, the Central Limit Theorem implies:

$$S \sim \mathcal{N}(2000\mu_X, 2000\sigma_X^2)$$

To determine the premium P per policy such that

$$Pr(S > 2000P) \leq .025$$

Under the normal approximation, the 97.5th percentile of the aggregate distribution is:

$$s_{0.975} = 2000\mu_X + z_{0.975}\sqrt{2000}\sigma_X$$

Where  $z_{0.975}$  is defined as the 97.5th percentile of the standard normal distribution.

$$P = \mu_X + 1.96 \frac{\sigma_X}{\sqrt{2000}} \approx 161.42.$$

This risk-adjusted premium incorporates a margin for adverse deviation sufficient to limit the probability of aggregate underwriting loss to 2.5% under the stated assumptions.

## Prospective Reserve Analysis (20 Years After Issue)

To evaluate reserve adequacy over time, the prospective reserve is calculated 20 years after policy issuance. At this duration, the insureds have attained age 60, and 1,775 policies remain in force.

The prospective reserve is defined as the expected present value of future benefits minus the expected present value of future premiums:

$${}_tV = \text{EPV}(\text{Future Benefits}) - \text{EPV}(\text{Future Premiums}).$$

For a whole life insurance policy issued at age 40 and valued at age 60:

$${}_{20}V = 1000A_{60} - P\ddot{a}_{60}$$

Where:

- $A_{60} \approx 0.35636$  is the present value of a whole life insurance of 1 at age 60
- $\ddot{a}_{60} \approx 11.3710$  is the present value of a whole life annuity-due at age 60
- $P \approx 10.38$  is the previously determined net annual premium

Therefore,

$${}_{20}V \approx 238.35$$

Since 1,775 policies remain:

$$\text{Aggregate Reserve} = 1775 \times {}_{20}V \approx \$423,076$$

The reserve increases due to higher attained-age mortality and the shortening of the remaining premium payment period. As duration increases, the reserve accumulates to ensure sufficient funding of future death benefits under the original pricing assumptions.

## Capital Requirement for 98% Solvency Confidence

Although deterministic pricing produces zero expected present value loss per policy, variability in mortality results in uncertainty around realized outcomes. To ensure a high probability of solvency, additional capital must be held to absorb adverse deviations.

Let the loss random variable per policy be defined as

$$L = \text{PV(Benefits)} - \text{PV(Premiums)}.$$

Under the equivalence principle,

$$E[L] = 0 \quad \text{and} \quad \text{Var}(L) > 0.$$

We want to determine the capital buffer  $R$  such that:

$$\Pr(L + R \geq 0) \geq 0.98.$$

Rewriting:

$$\Pr(L \geq -R) \geq 0.98.$$

Assuming approximate normality,

$$\frac{L}{\sigma_L} \sim \mathcal{N}(0, 1).$$

This implies:

$$\frac{-R}{\sigma_L} \leq z_{0.02}.$$

Since

$$z_{0.02} \approx -2.05,$$

$$R \geq 2.05 \sigma_L.$$

Recall the second moment:

$$E[X^2] = 1000^2 A_{60}$$

$$\sigma_L^2 = E[L^2] - (E[L])^2$$

Since

$$E[L] = 0,$$

$$\sigma_L^2 = E[L^2].$$

Using previously computed values:

$$\sigma_L \approx 409.09.$$

**Required Per-Policy Capital Buffer (98% confidence):**  $R = 2.05(409.09) \approx 838.62$ .

This capital buffer ensures that, under the stated assumptions, the insurer maintains at least a 98% probability of non-negative present value outcomes per policy. The result highlights that even when pricing is actuarially fair in expectation, substantial capital is required to protect against tail risk arising from mortality variability.

## Risk Extensions and Monte Carlo Simulation

The preceding analysis relies on deterministic mortality and interest rate assumptions. While useful for baseline pricing and solvency assessment, these assumptions do not fully capture real-world uncertainty. A stochastic framework can provide a more comprehensive understanding of portfolio risk.

Mortality experience may deviate from life-table assumptions due to longevity improvements, pandemics, or demographic shifts. Instead of treating mortality as fixed, Monte Carlo simulation can model stochastic mortality rates or mortality improvement trends. By simulating thousands of mortality scenarios, actuaries can evaluate the distribution of surplus outcomes rather than relying solely on normal approximations.

Similarly, investment returns may fluctuate around the assumed 6% interest rate. Incorporating stochastic interest rate paths allows the insurer to assess sensitivity of reserves and capital requirements to economic volatility.

By generating repeated simulated scenarios, Monte Carlo methods produce a full distribution of portfolio outcomes, including tail behavior. This approach enables dynamic solvency testing, stress analysis, and capital adequacy assessment beyond the limitations of deterministic or closed-form approximations.

Incorporating stochastic modeling enhances risk management by quantifying uncertainty in both mortality and investment performance, supporting more robust pricing, reserving, and capital decisions.