Generation of a squeezed state using a nonlinear photonic crystal and its application to a heralded single-photon source

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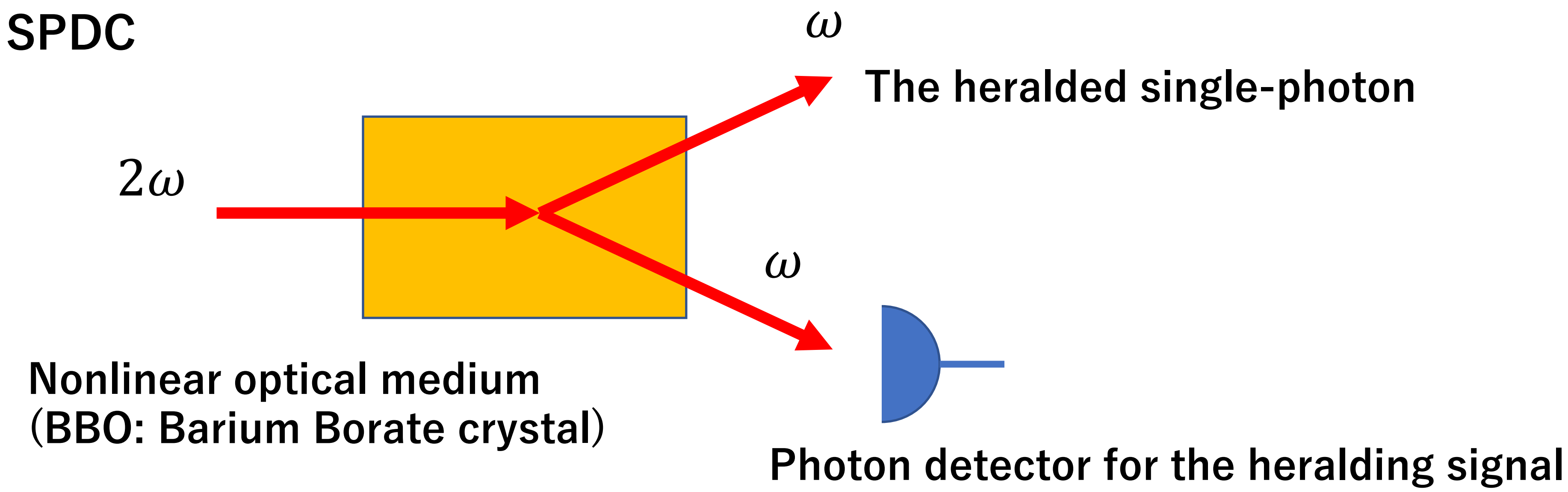
- We investigate how to generate squeezed light using a nonlinear photonic crystal.
- Because the photonic crystal reduces the group velocity of the incident light, if it is composed of a material with a second-order nonlinear optical susceptibility $\chi^{(2)}$, the interaction between the nonlinear material and the light passing through it increases, and the quantum state of the emitted light is largely squeezed.
- Thus, we can generate squeezed light with a nonlinear photonic crystal.
- Transforming this squeezed state into entangled light beams with a beam splitter, we discuss the implementation of a heralded single-photon source.

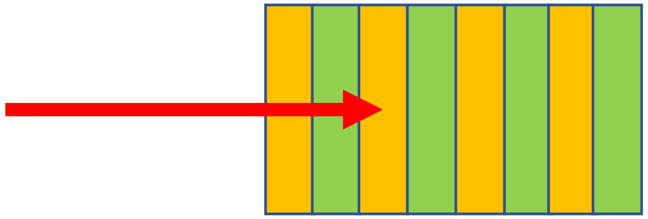
The on-demand and heralded single-photon emitters

- The on-demand single-photon emitter has not been realized experimentally yet.
- We cannot generate the single-photon source with a coherent light because the photon statistics of the coherent light are random (The Gaussian distribution).
- The heralded single-photon source can be realized with the spontaneous parametric down-conversion (SPDC), but its production rate is very low ($\mathbf{4 \times 10^{-6}}$ per pump photon at most).

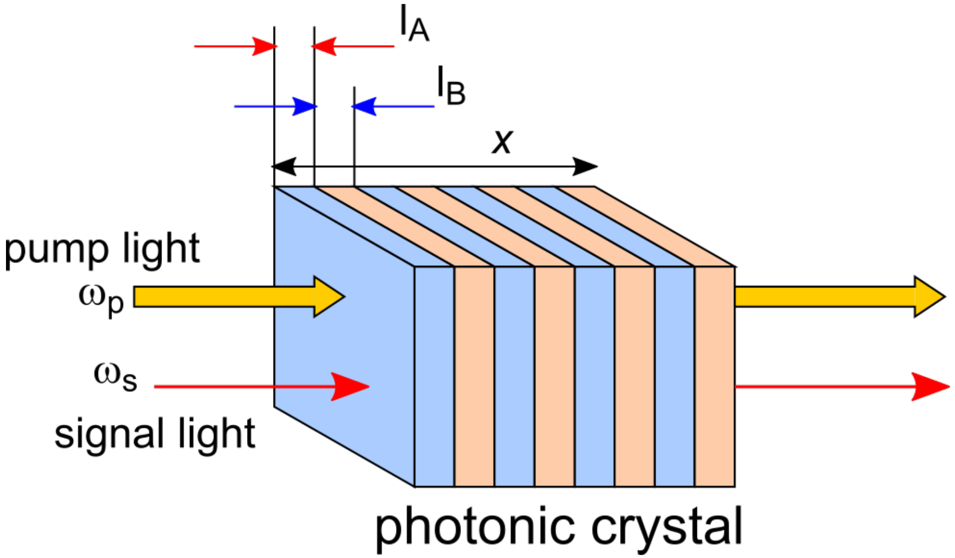


The photonic crystal

- The photonic crystal is a periodic structure composed of two materials whose refractive indices are different from each other.
- A light beam with a specific frequency cannot pass through the photonic crystal because of interferences inside the crystal.
- If we change the frequency of the photon gradually, its group velocity decreases and finally it reaches zero so that the photon cannot pass through the photonic crystal.
- If the group velocity of the photon is very slow, the photon's time-of-flight in the crystal becomes longer.
- If we make the photonic crystal out of a nonlinear material, the long time-of-flight amplifies the nonlinear interaction between the material and the photon.



Generation of squeezed light with a nonlinear photonic crystal



One-dimensional nonlinear photonic crystal, and incident pump and signal light beams.
The materials A and B are $LiNbO_3$ and air, respectively.

Pump light beam: classical strong coherent light $\boldsymbol{E}_{\mathrm{p}} = (0, E_{\mathrm{p}}, 0)$

Signal light beam: quantum weak coherent light $\hat{\boldsymbol{E}}_{\mathrm{s}} = (0, \hat{E}_{\mathrm{s}}, 0)$

Angular frequencies $\omega_{\mathrm{p}} = 2\omega_{\mathrm{s}}$

Wave vectors $\boldsymbol{k}_{\mathrm{p}} = (k_{\mathrm{p}}, 0, 0),\ \boldsymbol{k}_{\mathrm{s}} = (k_{\mathrm{s}}, 0, 0),\ k_{\mathrm{p}} = 2k_{\mathrm{s}}$

$$E_{\mathrm{p}}(x,t) = iA\{\exp[i(k_{\mathrm{p}}x - \omega_{\mathrm{p}}t)] - \exp[-i(k_{\mathrm{p}}x - \omega_{\mathrm{p}}t)]\}$$

$$\hat{E}_{\mathrm{s}}(x,t) = i\sqrt{\frac{\hbar\omega_{\mathrm{s}}}{2\epsilon_0 V}}\{\hat{a}_{\mathrm{s}}\exp[i(k_{\mathrm{s}}x - \omega_{\mathrm{s}}t)] - \hat{a}_{\mathrm{s}}^{\dagger}\exp[-i(k_{\mathrm{s}}x - \omega_{\mathrm{s}}t)]\}$$

Adjusting ω_{s} and ω_{p}, we can let the group velocity of signal photons in the photonic crystal be largely reduced.
This effect enlarges the nonlinear interaction of $LiNbO_3$.
Because $LiNbO_3$ has a large second-order nonlinear optical susceptibility $\chi^{(2)}$, this photonic crystal transforms the incident weak coherent light into the squeezed light.

The signal light that has passed the photonic crystal of width x is transformed into the squeezed signal light in the form,

$$\hat{E}_{\mathrm{s}}(x,t) = i\sqrt{\frac{\hbar\omega_{\mathrm{s}}}{2\epsilon_0 V}}\{\hat{b}_{\mathrm{s}}(x)\exp[i(k_{\mathrm{s}}x - \omega_{\mathrm{s}}t)] - \hat{b}_{\mathrm{s}}^{\dagger}(x)\exp[-i(k_{\mathrm{s}}x - \omega_{\mathrm{s}}t)]\}$$

$$\hat{b}_{\mathrm{s}}(x) = \hat{a}_{\mathrm{s}}\cosh(\beta x) + \hat{a}_{\mathrm{s}}^{\dagger}\sinh(\beta x)$$

$$\beta = \frac{\omega_{\mathrm{s}} A \chi^{(2)}}{\epsilon_0 v_{\mathrm{g}}}$$

A: the amplitude of the pump light
v_g: the group velocity of the signal photons in the photonic crystal
The value of β is real and positive.

Squeeze operator: $\hat{S}(r) = \exp[-\frac{r}{2}(\hat{a}_{\mathrm{s}}^{\dagger 2} - \hat{a}_{\mathrm{s}}^{2})])$

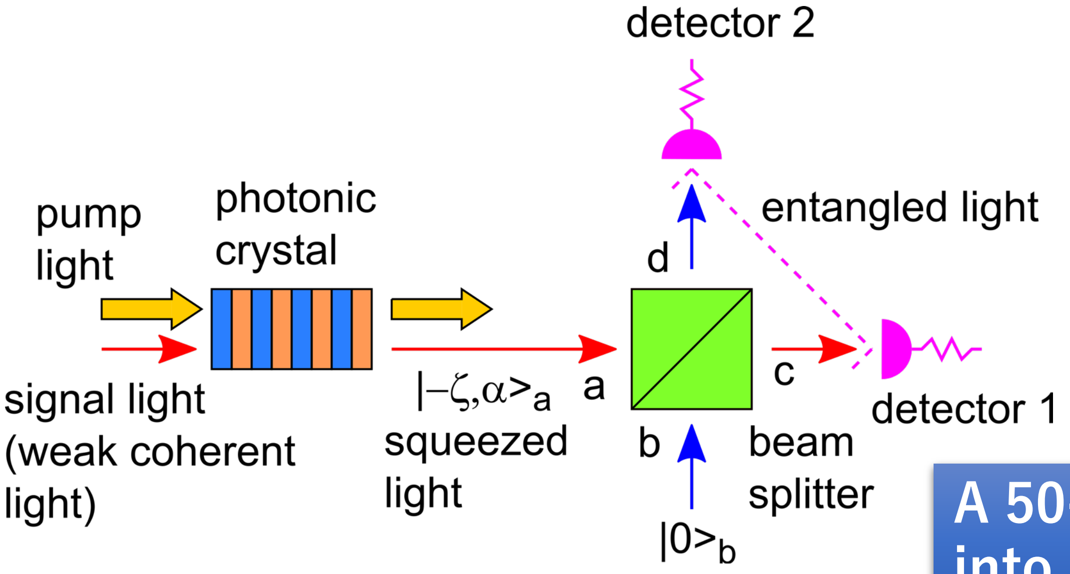
Real squeeze parameter: $r = \beta x$ x : total length of the photonic crystal

$\hat{S}(r)\hat{a}_{\mathrm{s}}\hat{S}^{\dagger}(r) = \hat{b}_{\mathrm{s}}(x)$ An operator $\hat{b}_s(x)$ is obtained by the Bogoliubov transform of $\hat{a}_s$.

Thus, if we inject a coherent light $|\alpha\rangle$ into the photonic crystal, a squeezed coherent light $|-r,\alpha\rangle$ is emitted from it in the form,

$$|-r,\alpha\rangle = \hat{S}^{\dagger}(r)|\alpha\rangle = \hat{S}(-r)|\alpha\rangle$$

How to generate entangled light beams by injecting the squeezed light into a beam splitter



The method for generating entangled light beams with the photonic crystal and beam splitter, and the method to detect entanglement by photon counting. The squeezed light $|-r,\alpha\rangle_a$ is injected into port a of the beam splitter.
Because we do not consider port b, we describe its state as $|0\rangle_b$.

A 50-50 beam splitter transforms the squeezed state into entangled light beams as

$$\hat{B}^{\dagger}|-r,\alpha\rangle_a|0\rangle_b = \hat{S}_{ab}(-\frac{1}{2}r)\hat{S}_a(-\frac{1}{2}r)\hat{S}_b(-\frac{1}{2}r)\hat{D}_a(\frac{1}{\sqrt{2}}\alpha)\hat{D}_b(\frac{1}{\sqrt{2}}\alpha)|0\rangle_a|0\rangle_b$$

$$\hat{S}_{ab}(-\frac{1}{2}r) = \exp[\frac{r}{2}(\hat{a}^{\dagger}\hat{b}^{\dagger} - \hat{b}\hat{a})] \qquad \hat{S}_a(-\frac{1}{2}r) = \exp[\frac{r}{4}(\hat{a}^{\dagger 2} - \hat{a}^2)] \qquad \hat{D}_a(\frac{1}{\sqrt{2}}\alpha) = \exp[\frac{\alpha}{\sqrt{2}}(\hat{a}^{\dagger} - \hat{a})]$$

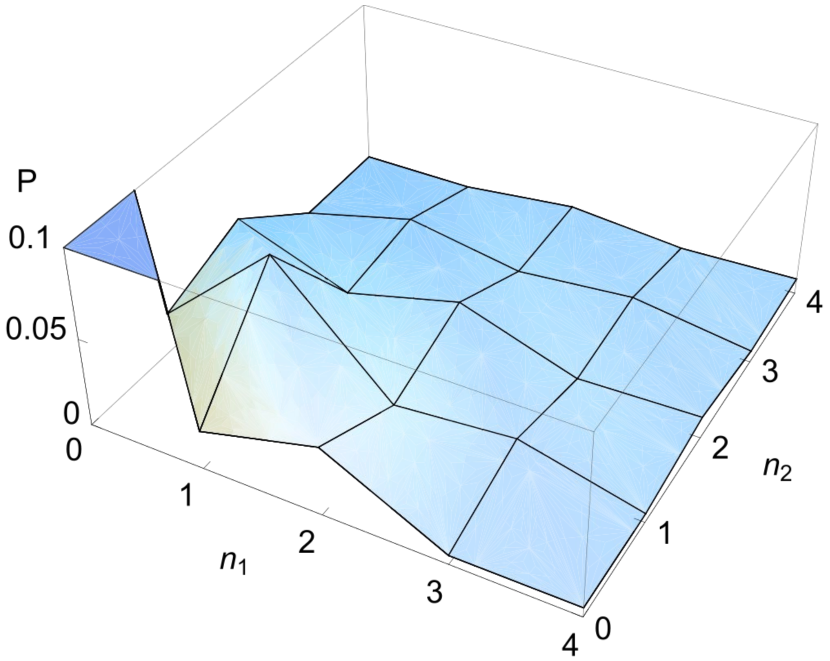
$\hat{S}_{ab}(-\frac{r}{2})$ is an origin of the entanglement.

Numerical calculations of photon statistics that reveal entanglement

The probability that the detectors 1 and 2 detect n_1 and n_2 photons respectively is given by

$$P(n_1, n_2) = |\langle n_1, n_2|\hat{B}^\dagger(0)| - r, \alpha\rangle_a |0\rangle_b|^2$$

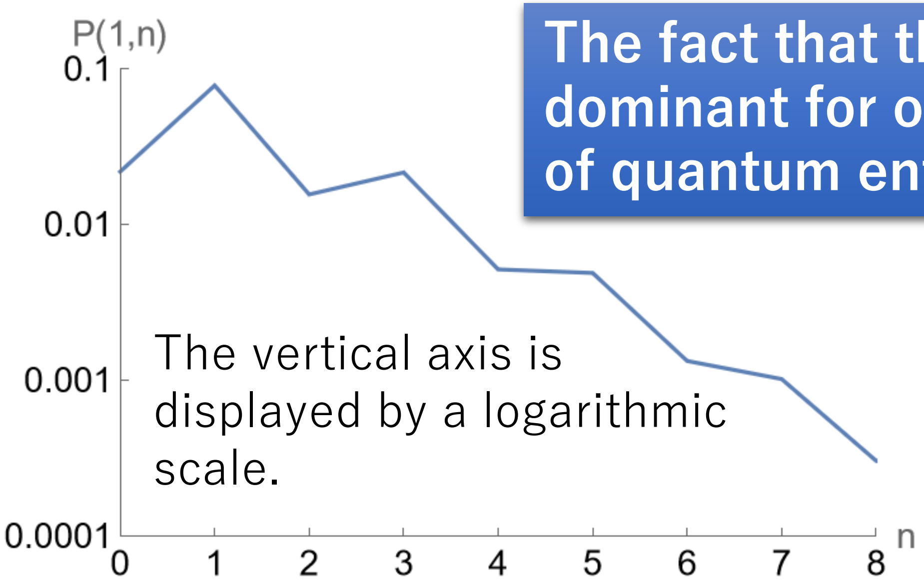
$P(1, n)$: the probability that the detectors 1 and 2 detect one and n photons, respectively.



Graph of $P(n_1, n_2)$ for $\alpha = 1/2$ and $r = 1$.
The highest peak is $P(1,1)$ except for $P(0,0)$.

Squeeze parameter $r = 1$
→8.69 dB

The fact that the probability is dominant for odd n is a reflection of quantum entanglement.



Graph $P(1, n)$ as a function of n for $\alpha = 1/2$ and $r = 1$.
The probability $P(1, n)$ is dominant at odd n and suppressed at even n.

$$P_1 = \sum_{n=0}^{\infty} P(1,n)$$

The probability that the detector 1 detects only a single photon as a heralding signal. In this probability, we do not know the number of photons the detector 2 detects.

$$P(n) = P(1,n)/P_1$$

Renormalized probability

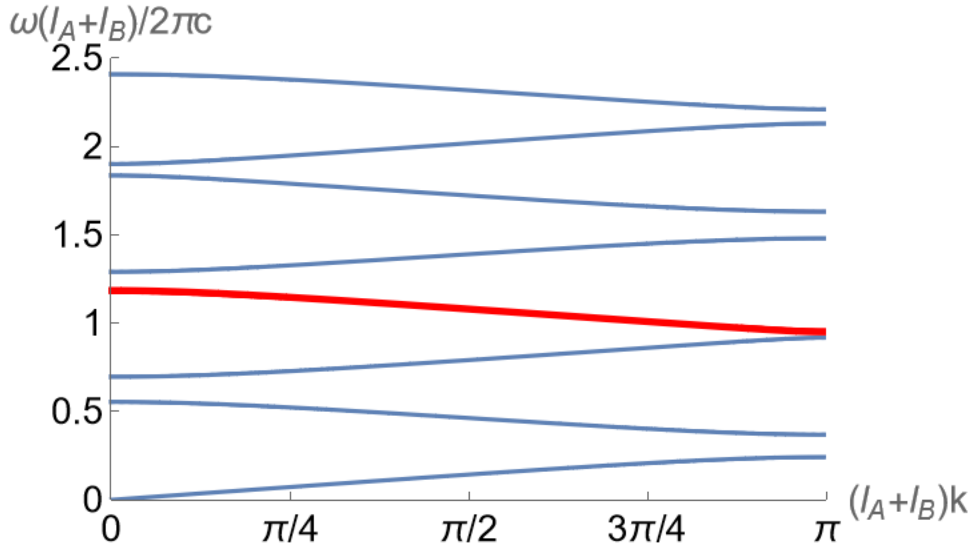
n	0	1	2	3	4	5	6
$P(n)$	0.145	0.520	0.104	0.144	0.0343	0.0325	0.00885

The probability $P(1)$ is greater than $\frac{1}{e} = 0.368$, which is the maximum probability of detecting only a single photon in a weak coherent light.

Thus, our proposed method for constructing the heralded single-photon source is superior to the method of the generation of a single photon from weak coherent light.

If we generate a pair of entangled photons by spontaneous parametric down-conversion (SPDC), the efficiency is 4×10^{-6} per pump photon at most.

Physical parameters of the photonic crystal for generating a squeezed light



Graphs of the dispersion relation for the photonic crystal.
Each curve corresponds to a conduction band.
We show eight conduction bands from the bottom. The red curve represents the fourth conduction band from the bottom.
The band gaps imply that light waves of certain angular frequency ranges are not allowed to pass through the photonic crystal.

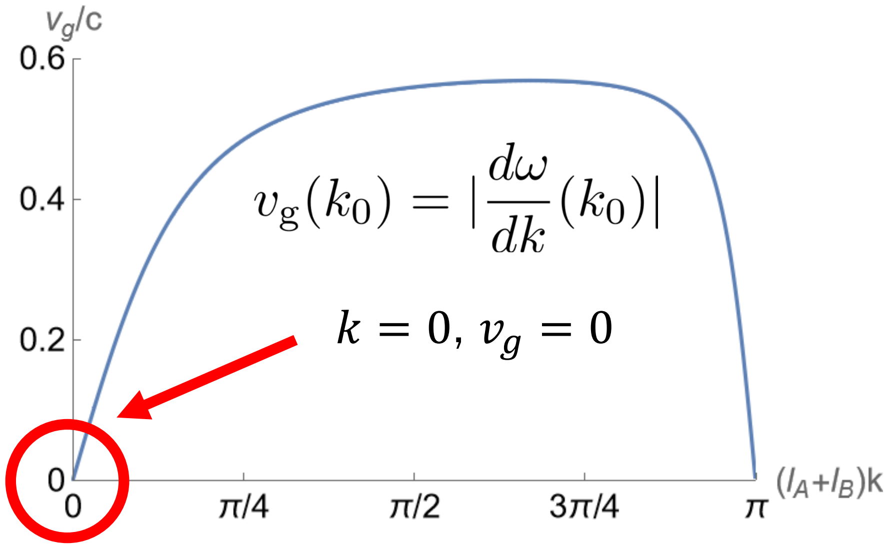
Material A: air
Material B: $\mathrm{LiNbO_3}$c

$$\varepsilon_{\mathrm{A}}/\varepsilon_0 = 1, \quad \varepsilon_{\mathrm{B}}/\varepsilon_0 = n_{\mathrm{B}}^2, \quad n_{\mathrm{B}} = 2.22$$

$$l_{\mathrm{A}} = l_{\mathrm{B}} = 5.5 \times 10^{-7}[\mathrm{m}]$$

We adjust the angular frequencies of the signal and pump photons, so that ω_{s} and ω_{p} belongs to the fourth and eighth conduction bands, respectively.

The group velocity of photons in the photonic crystal



Group velocity of the fourth conduction band from the bottom.
At both ends of the graph, v_g/c is equal to zero.

$$\omega_{\mathrm{s}}(l_{\mathrm{A}} + l_{\mathrm{B}})/(2\pi c) = 1.18$$

$$\lambda_{\mathrm{s}} = 2\pi c/\omega_{\mathrm{s}} = 9.29 \times 10^{-7}[\mathrm{m}]$$

$$\lambda_{\mathrm{p}} = 2\pi c/\omega_{\mathrm{p}} = 4.65 \times 10^{-7}[\mathrm{m}]$$

$LiNbO_3$ is transparent for the light beams of λ_s and λ_p .

$LiNbO_3$: $\chi^{(2)} = \varepsilon_0 \tilde{\chi}^{(2)}, \tilde{\chi}^{(2)} = 25.2 \times 10^{-12}$ [mV^{-1}]

$r = \beta l$

l: total length of the photonic crystal
$l = 5.0 \times 10^{-5}$[m]
We assume $r = 1$.

$$\beta = \frac{\omega_{\mathrm{s}} A \tilde{\chi}^{(2)}}{v_{\mathrm{g}}} \qquad A = \frac{v_{\mathrm{g}}}{\omega_{\mathrm{s}} \tilde{\chi}^{(2)} l} = 1.17 \times 10^8 \times \frac{v_{\mathrm{g}}}{c} \quad \mathrm{V/m}$$

Semiconductor laser
Radiant flux: 0.03 [W]
Radius of the laser beam: 5.0×10^{-6} [m]
Amplitude of the pump beam: $A = 5.36 \times 10^5$ [Vm^{-1}]

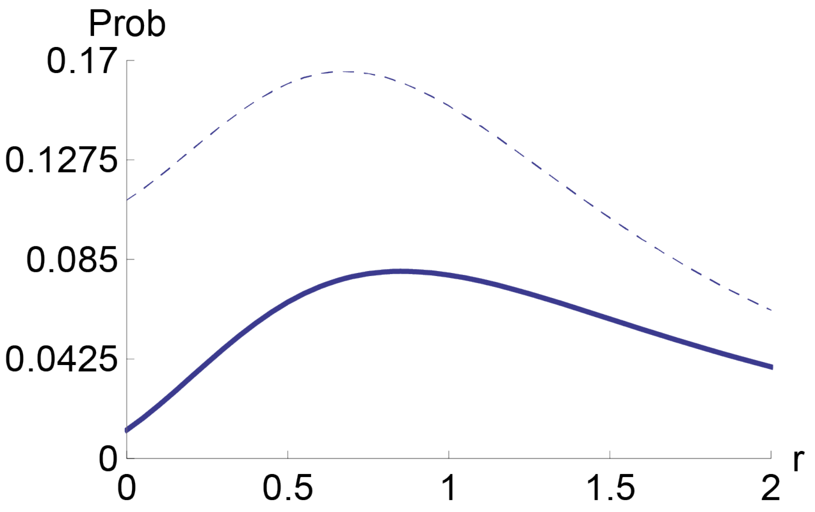
$$\frac{v_{\mathrm{g}}}{c} = 4.57 \times 10^{-3}$$

We must adjust the group velocity of the light passing through the photonic crystal with accurate numerical precision of order 10^{-3} to obtain the squeeze parameter $r = 1$.

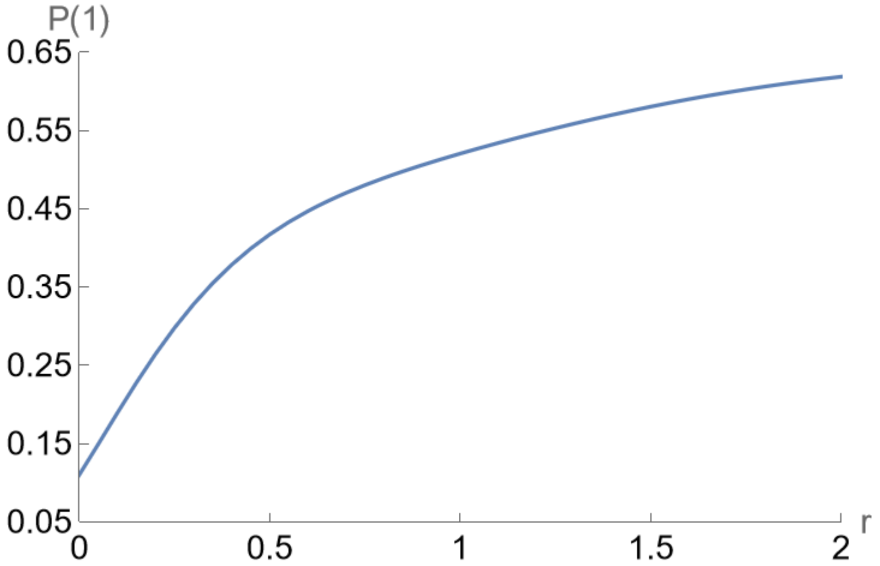
$\nu_s = 3.23 \times 10^{14}$ [Hz], $\Delta\nu_s = 3.11 \times 10^8$ [Hz]

Slightly higher than the Tera Hz wave

Photon statistics as functions of squeeze parameter r



Graphs of $P(1,1)$ and P_1 as functions of r for $\alpha = 1/2$.
The thick solid and thin dashed curves represent $P(1,1)$ and P_1, respectively.
The maximum value of $P(1,1)$ is given by 0.0799 at $r = 0.85$;
the maximum value of P_1 is equal to 0.165 at $r = 0.675$.



Graph of $P(1)$ as a function of r for $\alpha = 1/2$.
$P(1)$ increases with an increase in r.

Looking at this graph, we note that as the squeeze parameter gets larger, the probability of heralded single-photon emission becomes larger.

Summary

- We discuss how to generate the squeezed light with a nonlinear photonic crystal.
- We discuss how to generate entangled photons by injecting the squeezed light into a beam splitter.
- Detecting one of a pair of entangled photons, we can obtain the heralded single photon at the time of the detector's measurement.
- We estimate how much we should adjust accurately the group velocity of photons in the nonlinear photonic crystal to obtain enough squeeze level.