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Lower Bounds on Number of QAOA Rounds Required for Guaranteed Approximation Ratios

Naphan Benchasattabuse, Andreas Bärtschi, Luis Pedro García-Pintos, John Golden, Nathan Lemons, Stephan Eidenbenz
(arXiv:2308.15442)

15th December 2023
JFQI 2023 Workshop

This work was supported Laboratory Directed Research and Development program of Los Alamos National Laboratory under project number 20230049DR

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Combinatorial optimization problem

In the most general form, you're given

- Search space (we only consider binary string of length n; $x \in \{0,1\}^n$)
- Objective function $C(X) \mapsto \mathbb{R}$
- Feasible set (for constrained problem) $F \subseteq \{0,1\}^n$

The task is to find a feasible binary string which maximize the objective value.

Combinatorial optimization problem

In this work, we consider

- Search space (we only consider binary string of length n; $x \in \{0,1\}^n$)
- Objective function $C(X) \mapsto \mathbb{N}_0$
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If we allow good (not necessarily optimal) solution, the quantity of interest is

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The task is to find a feasible binary string which maximize the objective value.

$$C_{max} = \max_z C(z) \qquad C_{avg} = \frac{1}{N}\sum_z C(z) \qquad \sigma_C = \sqrt{\sum_z \frac{(C(z)-C_{avg})^2}{N}}$$

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"approximation ratio" - $\frac{C(\tilde{z})}{C_{max}}$

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Recast the problem in Hamiltonian

$$H_C = \sum_{z \in F} C(z) \left|z\right\rangle \left\langle z\right|$$

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more natural to find ground state and Hamiltonian with zero ground state energies

$$H_1 = \mathbb{1} C_{max} - H_C$$

Quantum Optimization

What quantum algorithms do we have to solve optimization problem?

- (exact) Grover Adaptive Search

  [Dürr, Høyer, arXiv:9607014] [Gilliam, Woerner, Gonciulea, Quantum 5, 428 (2021)]
- (exact/approximate) Quantum Annealing

  [Kadowaki, Nishimori, Phys. Rev. E 58, 5355 (1998)] [Farhi, Goldstone, Gutmann, Sipser, arXiv:0001106]
- (exact) Short-path algorithm

  [Hastings, Quantum 2, 78 (2018)] [Dalzell, Pancotti, Campbell, Brandão, STOC2023]
- (approximate) Quantum Approximate Optimization Algorithm (QAOA)

  [Farhi, Goldstone, Gutmann, arXiv:1411.4028] [Hadfield, Wang, O'Gorman, Rieffel, Venturelli, Biswas, Algorithms 12.2 (2019): 34]

Why QAOA?

Recent empirical results look promising

(arXiv:2308.02342)

Evidence of Scaling Advantage for the Quantum Approximate Optimization Algorithm on a Classically Intractable Problem

Ruslan Shaydulin,[1,*] Changhao Li,[1] Shouvanik Chakrabarti,[1] Matthew DeCross,[2] Dylan Herman,[1] Niraj Kumar,[1] Jeffrey Larson,[3] Danylo Lykov,[1,4] Pierre Minssen,[1] Yue Sun,[1] Yuri Alexeev,[4] Joan M. Dreiling,[2] John P. Gaebler,[2] Thomas M. Gatterman,[2] Justin A. Gerber,[2] Kevin Gilmore,[2] Dan Gresh,[2] Nathan Hewitt,[2] Chandler V. Horst,[2] Shaohan Hu,[1] Jacob Johansen,[2] Mitchell Matheny,[2] Tanner Mengle,[2] Michael Mills,[2] Steven A. Moses,[2] Brian Neyenhuis,[2] Peter Siegfried,[2] Romina Yalovetzky,[1] and Marco Pistoia[1]

[1] *Global Technology Applied Research, JPMorgan Chase, New York, NY 10017, USA*
[2] *Quantinuum, Broomfield, CO 80021, USA*
[3] *Mathematics and Computer Science Division, Argonne National Laboratory, Lemont, IL 60439, USA*
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Numerical Evidence for Exponential Speed-up of QAOA over Unstructured Search for Approximate Constrained Optimization

John Golden*†, Andreas Bärtschi*†, Daniel O'Malley‡, Stephan Eidenbenz†
† *Information Sciences (CCS-3), Los Alamos National Laboratory*, Los Alamos, NM 87544, USA
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* Corresponding authors: golden@lanl.gov, baertschi@lanl.gov

***Abstract*—Despite much recent work, the true promise and limitations of the Quantum Alternating Operator Ansatz** the optimal operators for each problem, and the amount of phasing and mixing for each round, generates a large number

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The Quantum Alternating Operator Ansatz for Satisfiability Problems

John Golden*†, Andreas Bärtschi*†, Daniel O'Malley‡, Stephan Eidenbenz†
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***Abstract*—We comparatively study, through large-scale numerical simulation, the performance across a large set of Quan-** others. Overall, the connection between mixers, phase separators, and QAOA performance is not well understood. In this

QAOA

Quantum approximate optimization algorithm or quantum alternating operator ansatz

- Two non-commuting Hamiltonians H_0(mixing) and H_1 (problem)
- Hyperparameter p
- Two set of angles (parameters) $\vec{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)$ $\vec{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_p)$

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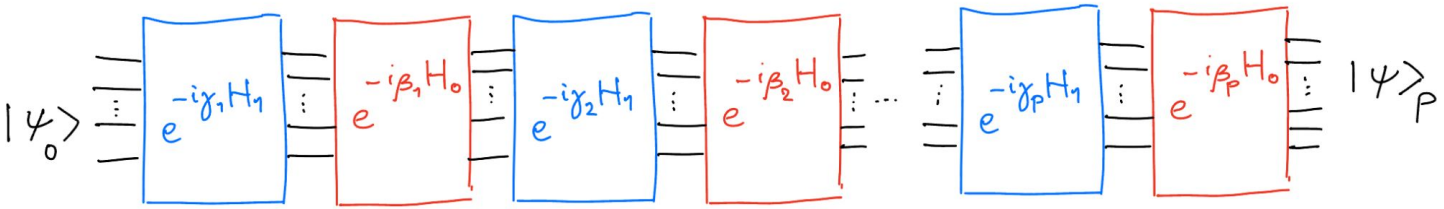

$$|\psi(\vec{\beta}, \vec{\gamma})\rangle_p = (e^{-i\beta_p H_0} e^{-i\gamma_p H_1})(e^{-i\beta_{p-1} H_0} e^{-i\gamma_{p-1} H_1}) \ldots (e^{-i\beta_1 H_0} e^{-i\gamma_1 H_1}) |\psi_0\rangle$$

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The goal is to get a good string " $\tilde{z}$"

And we use the expectation value to guide the parameter adjustment

$$|\psi(\vec{\beta}, \vec{\gamma})\rangle_p = (e^{-i\beta_p H_0} e^{-i\gamma_p H_1})(e^{-i\beta_{p-1} H_0} e^{-i\gamma_{p-1} H_1}) \ldots (e^{-i\beta_1 H_0} e^{-i\gamma_1 H_1}) |\psi_0\rangle$$

$$\lambda = \frac{\langle H_C \rangle_p}{C_{max}} = \max_{|\psi_p\rangle} \frac{\langle \psi_p | H_C | \psi_p \rangle}{C_{max}}$$

Quantum Annealing

Consists of (1) set of driving Hamiltonians (2) annealing schedule

- Two non-commuting Hamiltonians H_0(mixing) and H_1 (problem)
- Annealing schedule $g(t)$ ranges between 0 and 1

  $\text{that satisfies } g(0) = 0 \text{ and } g(t_f) = 1$

- The system is driven by time-dependent Hamiltonian

$$H(t) = (1 - g(t))H_0 + g(t)H_1$$

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Results on quantum annealing also applies to QAOA!

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Lower bound on annealing time

Theorem 1 (García-Pintos et al. [31]). *Given two driving Hamiltonians with zero ground state energies H_0 and H_1. For a quantum annealing protocol that starts in the ground state of H_0 and ends at time t_f, it holds that*

$$t_f \geq \tau_{anneal} := \frac{\langle H_0 \rangle_{t_f} + \langle H_1 \rangle_0 - \langle H_1 \rangle_{t_f}}{\|[H_1, H_0]\|}. \quad (7)$$

[García-Pintos, Brady, Bringewatt, Liu, Phys. Rev. Lett. 130, 140601]

$$\langle H \rangle_t = \langle \psi_t | H | \psi_t \rangle$$

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$$\sum_{j=1}^{p} (|\beta_j| + |\gamma_j|) \geq \frac{\langle H_0 \rangle_p - \langle H_1 \rangle_0 - \langle H_1 \rangle_p}{\|[H_1, H_0]\|}$$

for QAOA

Assumptions

- Periodic driving Hamiltonians $e^{iH(t+\theta)} = e^{iHt}$ (here $\theta = 2\pi$)
  - With $C(X) \mapsto \mathbb{N}_0$, we have periodicity for
  - Most mixers used in literature are periodic (e.g., transverse field, Grover-M, clique-M)
- Both QA and QAOA always start from a ground state of H_0

[Satoh et al., TQE.2020.3012068]

Lower bound on QAOA rounds

Derivations:

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2 pi period

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2 pi period

$$p(2\pi + 2\pi) \geq \frac{\langle H_0 \rangle_p - \langle H_1 \rangle_0 - \langle H_1 \rangle_p}{\| [H_1, H_0] \|}$$

$$p \geq \frac{\langle H_0 \rangle_p - \langle H_1 \rangle_0 - \langle H_1 \rangle_p}{4\pi \| [H_1, H_0] \|}$$

Lower bound on QAOA rounds

Theorem 2. *Given a classical objective function $C(x)$ for a maximization task, represented by the Hamiltonian H_C, encoded into the phase separator Hamiltonian $H_1 = \mathbb{1}C_{max} - H_C$ and a mixing Hamiltonian H_0, where all Hamiltonians are 2π periodic with zero ground state energies. Let C_{max} and C_{avg} denote the global maximum and the average of $C(x)$. If a QAOA protocol with p rounds driven by H_0 and H_1 that starts from the ground state $|\psi_0\rangle = |+\rangle^{\otimes n}$ of H_0 reaches a state with approximation ratio λ, then*

$$p \geq \frac{\langle H_0 \rangle_p + \lambda C_{max} - C_{avg}}{4\pi \|[H_C, H_0]\|}. \qquad (10)$$

We have a lower bound now what?

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If you're an expert in spectral graph theory or matrix analysis. We need your help!

What mixers do people use?

Unconstrained Problems

- Transverse field (go-to mixer)
- Grover-mixer

$$H_{TF} = \mathbb{1}\frac{n}{2} - \frac{1}{2}\sum_{j=1}^{n} X_j$$

$$\begin{aligned} H_{Grover} &= \mathbb{1} - |\psi_0\rangle\langle\psi_0| \\ &= \mathbb{1} - \frac{1}{N}\sum_{x}\sum_{y}|y\rangle\langle x| \end{aligned}$$

Constrained Problems (starts from a superposition of feasible states)

- Grover-mixer
- Clique-mixer (Hamming weight preserving)
- Ring-type mixer (Hamming weight preserving)

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$$p \geq \frac{1 - |\langle\psi_0|\psi_p\rangle|^2 + \lambda C_{max} - C_{avg}}{4\pi\sigma_C}$$

Examples (Grover-mixer)

Corollary 1. *If a QAOA protocol with p rounds finds an approximate solution with approximation ratio λ to Max-Cut of a graph with $|E|$ edges driven by the objective value phase separator H_1 and the Grover-mixer H_{Grover} that starts in the state $|\psi_0\rangle = |+\rangle^{\otimes n}$, then*

$$p \geq \frac{1 - |\langle \psi_0 | \psi_p \rangle|^2 + \lambda C_{max} - |E|/2}{2\pi\sqrt{|E|}}. \qquad (34)$$

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Consider a bipartite graph (easy)

$$p \geq \frac{1 - |\langle \psi_0 | \psi_p \rangle|^2 + \lambda |E| - |E|/2}{2\pi\sqrt{|E|}}$$
$$\geq \frac{(2\lambda - 1)}{4\pi}\sqrt{|E|}.$$

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state $|\psi_0\rangle = |+\rangle^{\otimes n}$ …

$$p \geq \frac{1 - |\langle\psi_0|\psi_p\rangle|^2 + \lambda C_{max} - |E|/2}{2\pi\sqrt{|E|}}.$$

Achieving constant approximation ratio requires a polynomial number of rounds for certain problems with Grover-mixer

Consider a bipartite graph (easy)

$$p \geq \frac{1 - |\langle\psi_0|\psi_p\rangle|^2 + \lambda|E| - |E|/2}{2\pi\sqrt{|E|}} \geq \frac{(2\lambda - 1)}{4\pi}\sqrt{|E|}.$$

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To get a non-trivial lower bound, we need (where m is the number of terms)

$$\|[H_C, H_{TF}]\| = o(m)$$

Is there any problem with this?

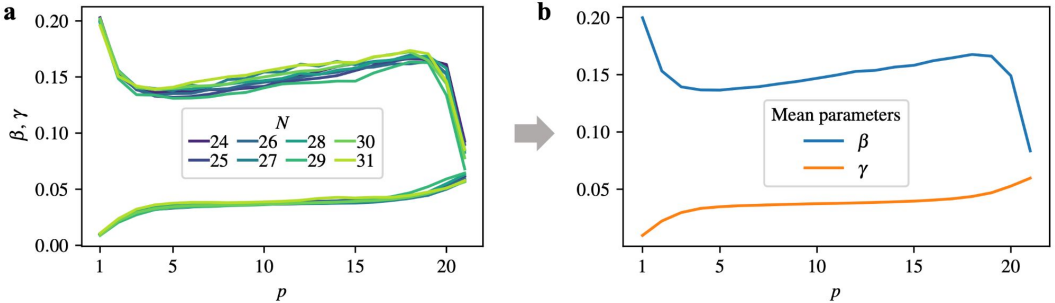
Are the lower bounds tight?

Grover-mixer:

- Tight for search (makes sense)
- Comb. opt? -> (numerical suggests exponential scaling but we get polynomial)
  - [Golden, Bärtschi, Eidenbenz, O'Malley, arXiv: 2202.00648] [Golden, Bärtschi, O'Malley, Eidenbenz, arXiv:2301.11292]

Transverse field mixer:

- Definitely not?



[Shaydulin et al., arXiv:2308.02342]

Warm-start QAOA

- Utilizing classical solver to get a starting point and continue with QAOA
  - [Tate, Farhadi, Herold, Mohler, Gupta, ACM TQC Vol 4, Issue 2 (9)] [Egger, Marecek, Woerner, Quantum 5, 479 (2021)]
- With modified mixers (the start states are ground states of the mixer)
- Initial states defined by (theta)
  - Small theta - starts as close to solution from solver as possible (high initial exp. val.)
  - Larger theta - starts from superposition from good set of candidates (lower initial exp. val.)
  - Theta = pi/2 - uniform superposition

Warm-start QAOA

- Utilizing classical solver to get a starting point and continue with QAOA
  - [Tate, Farhadi, Herold, Mohler, Gupta, ACM TQC Vol 4, Issue 2 (9)] [Egger, Marecek, Woerner, Quantum 5, 479 (2021)]
- With modified mixers (the start states are ground states of the mixer)
- Initial states defined by (theta)
  - Small theta - starts as close to solution from solver as possible (high initial exp. val.)
  - Larger theta - starts from superposition from good set of candidates (lower initial exp. val.)
  - Theta = pi/2 - uniform superposition

Numerical results show that “small” theta is bad.

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Lower bound evaluates to

$$p = \Omega(1/\theta)$$

"Theoretical Limitations of Warm-Started QAOA at Low Circuit Depth or Small Initialization Angle"
[Tate, Eidenbenz, (in preparation)]

Thank you for your attention!