Final Problem 1: Deer In Scotland.

An engineer, a physicist, and a mathematician are shown a pasture with a herd of sheep, and told to put them inside the smallest possible amount of fence.

The engineer is first. He herds the sheep into a circle and then puts the fence around them, declaring, "A circle will use the least fence for a given area, so this is the best solution."

The physicist is next. He creates a circular fence of infinite radius around the sheep, and then draws the fence tight around the herd, declaring, "This will give the *smallest* circle around the herd."

The mathematician is last. After giving the problem a little thought, he puts a small fence around himself and then declares, "I define myself to be on the outside."

In all seriousness though, grazing animals are a big problem. In Scotland, deer graze on young saplings killing them. This is preventing regeneration of Scotland's rare native woodlands and also means that new forestry nurseries have to be fenced in at high cost.

A forestry company wants you to model how the deer are eating our saplings and how fencing interacts with the deer/tree dynamic. The company's policy to protect half of however many saplings the current deer population would have eaten that year. The government is also thinking of introducing lynx to control deer numbers, once you have a viable model of trees/deer/fences, your job is to introduce lynx to see how this affects how much fence you need.
The cost of fencing is £3 per meter but reduces by 0.001 for each extra meter that is fenced up to a maximum discount of 80% off. These are special fences that last for three years then biodegrade- the length of time it takes a sapling to grow into a tree.

Only saplings are eaten by deer thus only saplings need to be fenced. You should track four classes of trees in your model, saplings of 1, 2, and 3 years of age and adult trees. This means you should track the number of each of the three sapling classes but also track how many of those are fenced and how many are unfenced since they have different mortality rates. Start with two million trees in each of the three sapling age classes, half fenced and half unfenced. There are four million adult trees to start with.

There is no mortality for fenced in saplings and the only lossed of unfenced saplings is to deer. There is no mortality of adult trees, adults are only removed by harvesting which is done at a rate of 10% of the adult trees per year. Each adult tree produces a viable sapling every other year. Saplings are eaten by deer at a rate of 300 saplings per deer *per month*.

Trees grow at a density of 100,000 trees per km squared. It takes 4km of fence to protect 1km squared (since each side of the square needs a 1km fence) thus every 4km of fence protects 100,000 trees. For this model we assume no fences are shared i.e. no two squares share a side so the amount of protected land is simply the length of fence divided by 4.

Deer are fast growing and reproduce at a rate of 0.9 x the number of female deer per year. We know that the deer population is 50/50 male/female. They all have a natural mortality of 0.006 *each month* but you should calculate how the deer population changes each year. There are no age classes in the deer population. The birth rate of deer stated above is affected by the carrying capacity in the same way that we limited red panda growth based on number of cabbages. You should use the information provided about how many saplings deer eat per year to link the birth rate of deer to the number of available (i.e. unfenced!) saplings. Start with a deer population around 500 individuals.

Deer graze randomly on all age classes of unfenced saplings in the proportion that they encounter them such that they eat unfenced saplings in the proportion of how many trees are in each sapling age class. The equation for how many one year old saplings the deer are eating is (total_saplings_eaten x (num_saps1_unfenced/total_saps_unfenced)) and the same for num_saps2_unfenced and num_saps3_unfenced. Saplings protected by fences do not get eaten at all. Remember that saplings are eaten by deer at a rate of 300 saplings per deer *per month*.

You should write a function to calculate the cost of fence for a given length of fence. The ODEs for all the classes of sapling, trees, and deer should be written in one function for odeint to work on.
One suggested management strategy is the reintroduction of lynx to Scotland. Carnivores breed slowly, and are very sensitive to changes in their prey population. Lynx produce 0.2 offspring per year but die at a rate of 0.14 per year.

Each lynx kills 2 deer per month and you need to start with at least one breeding pair. The reintroduction of lynx costs approximately £5000 per individual released.

Add an ODE describing how lynx prey on deer in to the model after you have a working model of trees/fences/deer. Tell me what the most cost effective strategy to keep costs down between fences and introduction of lynx.
Hint#####

if 2000m of new fence then

1m = 3

2m = 3 - 0.001

3m = 3 - 0.001x2

...

2000m = 3 - 0.001x (2000-1)

SO:

nth meter = 3 - 0.001 x n -1

up to a max discount of 80% (£2.40 off)

so all meters past 2,400m cost £0.60