DENSO
Crafting the Core

Enhancing Quantum Annealing in Digital-Analog Quantum Computing

(arXiv:2306.02059)

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This study begins with a simple question:

What would happen if D-Wave were to integrate the quantum annealer with quantum gates?

Another type of
Digital-Analog Quantum Device

arXiv:2306.02059

- Introduction
  - Digital Analog Quantum Computing
  - Counter Diabatic Driving
  - Quantum Greedy Optimization
- Proposed Method
  - Digital Analog Quantum Greedy Optimization
    - QA as a state preparation
    - A controlled QA block
- Results
  - Numerical simulations
  - Experiments on trapped ion
- Summary

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Digital Analog Quantum Computing (DAQC)

Dodd, et.al. 2002, Martin, el.al. 2020

single-qubit rotation($R_x(\phi) = e^{i\phi X/2},\ R_z(\phi) = e^{i\phi Z/2}$)

+ Ising Hamiltonian($R_{zz}(\phi) = e^{-i\phi ZZ/2}$)

=> Universal

Ex. a universal gate set $\{H, T, CNOT\}$ can be constructed.

$$H = e^{i\pi/2} R_z(\pi/2) R_x(\pi/2) R_z(\pi/2)$$
$$T = e^{i\pi/8} R_z(\pi/4)$$
$$CNOT = (I \otimes H) CZ (I \otimes H) = (I \otimes H) CR_z(\pi)(I \otimes H)$$

$$CR_z(\pi) = e^{i\pi/4}\big(R_z(\pi/2) \otimes R_z(\pi/2)\big)\underbrace{R_{zz}(-\pi/2)}_{\text{Ising}}$$

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Proof

$$R_z(\phi) = \begin{pmatrix} e^{-i\phi/2} & \\ & e^{i\phi/2} \end{pmatrix}, \qquad \underbrace{R_{zz}(\phi)}_{\text{Ising}} = \begin{pmatrix} e^{-i\phi/2} & & & \\ & e^{i\phi/2} & & \\ & & e^{i\phi/2} & \\ & & & e^{-i\phi/2} \end{pmatrix}$$

$$R_z(\phi/2) \otimes R_z(\phi/2)\, \underline{R_{zz}(-\phi/2)}$$

$$= \begin{pmatrix} e^{-i\phi/4} & \\ & e^{i\phi/4} \end{pmatrix} \otimes \begin{pmatrix} e^{-i\phi/4} & \\ & e^{i\phi/4} \end{pmatrix} \begin{pmatrix} e^{i\phi/4} & & & \\ & e^{-i\phi/4} & & \\ & & e^{-i\phi/4} & \\ & & & e^{i\phi/4} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-i\phi/4} & & & \\ & e^{-i\phi/4} & & \\ & & e^{-i\phi/4} & \\ & & & e^{3i\phi/4} \end{pmatrix} = e^{-i\phi/4} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & e^{i\phi} \end{pmatrix}$$

$= e^{-i\phi/4} CR_z(\phi)$ $\quad\rightarrow$ can be used to construct CNOT

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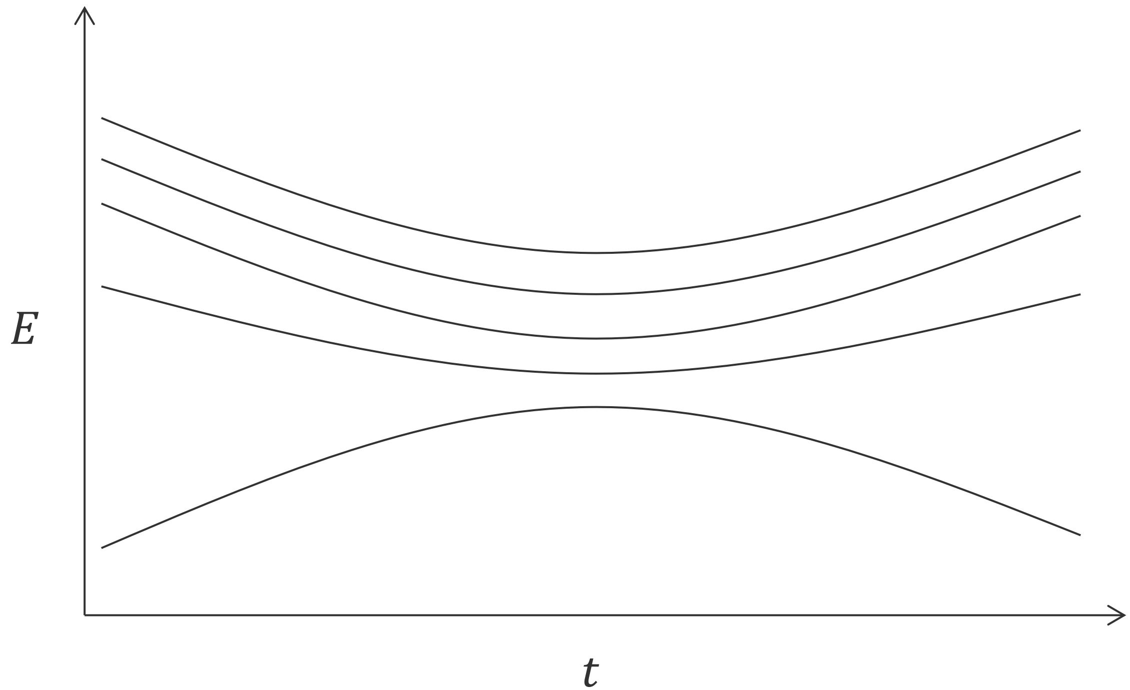
Quantum Annealing ~ Adiabatic Evolution



$$\mathcal{H}(0) = -\sum_i \sigma_i^x \qquad \mathcal{H}(\tau) = -\frac{1}{N-1}\sum_{i<j} \sigma_i^z \sigma_j^z$$

$$|\psi(0)\rangle = |+\rangle^{\otimes N} \qquad |\psi(\tau)\rangle = |GS\rangle$$

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Counter-Diabatic (CD) driving

$$\mathcal{H} = \underline{A(t)\mathcal{H}^z + B(t)\mathcal{H}^x} + \underline{\mathcal{H}^y}$$

$$\begin{cases} \mathcal{H}^z = -\frac{1}{N-1}\sum_{i<j}\sigma_i^z\sigma_j^z \\ \mathcal{H}^x = -\sum_i \sigma_i^x \end{cases}$$

: cost function

: uniform transverse field

$$\{ \mathcal{H}^y = -\sum_i C_i(t)\sigma_i^y$$

: non-uniform field in Y

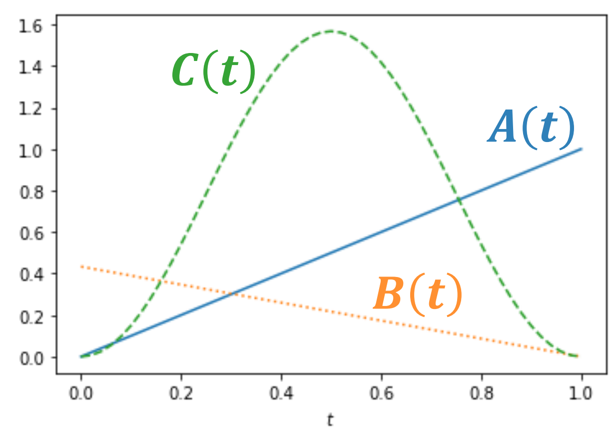
(counter-diabatic term)

where the coefficients are

$$A(t) = at/\tau$$

$$B(t) = b(1 - t/\tau)$$

$$C_i(t) = c_i \sin^2(\pi t/\tau)$$



Sels & Polkovnikov 2017, Takahashi 2017, Prielinger, et.al. 2021

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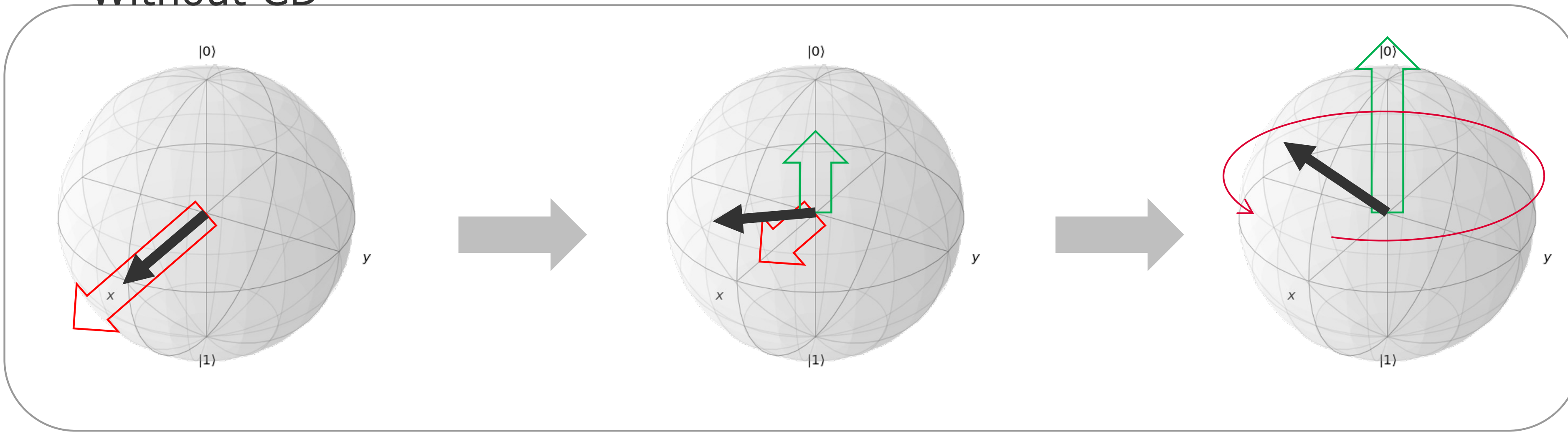
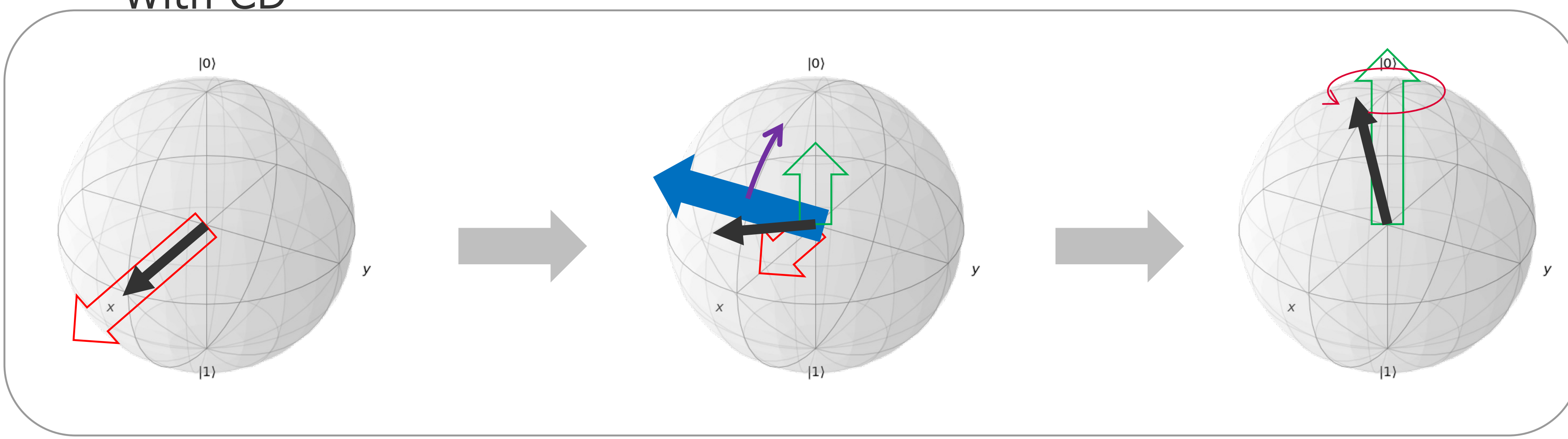
Time evolution of the CD system

Without CD



With CD



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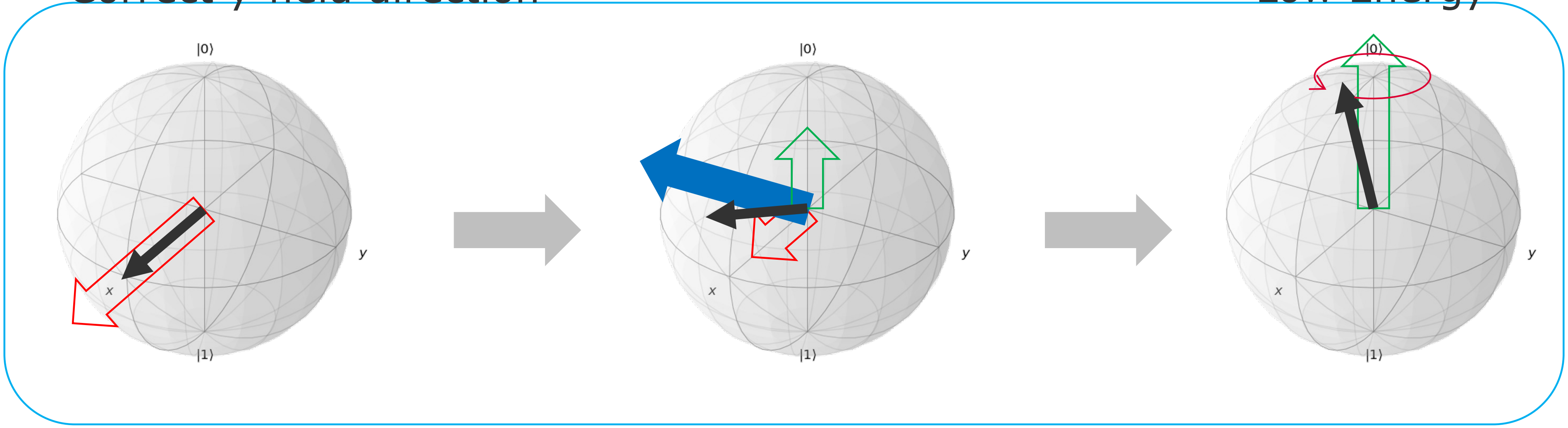
Sensitivity analysis

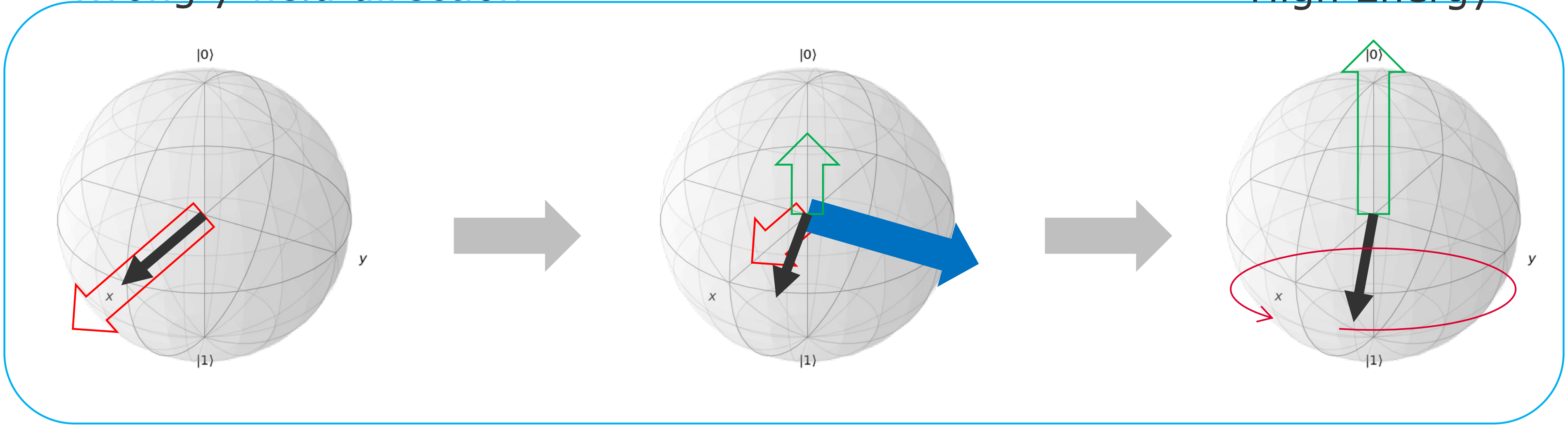
Correct y-field direction

Low Energy



Wrong y-field direction

High Energy



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Quantum Greedy Optimization (QGO)

Algorithm 1 QGO algorithm

Require: number of variables N, goodness measure $f(b, \boldsymbol{c})$, differentiation interval h, optimal values from ferromagnetic system b_{opt}^N and c_{opt}^N

Energy or other observables

Ensure: a solution of the cost function

$b \leftarrow b_{\text{opt}}^N$

$\boldsymbol{c} \leftarrow (0, \cdots, 0)$

repeat

$g_i \leftarrow f(b, c_1, \cdots, c_i + h, \cdots, c_N) - f(b, \boldsymbol{c})$

$k \leftarrow \arg\max_{j \in \{j | c_j = 0\}} |g_j|$

Sensitivity analysis finds the most sensitive parameter

$c_k \leftarrow -c_{\text{opt}}^N \operatorname{sgn} g_k$

Fix the most sensitive parameter to the correct direction

until $c_i \neq 0$ for all i

return $\operatorname{sgn} \boldsymbol{c}$

Kadowaki & Nishimori 2022 (AQC2021)

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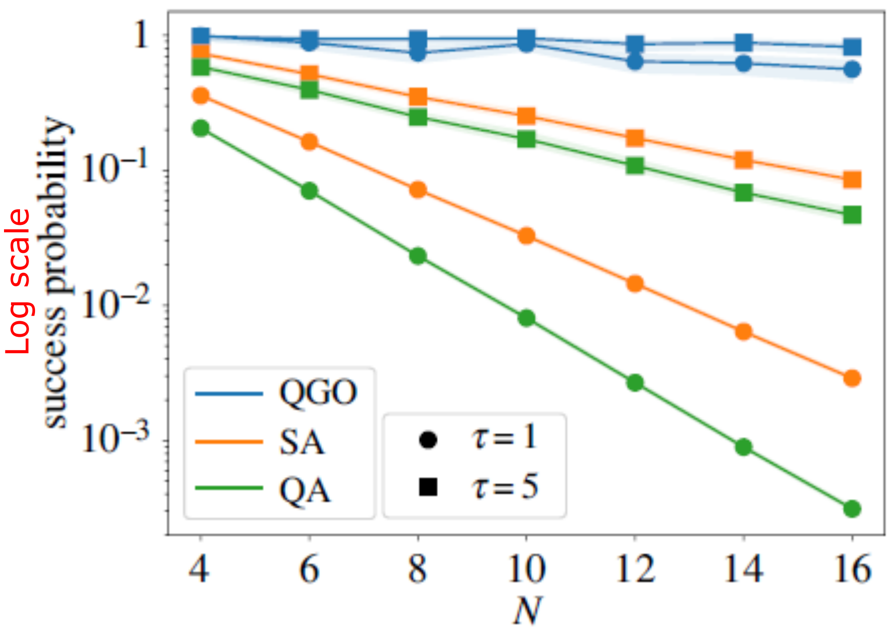
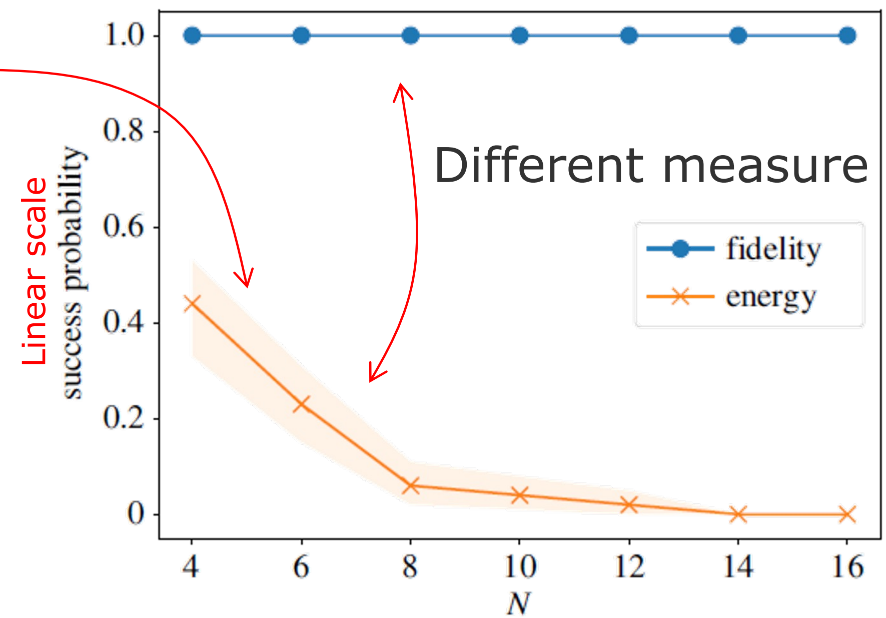
QGO performance

Kadowaki & Nishimori 2022
(AQC2021)

QGO outperforms the SA and QA in short annealing time.



Fidelity measure constantly finds the ground state.



We explore potential of QGO with other measures within DAQC framework.
(DAQC allow us to calculate other measures)

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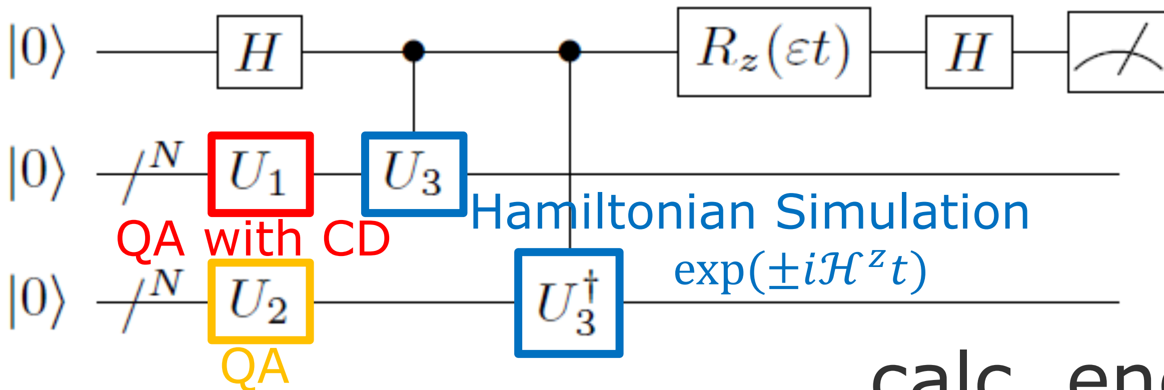
Digital Analog Quantum Greedy Optimization



DAQGO1

calc. energy diff. approx.

$$|\psi\rangle_F = \frac{1}{2}|0\rangle\left(|\psi_T\rangle|\psi_R\rangle + e^{i\varepsilon t}U_3|\psi_T\rangle U_3^\dagger|\psi_R\rangle\right) + \frac{1}{2}|1\rangle\left(|\psi_T\rangle|\psi_R\rangle - e^{i\varepsilon t}U_3|\psi_T\rangle U_3^\dagger|\psi_R\rangle\right)$$

$$P_0 = \frac{1}{2}\sum_{i<j}|\alpha_i|^2|\beta_j|^2\left(1+\cos(\Delta E_{ij}-\varepsilon)t\right)$$

Sensitivity analysis

$g_i \leftarrow f(b, c_1, \cdots, c_i + h, \cdots, c_N) - f(b, \mathbf{c})$

$k \leftarrow \arg\max_{j\in\{j|c_j=0\}}|g_j|$

$c_k \leftarrow -c_{\mathrm{opt}}^N \operatorname{sgn} g_k$

- Absolute value of g_i from $P_0(0)$
- Sign of g_i from $P_0(\varepsilon) - P_0(0)$

Assumption: two level system, $t \ll 1$

GS $\rightarrow 1-\delta$, excited δ

$$P_0(\varepsilon) = \frac{1}{2}\{2(1-\delta)+\delta(1+\cos(\Delta E-\varepsilon)t\} \sim 1-\delta\left(\frac{(\Delta E-\varepsilon)t}{2}\right)^2$$

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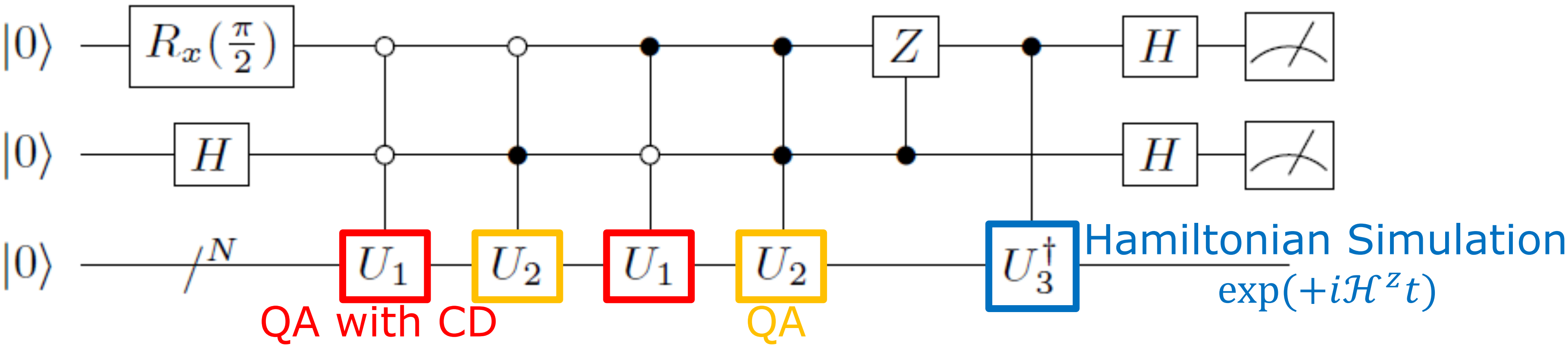
DAQGO2 calc. energy diff. more accurately



$$|\psi\rangle_F = |00\rangle\Big((|\psi_T\rangle + |\psi_R\rangle) - iU_3^\dagger(|\psi_T\rangle - |\psi_R\rangle)\Big)/4 \;+ |10\rangle\Big((|\psi_T\rangle + |\psi_R\rangle) + iU_3^\dagger(|\psi_T\rangle - |\psi_R\rangle)\Big)/4$$
$$+ |01\rangle\Big((|\psi_T\rangle - |\psi_R\rangle) - iU_3^\dagger(|\psi_T\rangle + |\psi_R\rangle)\Big)/4 \;+ |11\rangle\Big((|\psi_T\rangle - |\psi_R\rangle) + iU_3^\dagger(|\psi_T\rangle + |\psi_R\rangle)\Big)/4$$

$$P_{01}^{(0)} = P_0^{(0)} - P_1^{(0)} = \frac{1}{2}\,\mathrm{Im}\Big((\langle\psi_T| + \langle\psi_R|)U_3^\dagger(|\psi_T\rangle - |\psi_R\rangle)\Big)$$
$$\sim \frac{t}{2}(\langle\psi_T|\mathcal{H}^z|\psi_T\rangle - \langle\psi_R|\mathcal{H}^z|\psi_R\rangle) - \mathrm{Im}\,\langle\psi_T|\psi_R\rangle$$

$$g_i = (P_{01}^{(0)}(t) + P_{01}^{(1)}(t))/t$$

Sensitivity analysis

$g_i \leftarrow f(b, c_1, \cdots, c_i + h, \cdots, c_N) - f(b, \mathbf{c})$
$k \leftarrow \arg\max_{j\in\{j|c_j=0\}} |g_j|$
$c_k \leftarrow -c_{\mathrm{opt}}^N \,\mathrm{sgn}\, g_k$

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DAQGO3 and 4 employ $-\mathrm{Im}\,\langle\psi_T|\psi_R\rangle$

$$P_{01}^{(0)} = P_0^{(0)} - P_1^{(0)} = \frac{1}{2}\mathrm{Im}\Big(\big(\langle\psi_T| + \langle\psi_R|\big)U_3^\dagger\big(|\psi_T\rangle - |\psi_R\rangle\big)\Big)$$

$$\sim \boxed{\frac{t}{2}\big(\langle\psi_T|\mathcal{H}^z|\psi_T\rangle - \langle\psi_R|\mathcal{H}^z|\psi_R\rangle\big)\boxed{-\mathrm{Im}\,\langle\psi_T|\psi_R\rangle}}$$

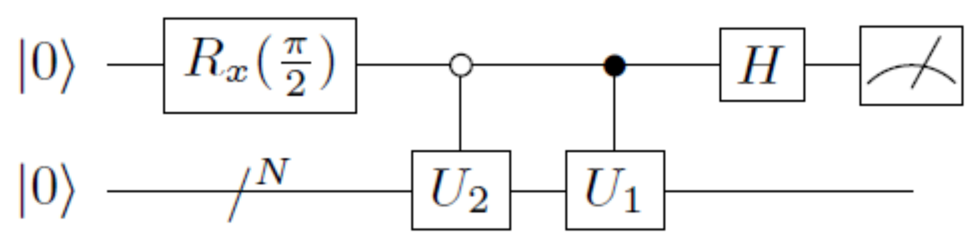
DAQGO3 (with energy diff.)

$$g_i = P_{01}^{(0)}(t)$$

DAQGO4

$$g_i = P_{01}^{(0)}(0)$$

equivalent circuit



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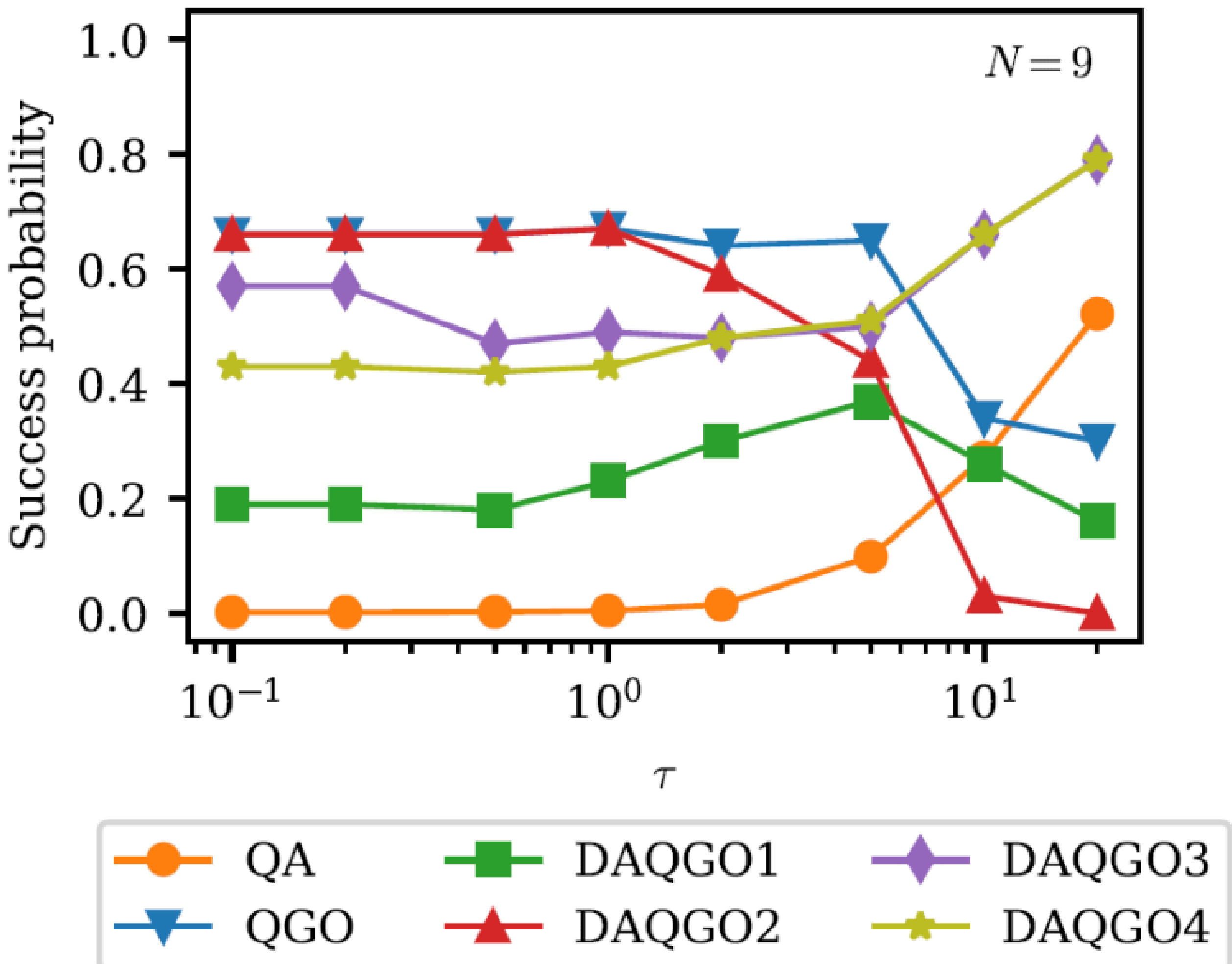
Results (Algorithm comparison)



Averaged among 100 random problems

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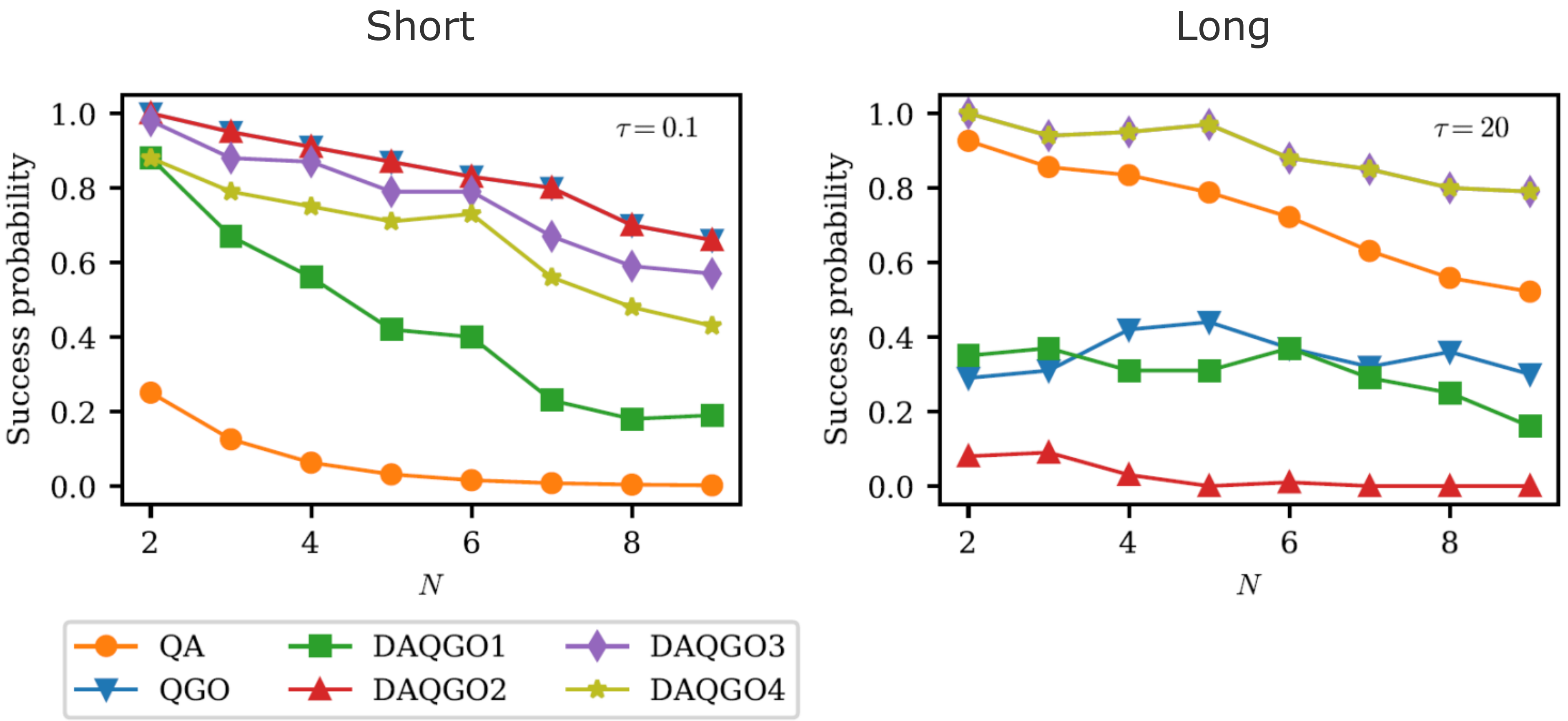
Results (Short vs long annealing times)



Averaged among 100 random problems

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Short number estimation

How to calculate the energy difference

QGO: directly calculates from the measurement → normal distribution
DAQGO: employs ancilla qubit → binomial distribution

Required shot number for

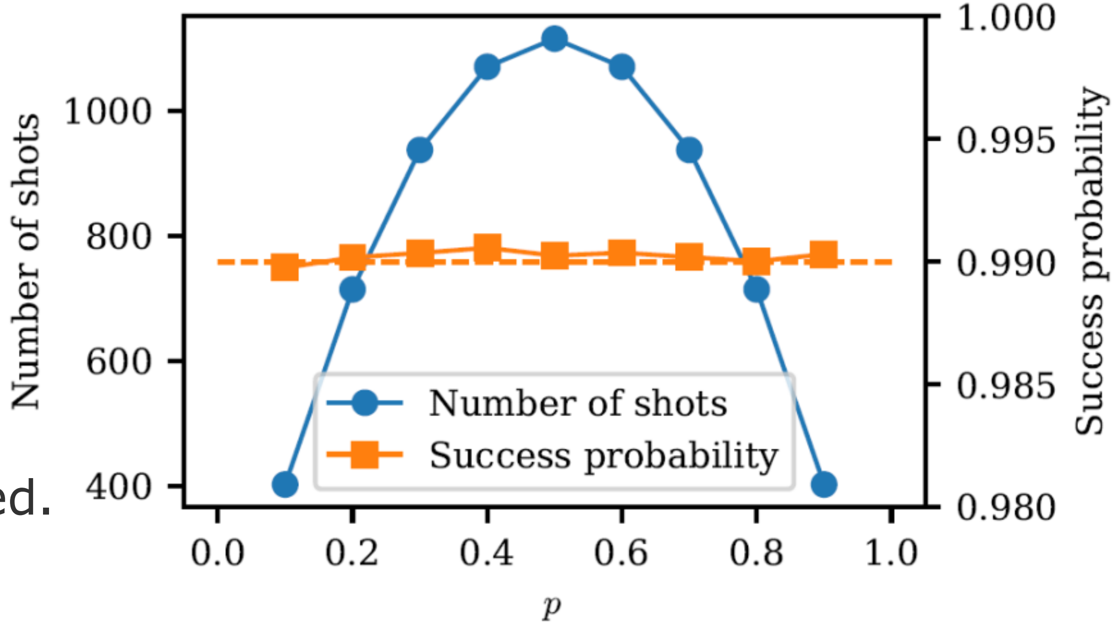
- Normal distribution

$$N \geq \frac{z^2(\sigma_1^2 + \sigma_2^2)}{d^2}$$

- Binomial distribution

$$N \geq \frac{z^2 p(1-p)}{d^2} \times \sqrt{2}$$

Only shot noise is considered.
Fidelity is not considered.



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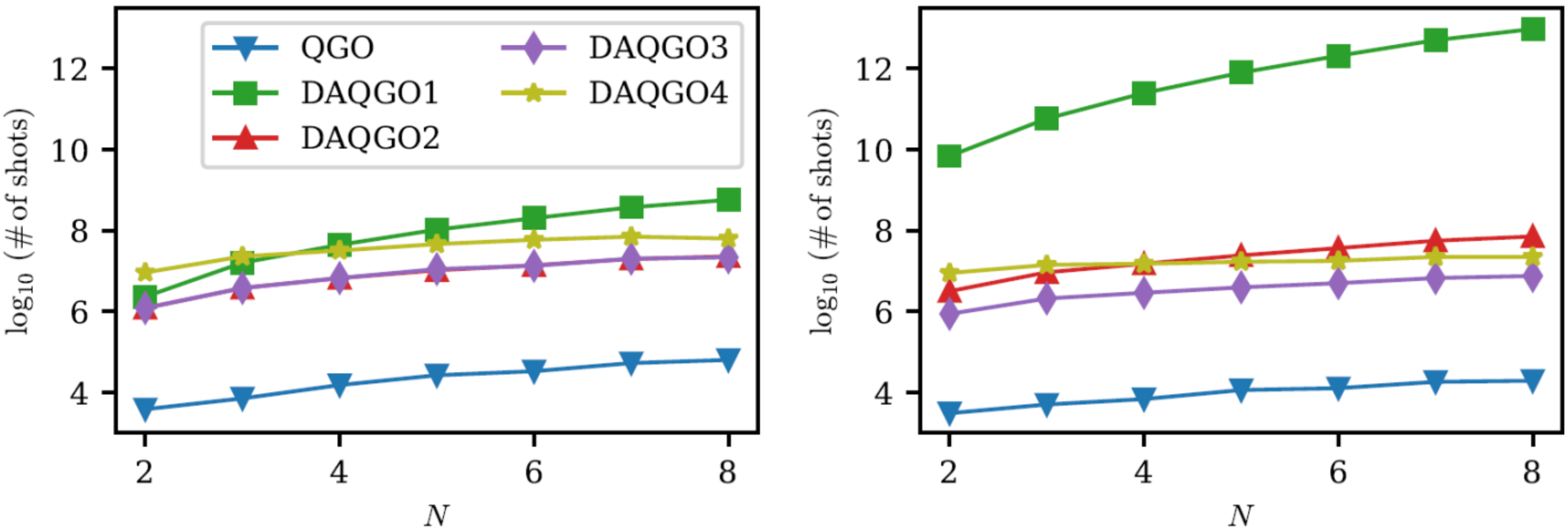
Results (Shot number estimation)



Site identification

$$g_i \leftarrow f(b, c_1, \cdots, c_i + h, \cdots, c_N) - f(b, \boldsymbol{c})$$
$$k \leftarrow \arg\max_{j \in \{j | c_j = 0\}} |g_j|$$

$$c_k \leftarrow -c_{\mathrm{opt}}^N \operatorname{sgn} g_k$$

Sign identification

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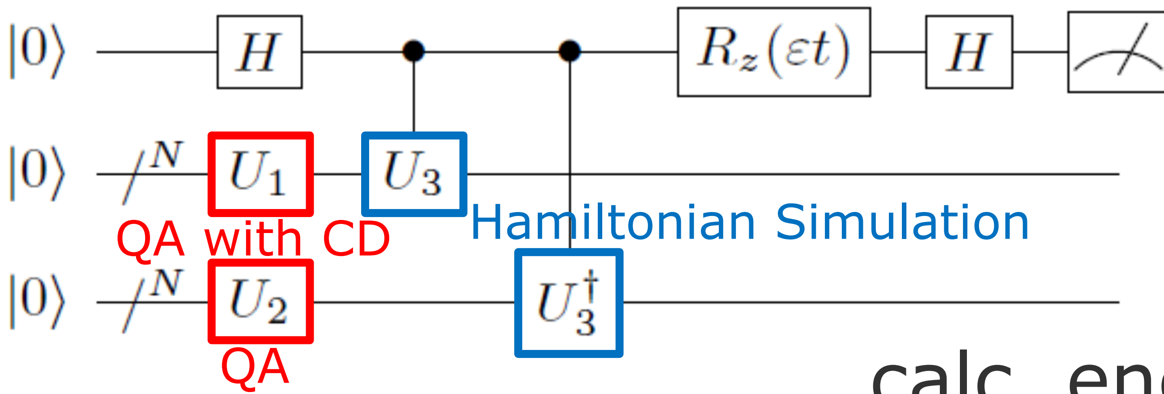
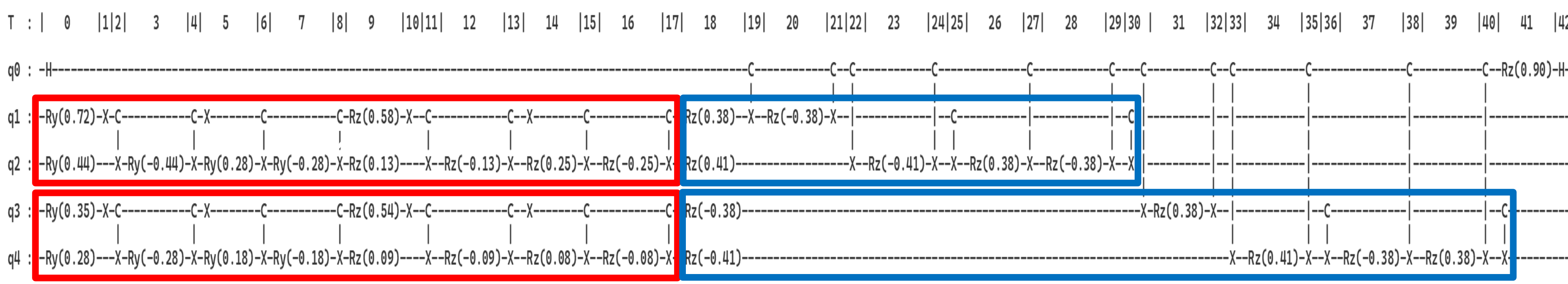
Experiments on a trapped ion device : DAQGO1



calc. energy diff. approx.



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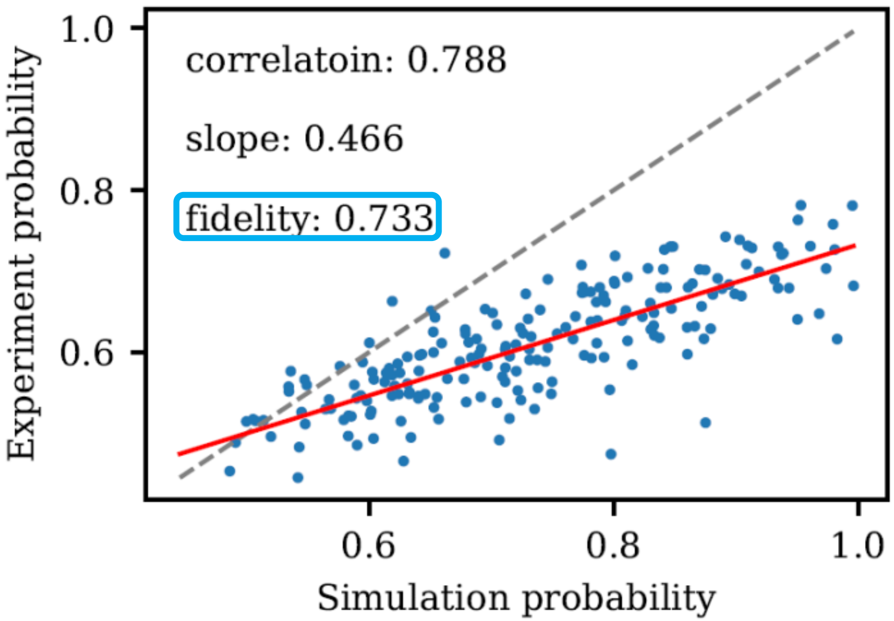
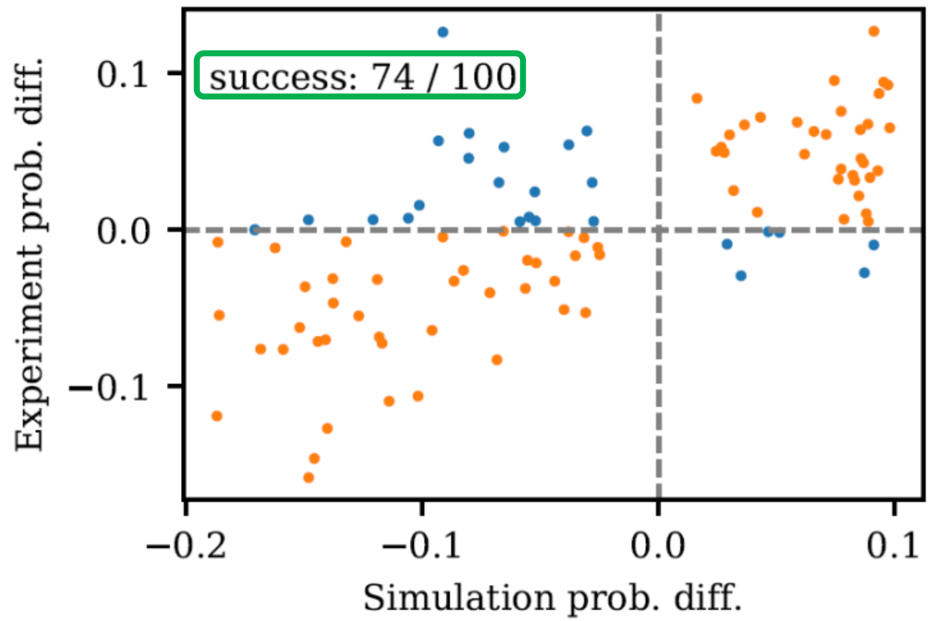
Results (DAQGO1 experiments)





Predicted fidelity : $0.9974^{20} \times 0.9898^{23}$ $\boxed{\sim 0.750}$
Predicted success probability : $\boxed{\sim 0.729}$

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Summary

- We study Quantum Annealing within Digital-Analog Quantum Computation.
- Digital-Analog Quantum Greedy Optimization repeatedly employs quantum circuits to evaluate y-field parameters. Then, fix the most sensitive parameter one by one.
- DAQGO1 (calc. energy diff. approx.) does not improve the performance.
- DAQGO2 (calc. energy diff. more accurately) shows the same performance as the original QGO.
- DAQGO3 and DAQGO4 (employ $-\mathrm{Im}\langle\psi_T|\psi_R\rangle$) demonstrate robust and improved performance.
- DAQGO1 on the real quantum device was tested. Outcomes were consistent with the simulation results.

★ This study shows an example that the quantum processing of quantum data could have advantages over the classical processing of quantum data.

$$\mathrm{Im}\langle\psi_T|\psi_R\rangle \text{ vs } \langle\psi_T|\mathcal{H}^z|\psi_T\rangle - \langle\psi_R|\mathcal{H}^z|\psi_R\rangle$$

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