DIFFERENT APPROACHES TOWARDS FORMALIZING & OPTIMIZING REVERSIBLE & QUANTUM CIRCUITS

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JFQI2023 WORKSHOP

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OUTLINE of the TALK

1 – INTRODUCTION: CIRCUITS

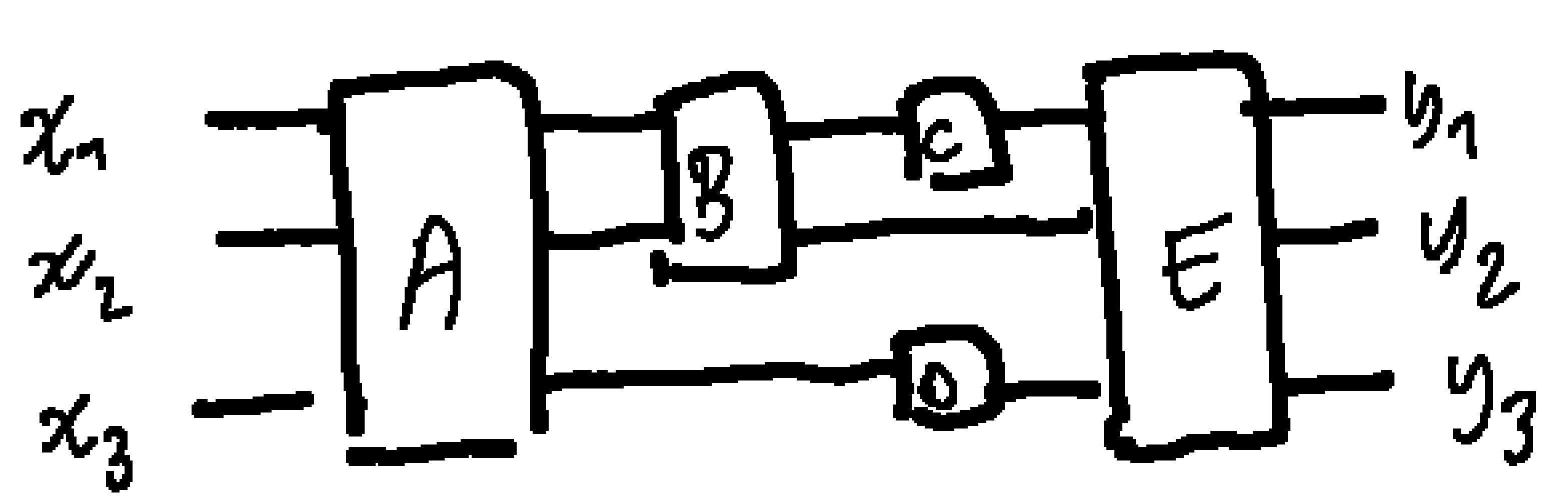
2 – FORMALIZING CIRCUITS AND REWRITINGS

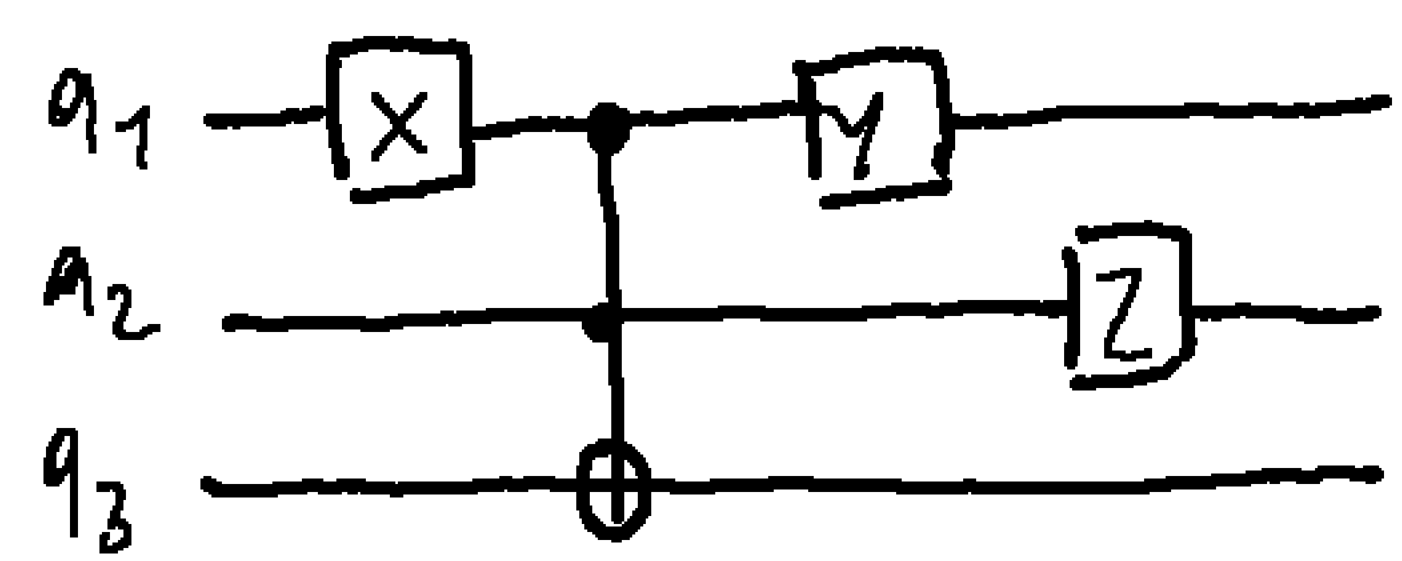
3 – BYPASSING LOGIC – APROXIMATE COMPILING

4 – REVERSIBILITY AND COMPOSITIONALITY

INTRODUCTION

- Reversible Boolean Circuits constitute a Computational primitive for Reversible Computing
- Quantum Computing in its most common formalization is circuit-based
- Circuits in a given gate base can REPRESENT boolean functions or unitary transformations.





SOME GENERAL QUESTIONS

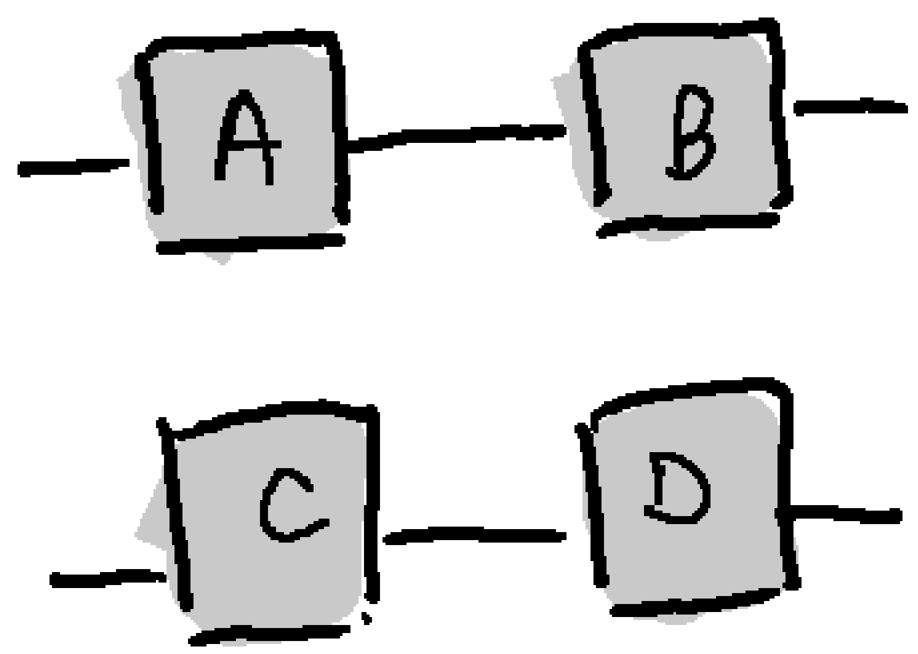
- **SYNTAX.** How do you "write" circuits? What are some structural rewriting rules circuits should be quotiented by?

- **SYNTHESIS.** What is the most efficient circuit for a given operation? Is there a general way to compile a circuit?

- **REPRESENTATION.** How can a boolean function be represented by a Reversible or Quantum circuit? How do you deal with ancillae & garbage in a compositional setting?

FORMALIZING the CIRCUITAL SYNTAX

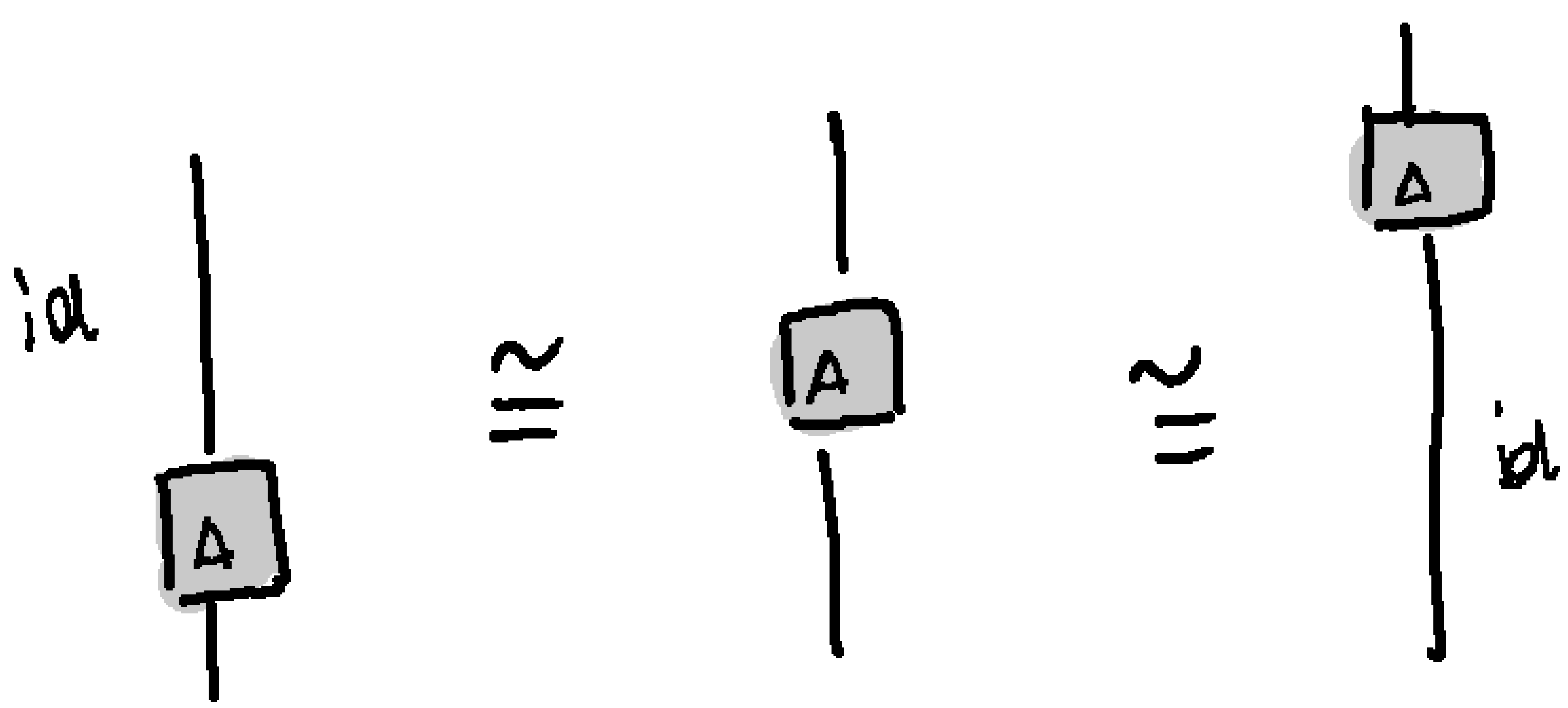
Diagrammatic (2-dimensional) syntax finds a "natural" formalization in the string diagrams for monoidal categories



sequential parallel

$$(A;B)\,\|\,(C;D) \cong (A\|C);(B\|D)$$

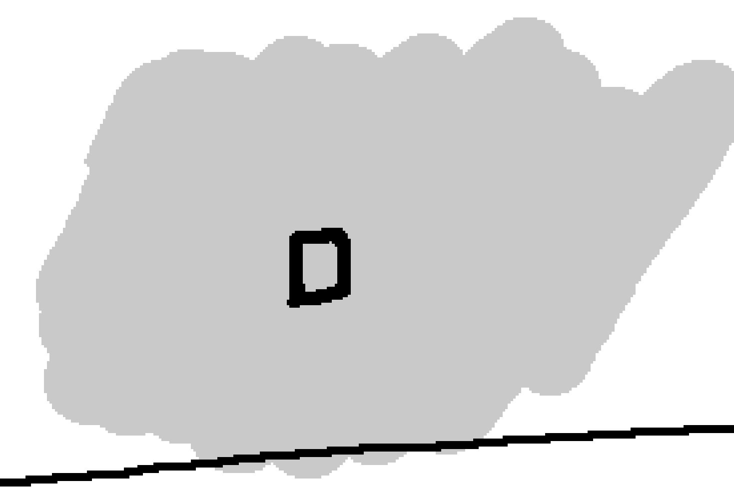
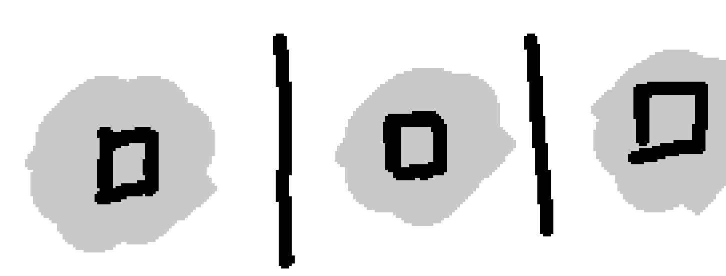
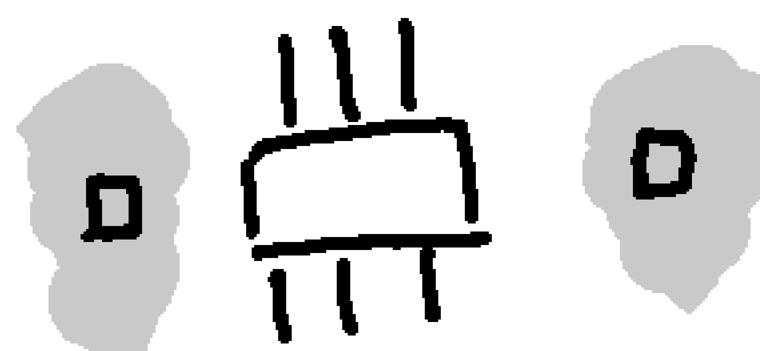
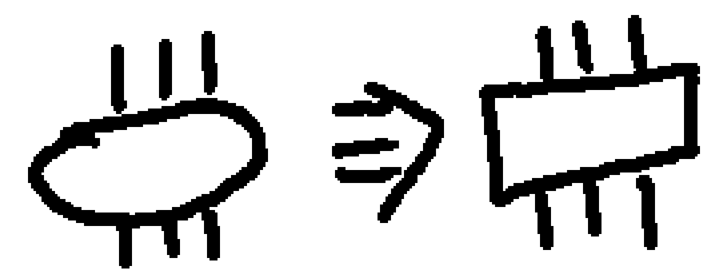
in what sense?

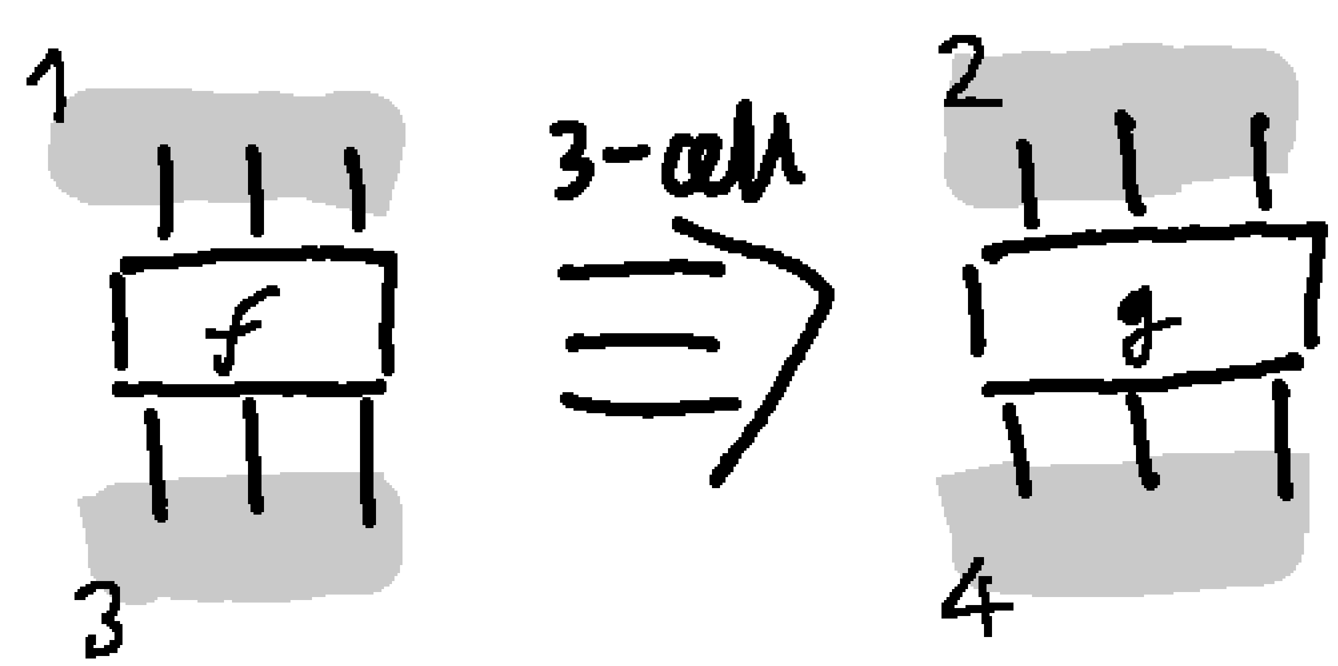


(Identity) wires can be deformed

A Burroni/Guiraud-inspired higher categorial formalization

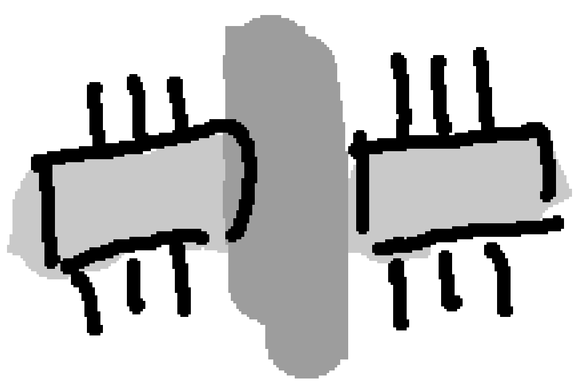
source & target maps

0-cells		blank space
1-cells		wires
2-cells		gates (boxes)
3-cells		rewriting rules

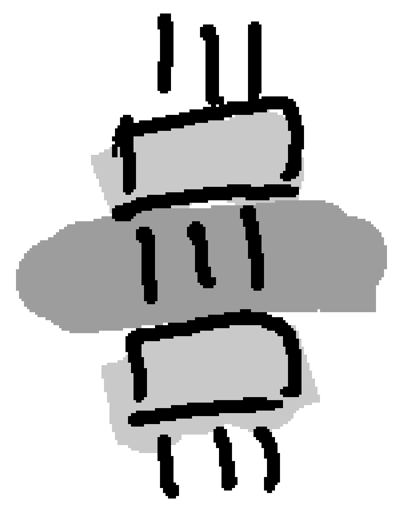
Globularity : 

1=2 , 3=4

The i-compositions attach two j-cells by their common "i-side"

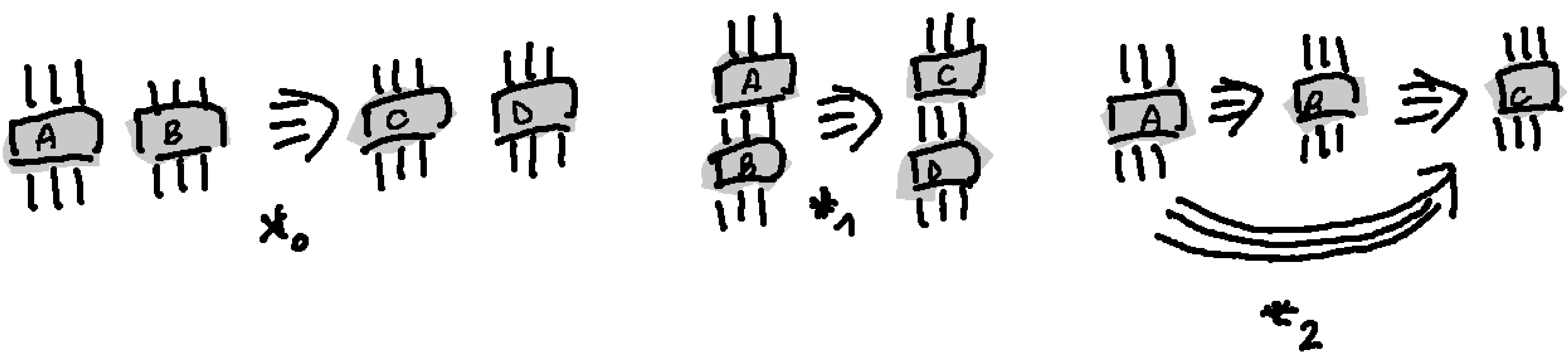


0-composition (parallel)



1-composition (sequential)

2-compositions of rewriting rules allow for simultaneous rewriting of parallel & sequential circuits and define multiple reduction steps.

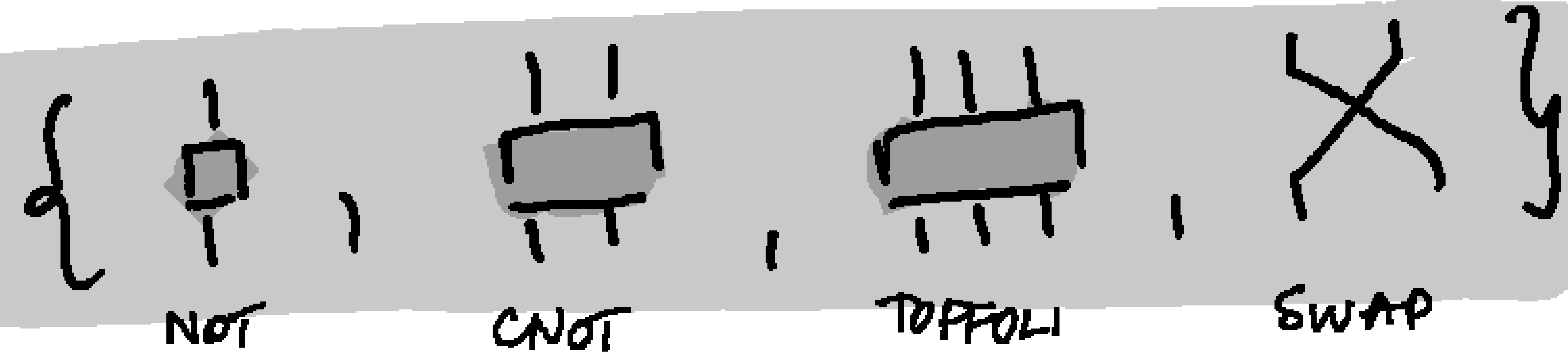


The Free 3-Category Toff

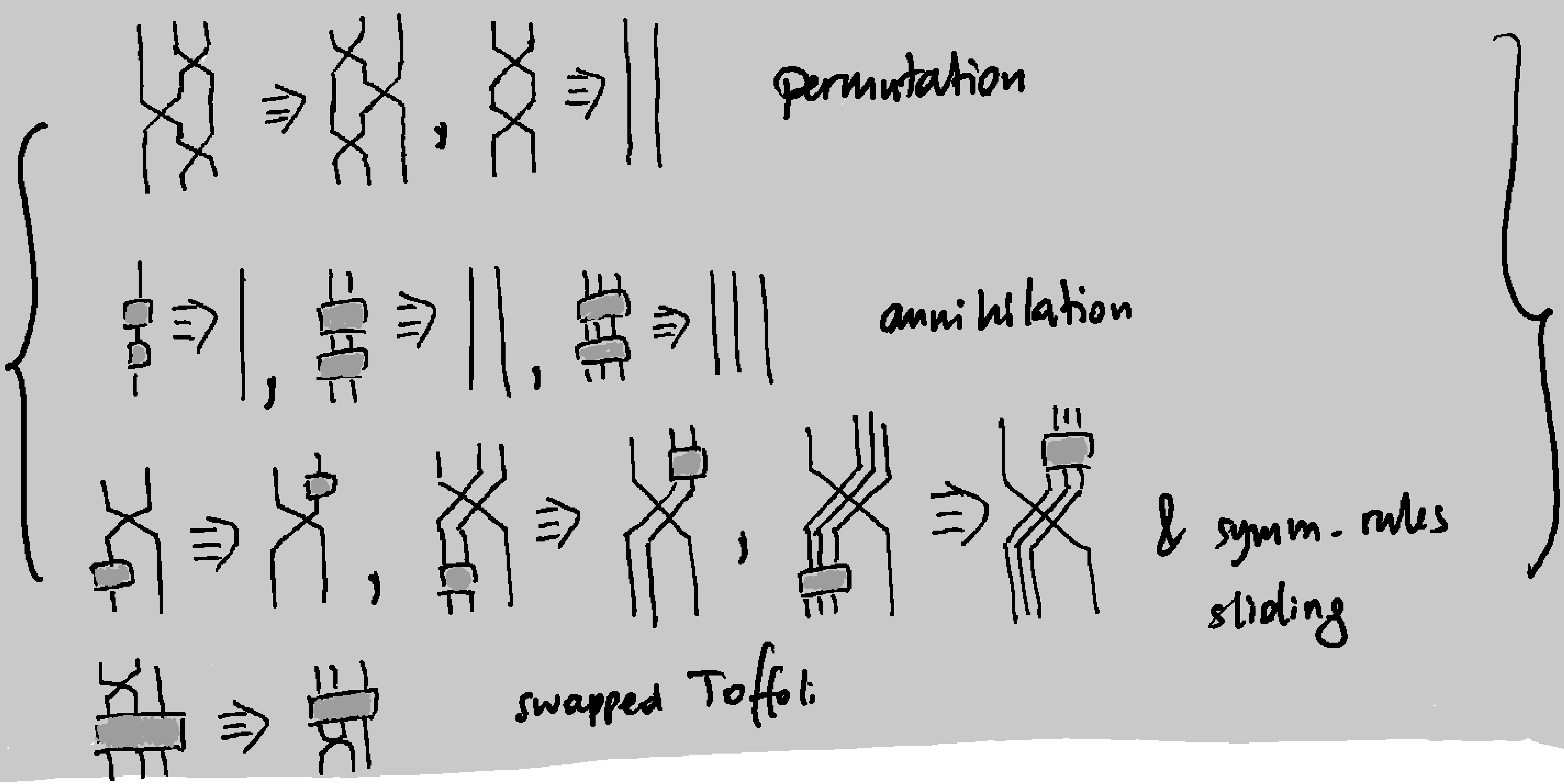
$G_0 = \{*\}$

$G_1 = \{ | \}$

$G_2 =$



$G_3 =$

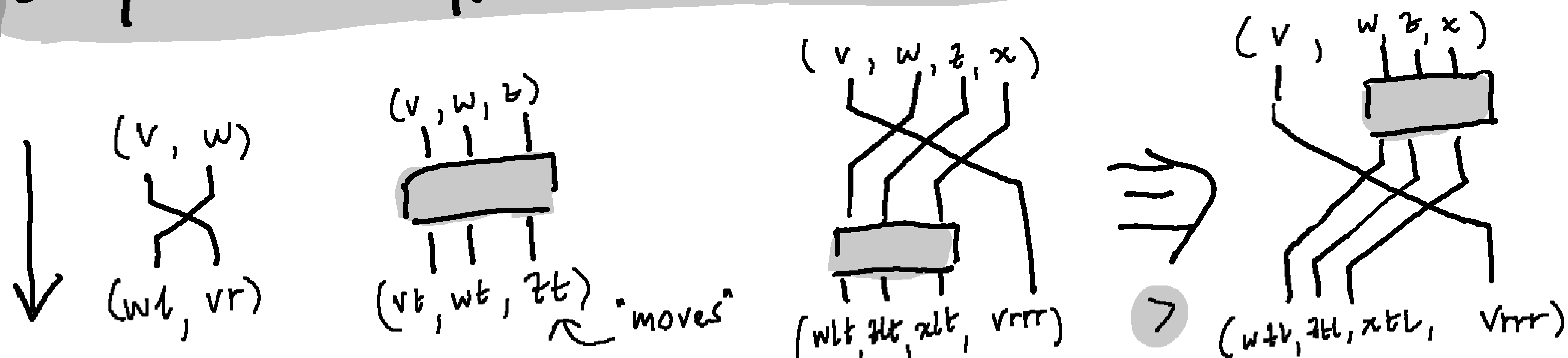


Theorem

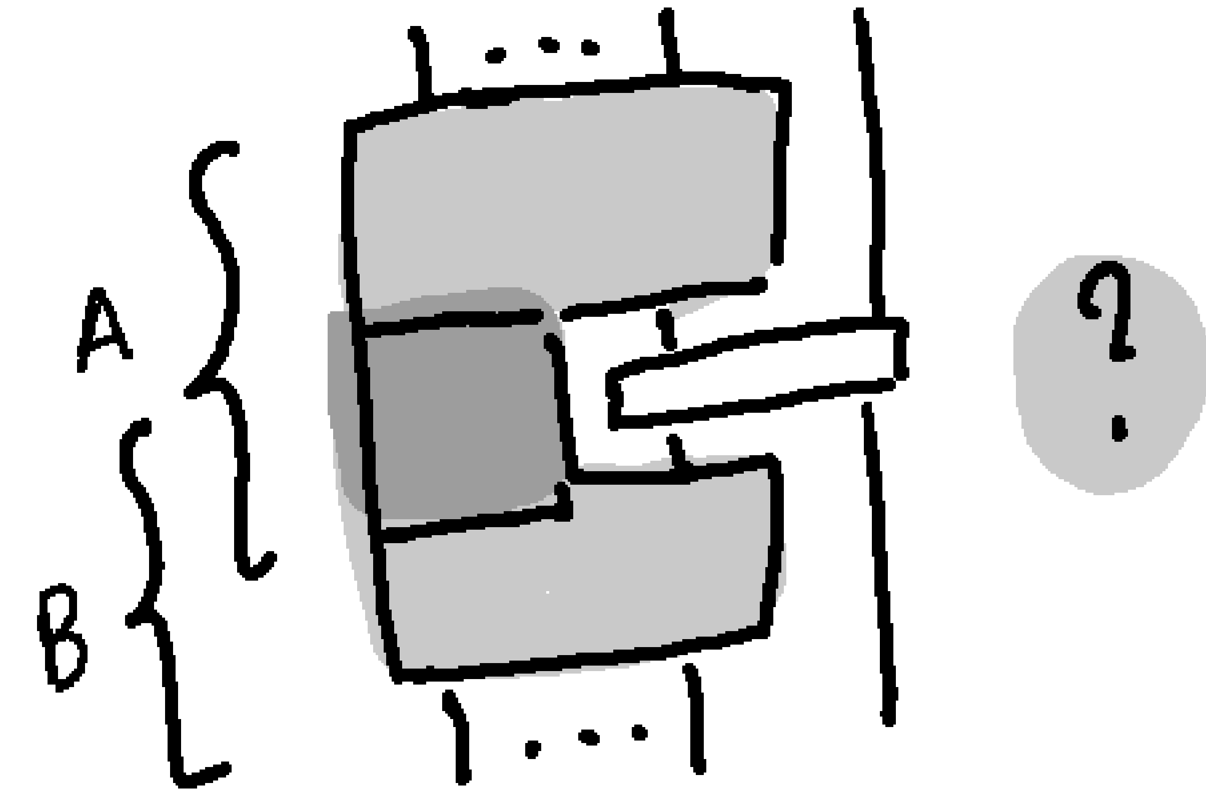
The rewriting system induced by G_3 on Toff terminates

Proof (sketch)

By defining a "measure" 3-category Move and a "measure" 3-functor Toff $\longrightarrow$ Move



Definitions of the structures involved guarantee that checking that the measure reduces on reduction rules in G_3 suffices.



non-trivial critical pairs

FURTHER STEPS

- The system is not convergent. In any case, convergence for "circuital" rewriting system is not trivial (Guiraud)
- Further semantic rules should be studied for specific Reversible/Quantum gate bases.
- A Proof Assistant formalization has been planned (Cubical Agda)

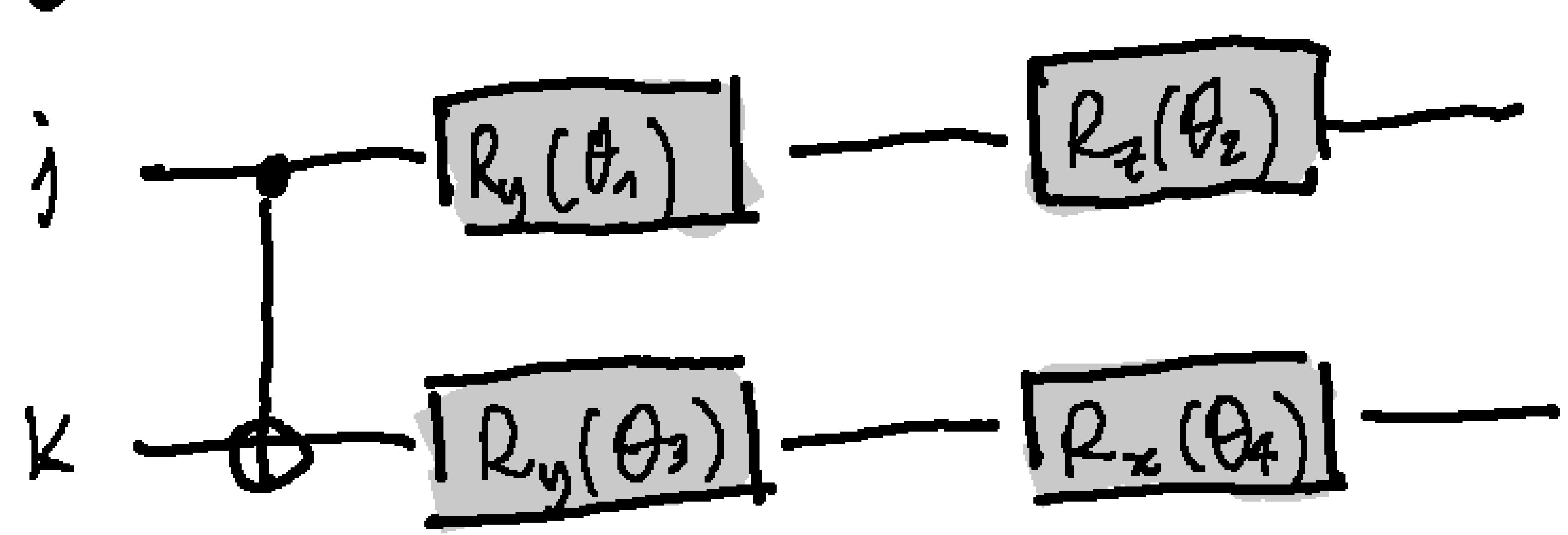
BYPASSING LOGIC – AQC

- The possibility for efficient approximate compilation of Quantum circuits is guaranteed by the Solovay-Kitaev theorem.

- AQC (Madden & Simonetto, 2021) uses Frobenius distance to find a unitary matrix V s.t.

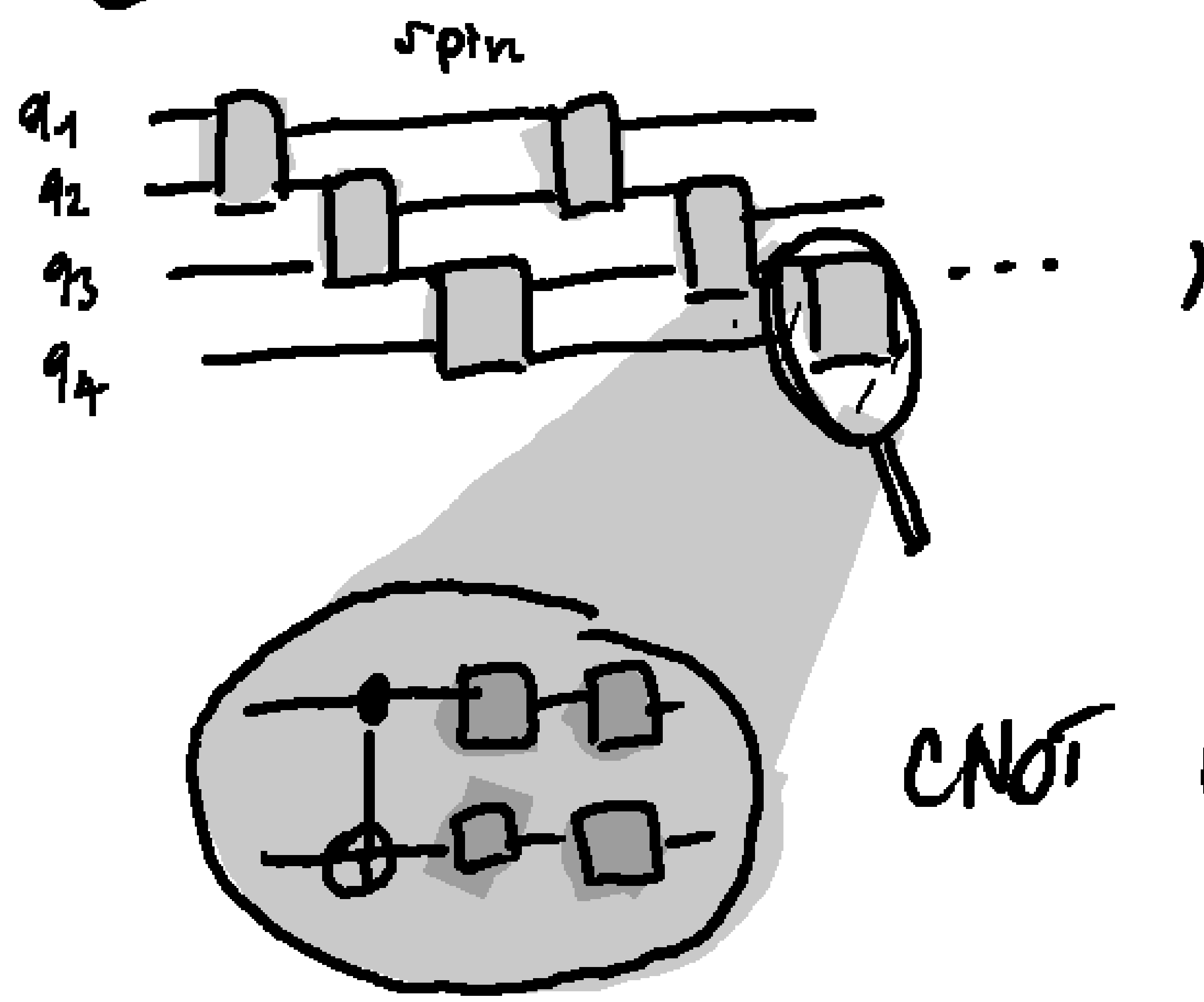
$$\boxed{d(V,U) = \frac{1}{2}|V-U|_F^2} \quad \text{is minimal}$$

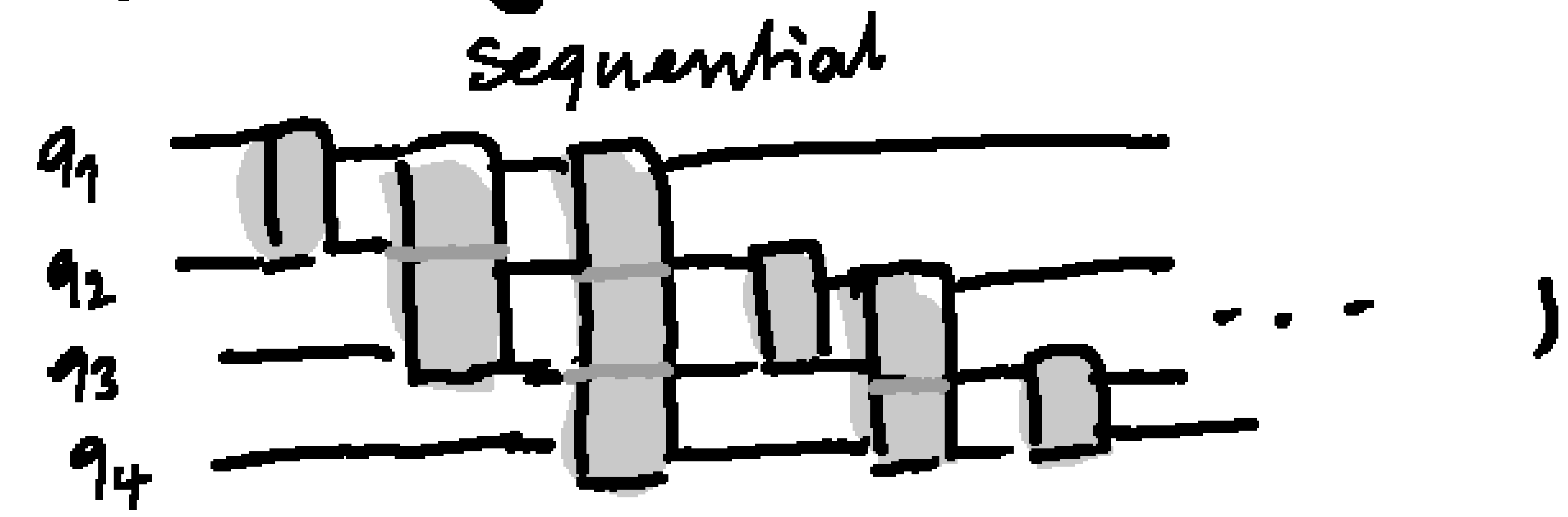
and compiles it on a network of CNOT units



CNOT & rotations base

CNOT structures (depending on HW constraints)





Cartan (Lie Algebra decomp.)

CNOT unit, 4 real parameters (angles) each

- The approximation problem becomes a gradient descent problem

$$\underbrace{f}_{\text{cost function}}{}_{\underbrace{ct}_{\text{fixed CNOT structure}}}(\underbrace{\underline{\theta}}_{\text{parameters}}) = \frac{1}{2}\left|\underbrace{V_{ct}(\underline{\theta})}_{\text{circuit compiled on structure } ct \text{ with parameters } \underline{\theta}} - \underbrace{U}_{\text{target matrix}}\right|_F^2$$

is **minimal**

- over param. $\underline{\theta}$
- on fixed struct. ct

The function $V_{ct}(\cdot)$ has some "nice" properties. Depending on the fixed length L, it might be surjective.

Ideally, convergence might be reached for

$$\boxed{L \geq \left\lceil \frac{1}{4}\left(4^n - 3n - 1\right) \right\rceil}$$

Theoretical Lower Bound for exact compilation

ISSUES

- It's an optimization problem (barren plateau)
- Convergence is still conjectured
- It's classically costly $(O(L^2 d^3 T))$

L = length
d = #qubits
T = #iterations

HOWEWER:

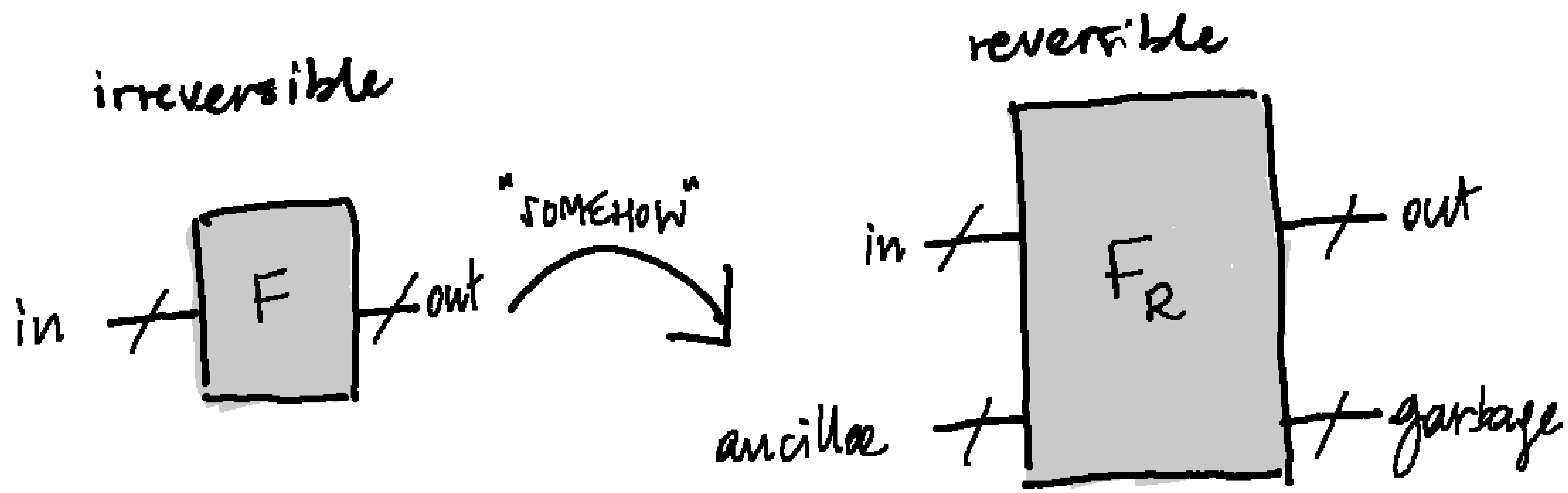
- It's already compiled in Qiskit
- Possibly "extreme" compressions in terms of length can be achieved

FURTHER STEPS

- Pre-generating compressed versions of recurrent operators (e.g. Grover operator)
- Changing metric and/or cost function?

REVERSIBILITY & COMPOSITIONALITY

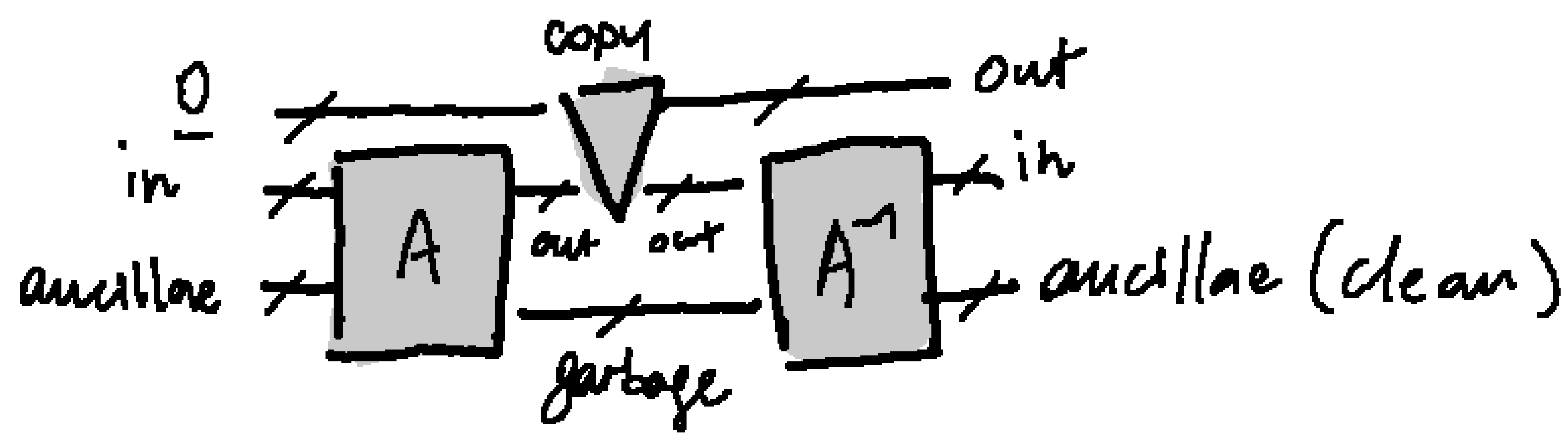
- Both in classical and Quantum computing theory, there exist some results that allow for REVERSIBLE REPRESENTATION of Boolean functions with ancillary data



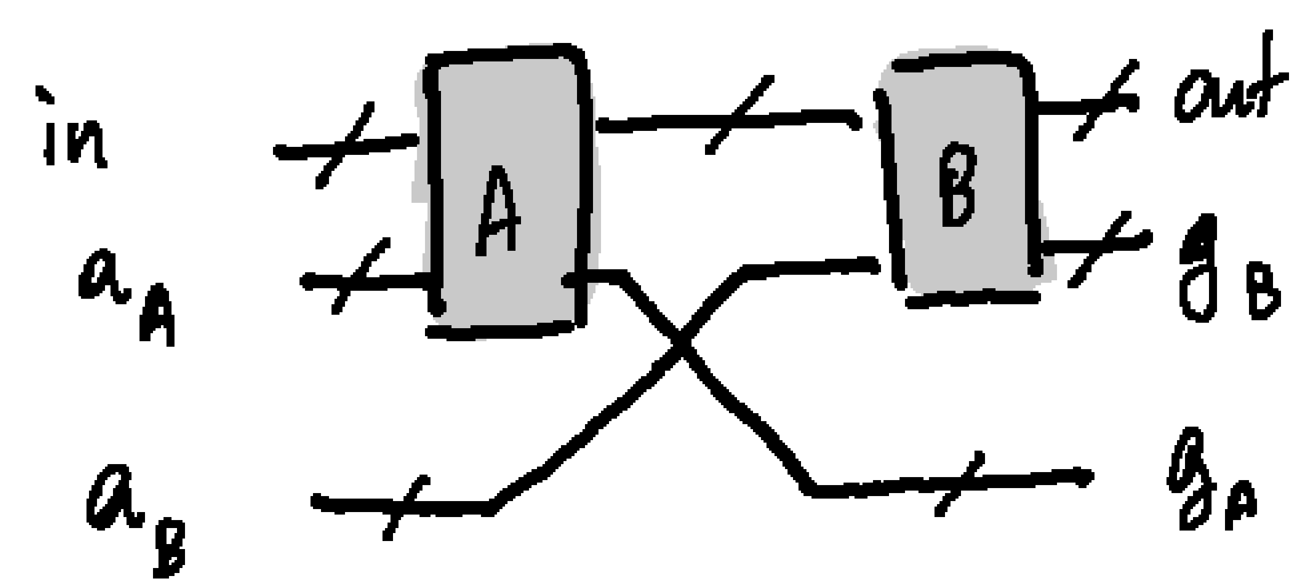
⚠ "SOMEHOW" = <u>INTENSIONALLY</u>

- There are several ways to synthetize a reversible (quantum) circuit that represents a given function.

- "Bennett's trick" allows for garbageless representation:

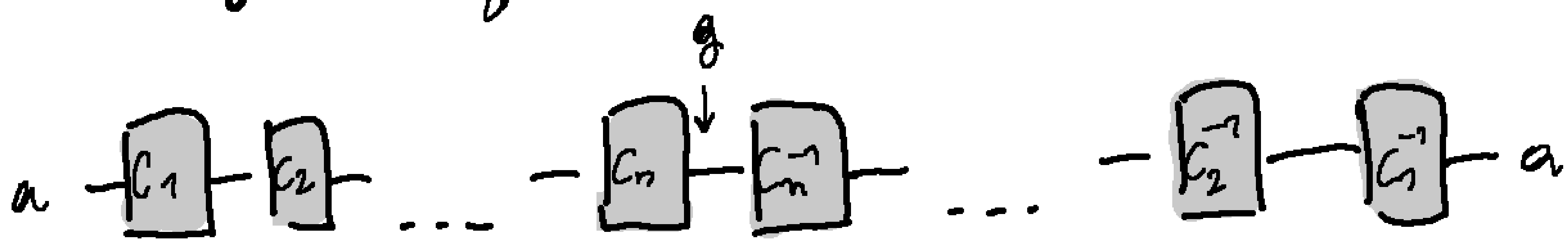


- What about translating a composition of functions? we would need to cross-combine the ancillae needed for both functions:



We can define ancillary data only "outside" of the boxes. (It's a contextual definition)

- When uncomputing garbage, can a more efficient compositional strategy be employed?
- For instance, should garbage generated by the last component go through all decomputation steps in order?



YEAH, PROBABLY.

- What if we use measurements?
- What if we get an approximation/compression method to work "good enough" for decomputation of garbage in a single step?

A common goal for these lines of research might be devising a "Versatile Quantum Functional Programming Language" with (possibly) the following features:

- (Higher) categorial interpretation
- "Flexible" irreversible → reversible translation
  → a "reversible monad"?
- "Compact" garbage-cleaning using approximate techniques

Thanks for the attention!

ご清聴ありがとうございます!

Merci pour l'attention!

Grazie per l'attenzione!

References

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- Guiraud, Yves and Malbos, Philippe*. Higher-dimensional categories with finite derivation type,* Theory and Applications of Categories [electronic only] 22 (2009), http://eudml.org/doc/222617.

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