A Linear Algebraic Framework for Dynamic Scheduling Over Memory-Equipped Quantum Networks[1]

arXiv:2307.06009

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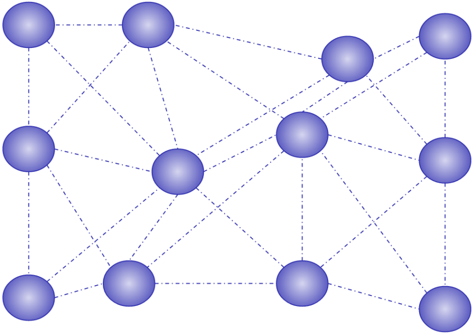
[1] P. F., Anastasios Giovanidis, and Frédéric Grosshans. “A Linear Algebraic Framework for Dynamic Scheduling Over Memory-Equipped Quantum Networks”. In: *IEEE Transactions on Quantum Engineering [Accepted for Publication]* (2023).

Outline

The Problem of Scheduling

Classical Networks

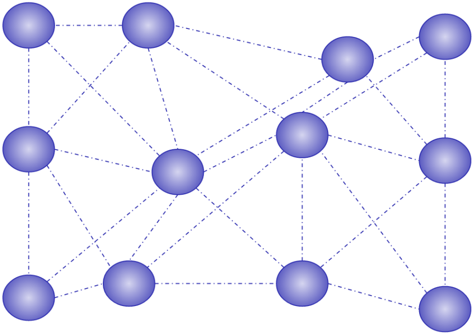


- Regulation of packet traffic along the network routes
- Standard solutions are well-known in the classical domain[2]

[2] L. Tassiulas and A. Ephremides. "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks". In: *IEEE Transactions on Automatic Control* 37.12 (1992), pp. 1936–1948.

The Problem of Scheduling

Quantum Networks



- Regulating which swapping operations happen at a given time
- Max Weight policies have recently been proven useful in quantum networks as well[3,4]

Thirupathaiah Vasantam and Don Towsley. "A throughput optimal scheduling policy for a quantum switch". In: *Quantum Computing, Communication, and Simulation II*. ed. by Philip R. Hemmer and Alan L. Migdall. Vol. 12015. International Society for Optics and Photonics. SPIE, 2022, p. 1201505.

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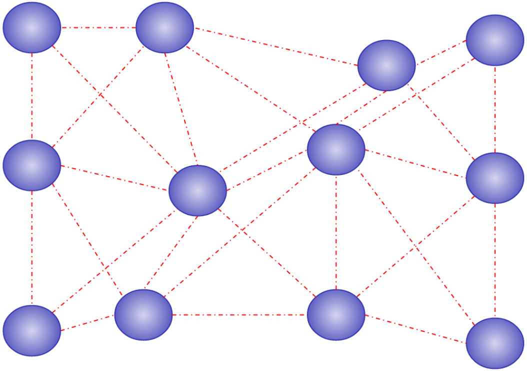
Contributions

- **Dynamically controlled scheduling framework for arbitrary topologies and multiple commodities**;
- Proposal of two classes of scheduling policies;
- Benchmarking of the proposed policies over nontrivial topologies;

Notation



Arbitrary connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$:

- Switches located at $\mathcal{V}$
- (Lossy) fiber links along $\mathcal{E}$
- Additionally, $\tilde{\mathcal{E}} = \mathcal{V} \times \mathcal{V}$ contains *all* possible edges

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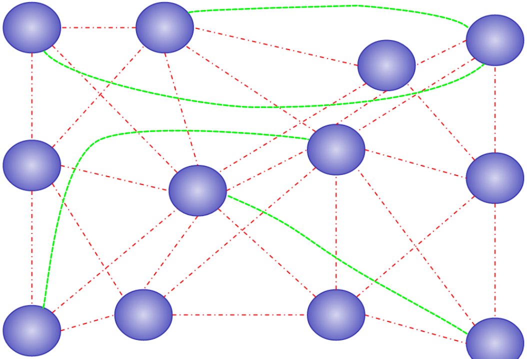

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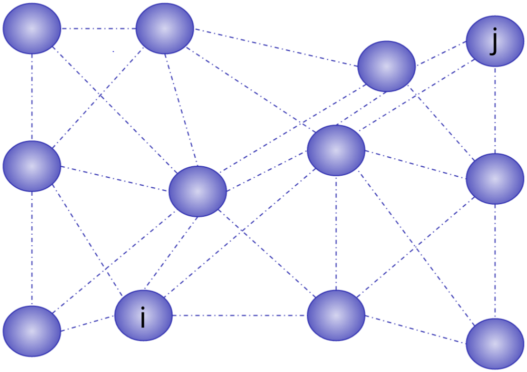
A Bird's-Eye View



$\forall i, j \in \tilde{\mathcal{E}}$:

- An *Ebit Queue* $q_{ij}(t)$ keeps track of the stored entangled pairs
- A *Demand Queue* $d_{ij}(t)$ stores user demand (if any).

Objective

Linear equations that describe their intertwined evolution.

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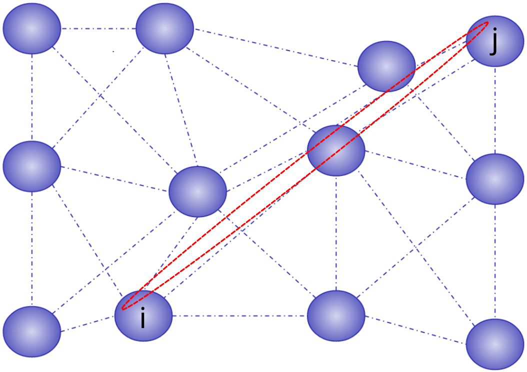

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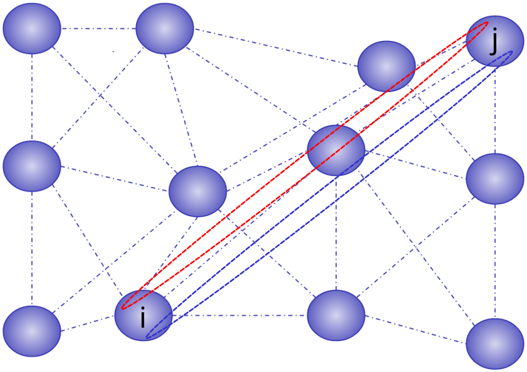

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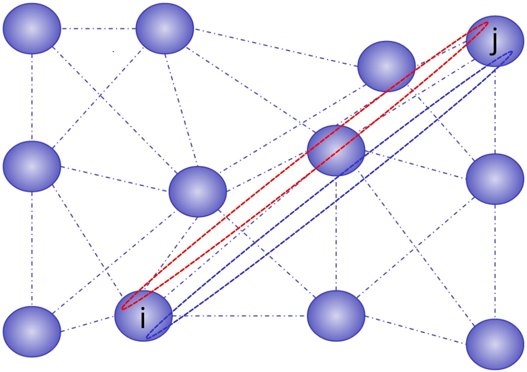

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Ebit Queues

$\forall i, j \in \tilde{\mathcal{E}}$:

$$\begin{bmatrix} q_{AB}(t+1) \\ q_{BC}(t+1) \\ q_{AC}(t+1) \\ \cdots \end{bmatrix} = \begin{bmatrix} q_{AB}(t) \\ q_{BC}(t) \\ q_{AC}(t) \\ \cdots \end{bmatrix} + \begin{bmatrix} a_{AB}(t) \\ a_{BC}(t) \\ a_{AC}(t) \\ \cdots \end{bmatrix} - \begin{bmatrix} \ell_{AB}(t) \\ \ell_{BC}(t) \\ \ell_{AC}(t) \\ \cdots \end{bmatrix} \pm \text{scheduling}$$

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Ebit Queues

Notation for Swapping

$$r_{i[j]k}(t) = n \in \mathbb{N}$$

Example

$r_{A[B]C}(2) = 3 \rightarrow$ Three swapping operations at node B from queues AB, BC to AC are scheduled to happen at time step 2.

Ebit Queues

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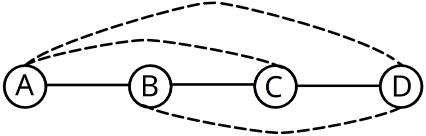
$$\boldsymbol{r}(t) = \begin{bmatrix} r_{A[B]C}(t) & r_{B[C]D}(t) & \dots \end{bmatrix}$$

Ebit Queues

The $\boldsymbol{M}$ Matrix



	$A[B]C$	$B[C]D$	$A[B]D$	$A[C]D$
AB	-1	0	-1	0
BC	-1	-1	0	0
CD	0	-1	0	-1
AC	$+1$	0	0	-1
BD	0	$+1$	-1	0
AD	0	0	$+1$	$+1$

$$\boldsymbol{q}(t+1) = \boldsymbol{q}(t) - \boldsymbol{\ell}(t) + \boldsymbol{a}(t) + \boldsymbol{M}\boldsymbol{r}(t)$$

Consuming Pairs to Serve Demands

$$\tilde{\boldsymbol{M}} = \begin{bmatrix} \boldsymbol{M} & | & -\mathbb{I}_{N_{\text{queues}}} \end{bmatrix}$$

	$A[B]C$	$B[C]D$	$A[B]D$	$A[C]D$	AB	BC	CD	AC	BD	AD
AB	−1	0	−1	0	−1	0	0	0	0	0
BC	−1	−1	0	0	0	−1	0	0	0	0
CD	0	−1	0	−1	0	0	−1	0	0	0
AC	+1	0	0	−1	0	0	0	−1	0	0
BD	0	+1	−1	0	0	0	0	0	−1	0
AD	0	0	+1	+1	0	0	0	0	0	−1

$$\boldsymbol{r}(t) = \begin{bmatrix} r_{A[B]C}(t) & r_{B[C]D}(t) & \dots & | & r_{AB}(t) & r_{BC}(t) & \dots \end{bmatrix}$$

Demand Queues

$$\boldsymbol{q}(t+1) = \boldsymbol{q}(t) + \boldsymbol{a}(t) - \boldsymbol{\ell}(t) + \tilde{\boldsymbol{M}}\boldsymbol{r}(t)$$

- $$\boldsymbol{d}(t+1) = \boldsymbol{d}(t) + \boldsymbol{b}(t) \qquad + \tilde{\boldsymbol{N}}\boldsymbol{r}(t)$$
- $$\tilde{\boldsymbol{N}} = \left[\mathbb{O}_{N_{\text{queues}} \times N_{\text{transitions}}} \quad | \quad -\mathbb{I}_{N_{\text{queues}}}\right]$$

Recap

- $\forall (i,j) \in \tilde{\mathcal{E}}$ there is an ebit queue and a demand queue.
- Scheduling $\rightarrow$ Deciding $\boldsymbol{r}(t)$ given some information about $\boldsymbol{q}$ and $\boldsymbol{d}$
- Information about routing and topology is encoded in the $\boldsymbol{M}$ matrix.

Deriving a Scheduling Policy

The Lyapunov Function

$$V(t) = \frac{1}{2}\boldsymbol{d}^T(t)\boldsymbol{d}(t) = d_{AB}^2 + d_{BC}^2 + d_{CD}^2 \cdots$$

- Scalar measure of the overall system stability;
- Standard tool in classical Network Science;
- Akin to a potential in physics;

Deriving a Scheduling Policy

Lyapunov Drift

$$V(t) = \frac{1}{2}\boldsymbol{d}^T(t)\boldsymbol{d}(t)$$

$$\Delta V = \frac{1}{2}\mathbb{E}[\boldsymbol{d}^T(t+1)\boldsymbol{d}(t+1) - \boldsymbol{d}^T(t)\boldsymbol{d}(t)\big|\mathcal{I}(t)]$$

Expanding the mathematics, we obtain a **controllable expression** that gauges the effects of a given scheduling decision on the system.

Deriving a Scheduling Policy

The Full Information Quadratic Scheduler

$$\Delta V = \frac{1}{2}\mathbb{E}[\boldsymbol{d}^T(t+1)\boldsymbol{d}(t+1) - \boldsymbol{d}^T(t)\boldsymbol{d}(t)\big|\mathcal{I}(t)] \rightarrow \begin{cases} \boldsymbol{r}(t) = \arg\min\Big(\boldsymbol{w}^T(t)\cdot\boldsymbol{r}(t) + \\ \qquad +\frac{1}{2}\boldsymbol{r}(t)^T\tilde{\boldsymbol{N}}^T\tilde{\boldsymbol{N}}\boldsymbol{r}(t)\Big) \\ \text{s.t. } \boldsymbol{r}(t) \in \mathcal{R}(t) \\ \boldsymbol{w}(t) = (\boldsymbol{d}(t) + \boldsymbol{b}(t))^T\tilde{\boldsymbol{N}} \end{cases}$$

$$\mathcal{R}(t) = \left\{\boldsymbol{r}(t) \in \mathbb{N}^d \;\middle|\; \begin{array}{l} -\tilde{\boldsymbol{M}}\boldsymbol{r}(t) \leq \boldsymbol{q}(t) - \boldsymbol{\ell}(t) + \boldsymbol{a}(t) \\ -\tilde{\boldsymbol{N}}\boldsymbol{r}(t) \leq \boldsymbol{d}(t) \qquad\qquad + \boldsymbol{b}(t) \end{array}\right\}$$

- At every time step, solve the optimization problem

\+ High performance

– Large communication requirements

– High computational cost

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Alternative Scheduling Policies

PROBLEMS

- Quadratic integer optimization is computationally expensive;
- Full-Information scheduling has impossible communication constraints;

SOLUTIONS

- Suppress the quadratic term in the optimization problem $\longrightarrow$ **MAX-WEIGHT SCHEDULING**;
- Replace all the information we don't have with educated guesses $\longrightarrow$ **LOCAL INFORMATION SCHEDULING**

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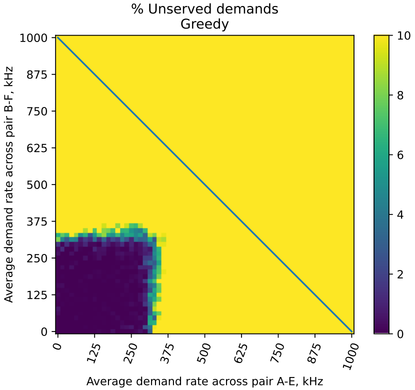
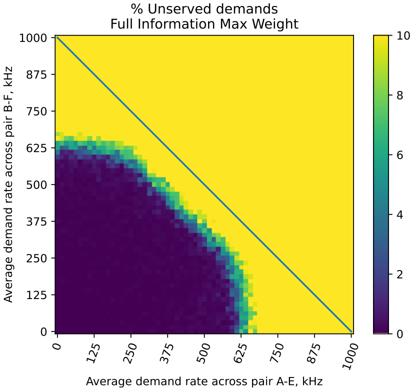
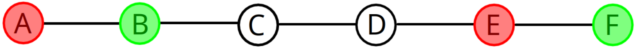
Result Example

FIMW Scheduler, Simple Topology

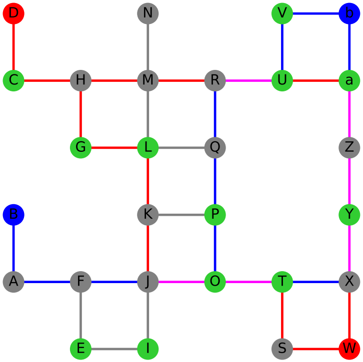


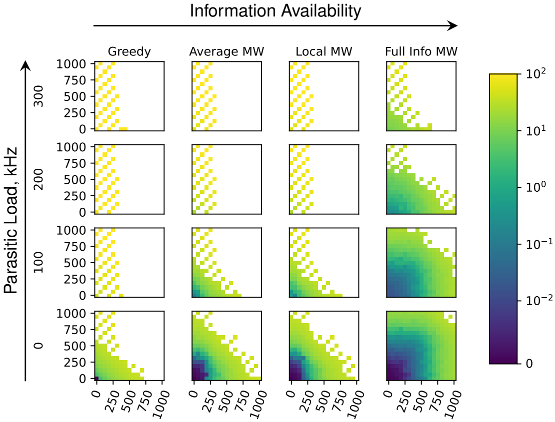




Results

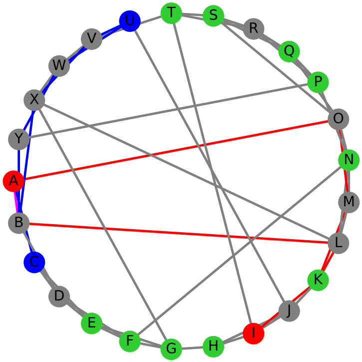
Grid with Holes

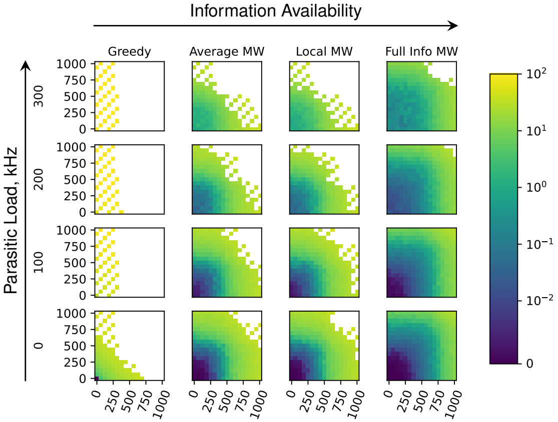




Results

Watts-Strogatz Graph





Summary

- We introduced a multicommodity dynamically controlled framework for scheduling on general topologies;
- We derived a class of scheduling policies for quantum networks that minimizes the square norm of the demand backlog;
- Our framework was demonstrated to be useful as a benchmarking tool for arbitrary policies over non-trivial topologies.
- Outlook
  - Introducing quantum noise in the framework;
  - Provide analytical demonstration of stability and optimality of policies.

References

- **F., P.**, Anastasios Giovanidis, and Frédéric Grosshans. "A Linear Algebraic Framework for Dynamic Scheduling Over Memory-Equipped Quantum Networks". In: *IEEE Transactions on Quantum Engineering* ***[Accepted for Publication]*** (2023).
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