Linear optical logical Bell measurements

constructions & optimality

arXiv:2101.11082 & arXiv:2302.07908

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with Paul Hilaire, Yaron Castor, Edwin Barnes, Sophia E. Economou

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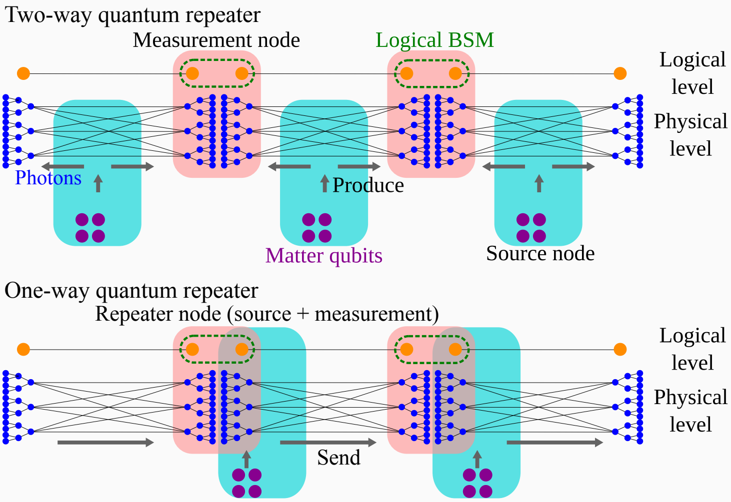
Our goal: teleport qubits far away



Linear Optical Bell Measurement

A family of photonic tree codes

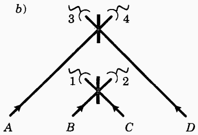
Optimal loss-tolerant LO-BSMs

What’s next?

Linear Optical Bell Measurement

Physical Bell State Measurement

[Weinfurter '94] „Innsbruck scheme"



$\boxed{\Phi^+ \qquad \Phi^-}$

$\boxed{\Psi^+}$ $\boxed{\Psi^-}$

- $\Pr(\text{success}) = \frac{1}{2}$
- known to be optimal

  [Calsamiglia&Lütkenhaus '99]

Physical Bell State Measurement

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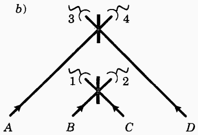

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Physical Bell State Measurement

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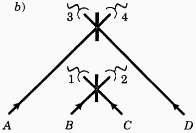

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[Calsamiglia&Lütkenhaus '99]

$X_a X_b =$	$+1$	-1
$Z_a Z_b = +1$	Φ^+	Φ^-
$Z_a Z_b = -1$	Ψ^+	Ψ^-

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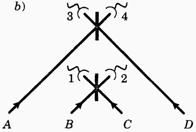

- $\Pr(\text{success}) = \frac{1}{2}$
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  [Calsamiglia&Lütkenhaus '99]

$$\begin{array}{lcc} X_aX_b = & +1 & -1 \\ Z_aZ_b = +1 & \multicolumn{2}{c}{\boxed{\Phi^+ \qquad \Phi^-}} \\ Z_aZ_b = -1 & \boxed{\Psi^+} & \boxed{\Psi^-} \end{array}$$

Actually equivalent to

Measure Z_aZ_b. If

$Z_aZ_b \overset{?}{=} +1$: Measure X_aX_b

$Z_aZ_b \overset{?}{=} -1$: Measure Z_a

Physical Bell State Measurement

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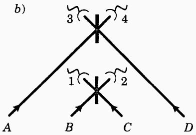

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$X_aX_b =$	$+1$	-1
$Z_aZ_b = +1$	$\boxed{\Phi^+}$	$\boxed{\Phi^-}$
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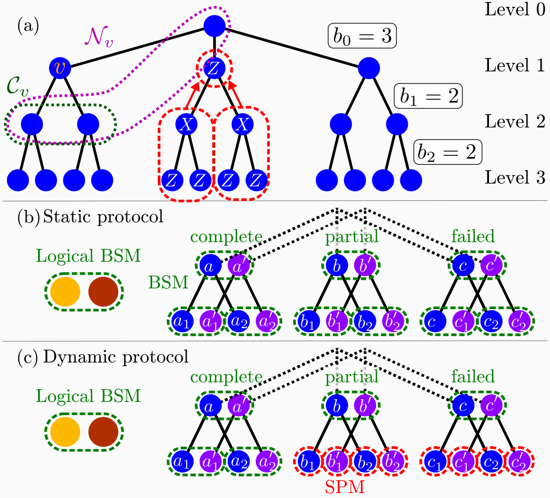
Actually equivalent to

Measure Z_aZ_b. If

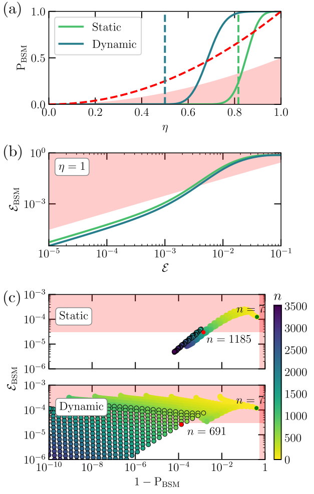
$Z_aZ_b \stackrel{?}{=} -1$: Measure X_aX_b

$Z_aZ_b \stackrel{?}{=} +1$: Measure Z_a

A family of photonic tree codes


(a)
$\mathcal{N}_v$
$\mathcal{C}_v$
v
Z
X
X
Z
Z
Z
Z
$b_0 = 3$
$b_1 = 2$
$b_2 = 2$
Level 0
Level 1
Level 2
Level 3
(b) Static protocol
Logical BSM
BSM
complete
partial
failed
a a'
b b'
c c'
a_1 a_1' a_2 a_2'
b_1 b_1' b_2 b_2'
c c_1' c_2 c_2'
(c) Dynamic protocol
Logical BSM
complete
partial
failed
a a'
b b'
c c'
a_1 a_1' a_2 a_2'
b_1 b_1' b_2 b_2'
c_1 c_1' c_2 c_2'
SPM

GOOD PERFORMANCES



- (a-b) for 15-15-2 tree (691 qubits)

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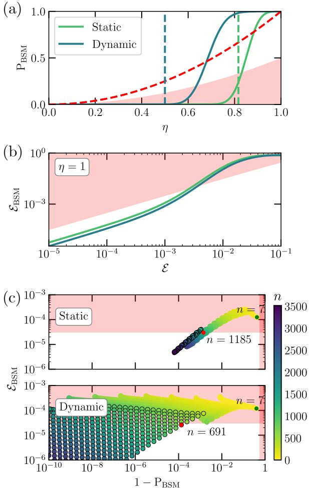
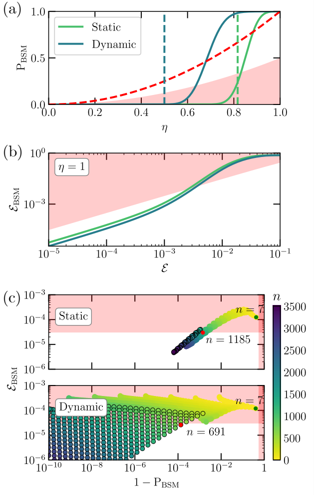


- (a-b) for 15-15-2 tree (691 qubits)
- For $n \to \infty$, numerics show $P_{\mathrm{BSM}} \to \begin{cases} 1 \\ 0 \end{cases}$ around
    - $\eta^{\text{min}}_{\text{stat}} \to .81640568 \cdots \approx \sqrt{\frac{2}{3}}$
    - $\eta^{\text{min}}_{\text{dyn}} \to .50000000 \cdots \approx \frac{1}{2}$

Too good performances to be true ?



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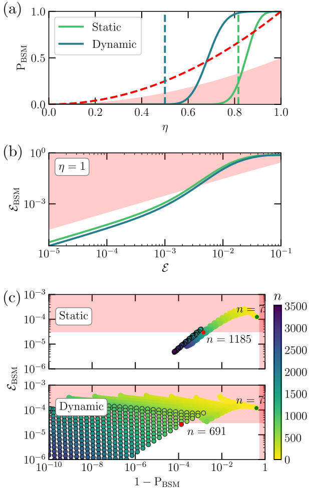
  - $\eta_{\text{stat}}^{\text{min}} \to .81640568 \cdots \approx \sqrt{\frac{2}{3}}$
  - $\eta_{\text{dyn}}^{\text{min}} \to .50000000 \cdots \approx \frac{1}{2}$
- but [Lee, Ralph, Jeong '19] showed
  - $\eta_{LO}^{\text{min}} > \frac{1}{\sqrt{2}} \approx .70710681 \ldots$



- (a-b) for 15-15-2 tree (691 qubits)
- For $n \to \infty$, numerics show P_{BSM} bifurcates to 1 / 0 around
  - $\eta_{\text{stat}}^{\text{min}} \to .81640568 \cdots \approx \sqrt{\frac{2}{3}}$
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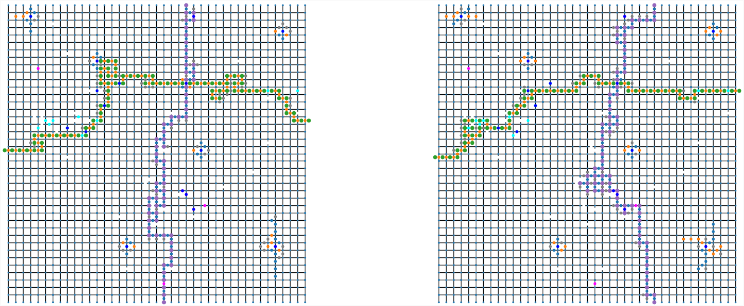
- but [Lee, Ralph, Jeong '19] showed
  - $\eta_{LO}^{\text{min}} > \frac{1}{\sqrt{2}} \approx .70710681 \ldots$
- [Lee, Ralph, Jeong '19]'s proof assumed BSMs measurement but single-photon measurements are useful!

Can we do better with surface codes?



- Surface code are more efficient that tree codes
- ...and we can use the same tricks

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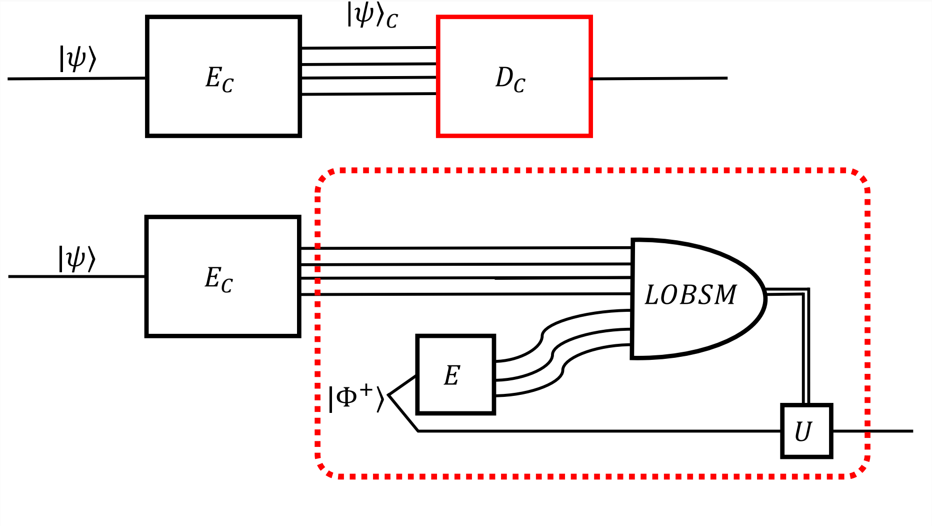
  Yaron proved us wrong
- Easier to find analytical bounds

Optimal loss-tolerant LO-BSMs

NO CLONING BOUND AND LOGICAL BSM

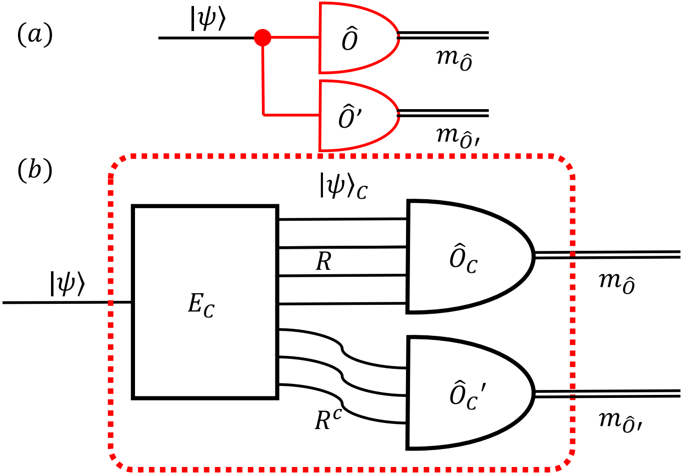


Data only depends on the product $\eta_a \eta_b$ so
no cloning condition has to be ensured by $\eta_a \eta_b$

$$\forall \eta_a, \eta_b : \eta_a \eta_b > \frac{1}{2} \Rightarrow \begin{cases} \frac{1}{2} < \eta_a < 1 & \nexists \text{ copy of } a \\ \frac{1}{2} < \eta_b < 1 & \nexists \text{ copy of } b \end{cases}$$

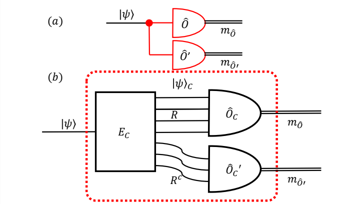
$$\exists \eta_a, \eta_b : \eta_a \eta_b \leq \frac{1}{2} \Leftarrow \begin{cases} \eta_a \leq \frac{1}{2} & \exists \text{ copy of } a \\ \eta_b = 1 & \end{cases}$$

This is the $\frac{1}{\sqrt{2}}$ bound of [Lee, Ralph, Jeong '14] and they provide an explicit code achieving it



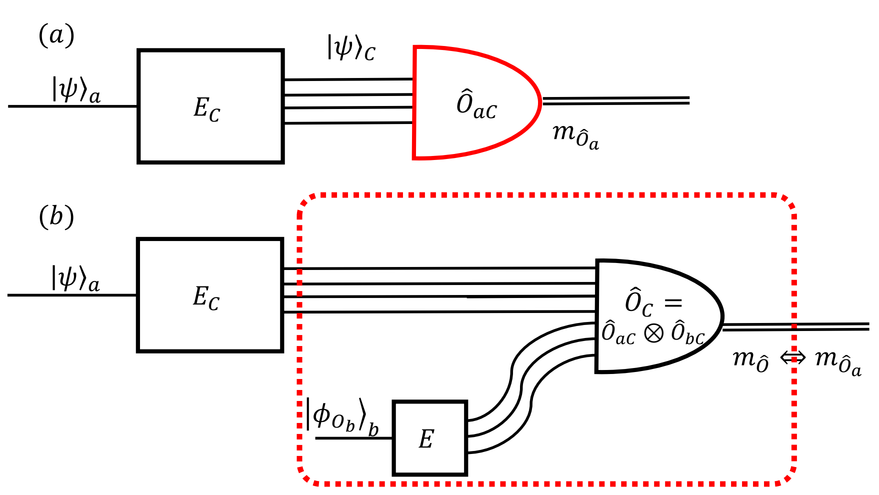
(a)
$|\psi\rangle$
$\hat{O}$
$m_{\hat{O}}$
$\hat{O}'$
$m_{\hat{O}'}$
(b)
$|\psi\rangle_C$
$|\psi\rangle$
E_C
R
$\hat{O}_C$
$m_{\hat{O}}$
R^c
$\hat{O}_C{}'$
$m_{\hat{O}'}$

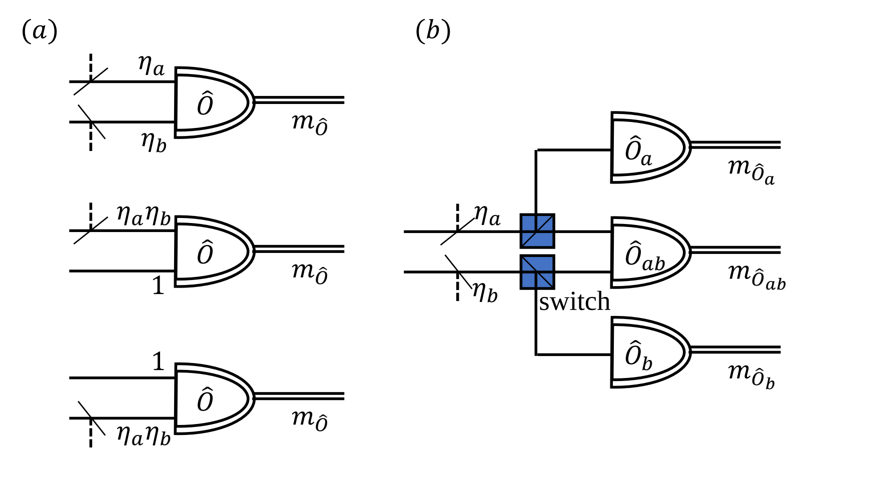
Local observable decoding



If $[\hat{O}_C, \hat{O}'_C] \neq 0$, $\eta^{\mathrm{min}}_{O_C} + \eta^{\mathrm{min}}_{O'_C} > 1$

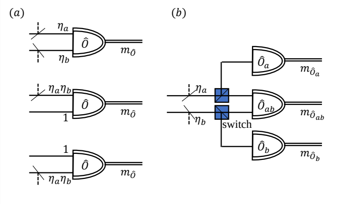
Using static BSMs to measure X and Z





(a)
η_a
$\hat{O}$
$m_{\hat{O}}$
η_b
$\eta_a\eta_b$
$\hat{O}$
$m_{\hat{O}}$
1
1
$\hat{O}$
$m_{\hat{O}}$
$\eta_a\eta_b$
(b)
$\hat{O}_a$
$m_{\hat{O}_a}$
η_a
$\hat{O}_{ab}$
$m_{\hat{O}_{ab}}$
η_b
switch
$\hat{O}_b$
$m_{\hat{O}_b}$

Using static BSMs to measure X and Z



Success if $\eta_a \Pr(XX) > \eta_X^{\min}$ & $\eta_a \Pr(ZZ) > \eta_Z^{\min}$

$\Rightarrow \eta_a\eta_b > \max\left(\frac{\eta_X^{\min}}{\Pr(XX)}, \frac{\eta_Z^{\min}}{\Pr(ZZ)}\right) \geq \frac{2}{3}$ since $\Pr(XX) + \Pr(ZZ) = \frac{2}{3}$ and $\eta_X^{\min} + \eta_Z^{\min} > 1$

Using static BSMs to measure X and Z

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USING STATIC BSMS TO MEASURE X AND Z

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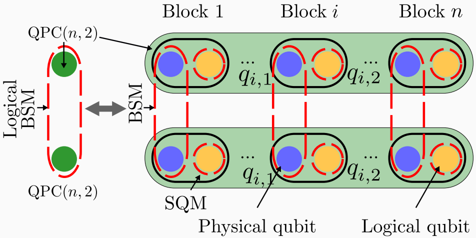
Limit achieved with random BSMs $(\Pr\{XX\} = \Pr(ZZ) = \frac{3}{4})$

and surface codes with $\frac{1}{2}$ threshold[1]

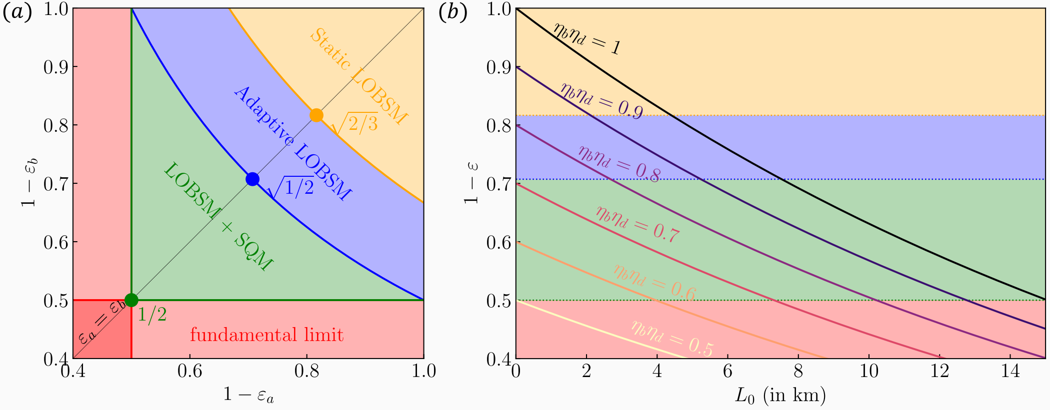
[1]or our trees if $.5000000\cdots = \frac{1}{2}$

There is a concatenated $(2, n)$-Shor code allowing to make BSMs when

$$\min(\eta_b, \eta_a) > \frac{1}{2}$$



Performances



What's next?

What's next?

- Look at actual good QECC (finite number of qubits, finite efficiency)
- Take into account the preparation cost of the codes
- Static LOBSM setting if we allow more general linear-optical information processing ?
- Find (and implement) a realistic one !