

Scientific Computing in Python 6 – More ODE Modelling.

So previously you managed to model a population using an ODE. You, hopefully, created a system of 3 equations that fed into each other and were solvable as a whole.

Today we will do a similar thing with a Lorenz system, which models a particle's chaotic movement through 3D space.

I'll give you less code for you to try and get things working more on your own. First of all, we need to describe the equations of movement across the 3 axes.

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

In reality these equations were originally used to describe atmospheric convection. And the constants σ β ρ are representatives of this system's dimensions. It has been used to describe 3d motion chaotically in a number of systems. The equations should be fairly simple for you to get into a single function with a y and a t input.

You also need to define your constants. I used:

```
rho = 28.0
sigma = 10.0
beta = 8.0 / 3.0
```

Then set the starting y (I called it state) to

```
state0 = [1.0, 1.0, 1.0]
t = np.arange(0.0, 40.0, 0.01)
#Run the ODE
states = odeint(f, state0, t)
```

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
```

Lets bring in these two packages and then we will try and plot our movement through 3D space

```
fig = plt.figure()
ax = fig.gca(projection='3d')
ax.plot(states[:,0], states[:,1], states[:,2])
plt.show()
```

Our particle (or whatever it is we modelled) started at 1,1,1 and then moved around through that pattern. Isn't that cool. Our last model plotted 3 things through one dimension of population size, but as we see here we can define an equation to be an axis of movement and move 1 thing through 3 of them.

How about we return to our temperature and sea level rise model.

Problem 1: Previously you had to add a single equation to an existing system this time you will design the whole system yourself based on the parameters I provide.

If we remember temperature increase led to sea level rise, however in reality this works through several steps:

Sea level rise is actually described by ice cap melt, causing sea levels to rise.

Ice cap melting is caused by increased temperatures.

Increased temperatures is caused by increased CO₂ in the atmosphere.

The actual values are not important as you will trial parameters in your model until it becomes realistic.

Start by creating your own equations to describe how you think these things interact. Once you have equations try some parameters in them solving the ODE to get to a fairly realistic point.

Challenge problem

We have a system of 3 ODEs to describe climate change with the most basic important part being the amount of CO₂ in the atmosphere. I want you to do some research and find out the breakdown of CO₂ being produced from the major sources: Energy Production, Agriculture and Transport. And relate these to their economic output. Whatever data you can find for this will do. Design an equation that relates the output of emissions from these sectors directly to their profit, where reduced emissions means reduced profits.

Time to save Cornwall. Much of Penzance in Cornwall is less than 2m above sea level. I want you to add this new equation of emission sources and profit in to the previous system then trial scenarios of reducing emissions from those sources until you find the cheapest reduction that limits sea level rise below 2m.

Hint: once you design the equation you want to trial a lot of parameters. You can use a for loop to trial model parameters and get it to return just the things you are interested in.