Energy-Consumption Advantage of Quantum Computation

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14th December, 2023

Florian Meier, Hayata Yamasaki, arXiv:2305.11212

About me

Social implementation

Advance of IT society by quantum technology

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Useful quantum algorithm
Quantum machine learning with high speed/applicability

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Theoretical foundation = my works

Implementation of QC
Low-overhead/scalable fault-tolerant QC (FTQC)

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Efficient Q operations
Quantitative analysis of use of quantum resources

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Experimental foundation

Advance of quantum technology

- **Provable quantum advantage** in solving learning tasks arXiv:2305.11212 (**this talk**), arXiv:2312.03057
- Quantum machine learning (QML) using **exponential speedup without sparse or low-rank matrices** arXiv:2004.10756 (NeurIPS2020), arXiv:2106.09028 arXiv:2301.11936 (ICML2023)
- **Time-efficient constant-space-overhead FTQC** arXiv:2207.08826 (To appear in Nature Physics)
- Analysis of **GKP Code** arXiv:1910.08301 (PRA2020) arXiv:1911.11141 (PRR2020) arXiv:2006.05416
- Practical **testing** of entangled states arXiv:2201.11127 arXiv:2202.13131 (PRL2022)
- **Distributed** quantum information processing arXiv:2106.01372 (Quantum2022) etc.

Energy Consumption in Computation

<u>Energy consumption</u>: A part of **performance measures** of modern computers

Sustainability

https://blogs.nvidia.com/blog/what-is-green-computing/

<u>Foundation of quantum mechanics</u>

Quantum computation is substantially advantageous over classical in

- Time complexity
- Query complexity
- Communication complexity

But not in memory consumption (up to polynomial)

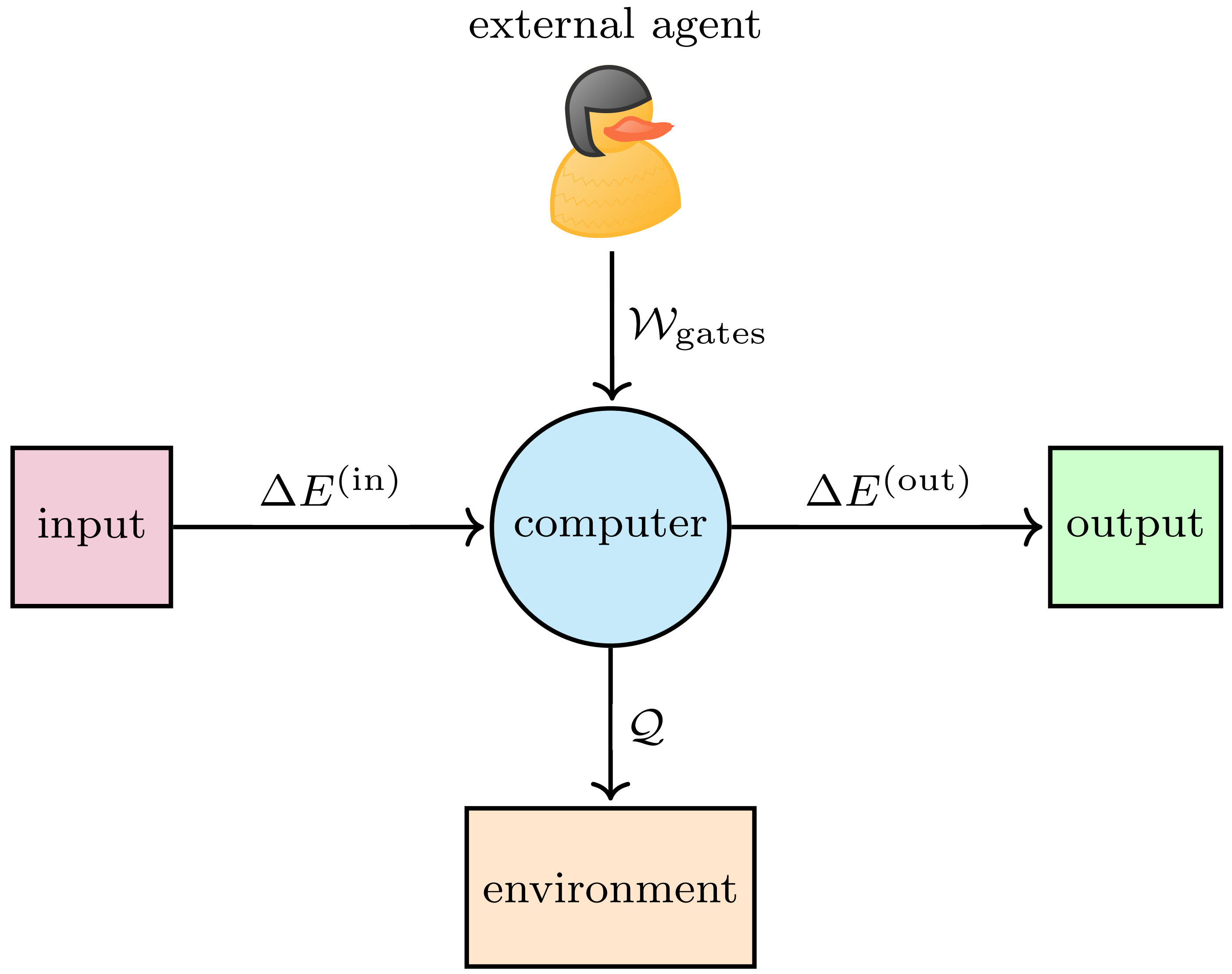
Watrous, computational complexity 12, 48 (2003)

Q: Can quantum computation offer a substantial **energy-consumption advantage** over classical?

Summary of Main Results

Existing works on quantum thermo: Cost of operation ⟺ **This work**: Cost of **overall** computation

1: Framework for thorough analysis



2: Fundamental bounds on energy consumption

	General bound	For Simon's problem
Quantum	$\mathcal{W} \leq \mathcal{W}^{(\mathrm{Q})}$	$\mathcal{W}^{(\mathrm{Q})} < O(\mathrm{poly}(N))$
Classical	$\mathcal{W} \geq \mathcal{W}^{(\mathrm{C})}$	$\mathcal{W}^{(\mathrm{C})} > \exp(\Omega(N))$

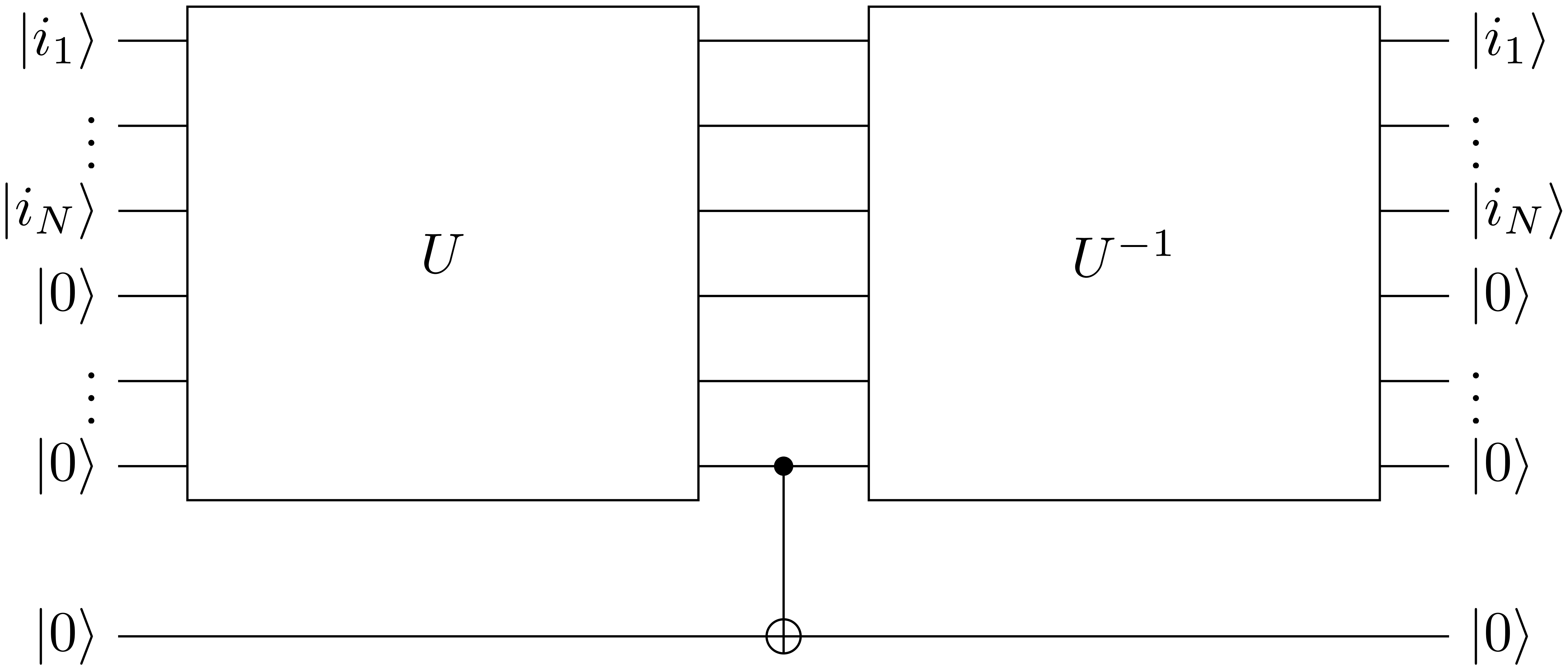
For N-bit input

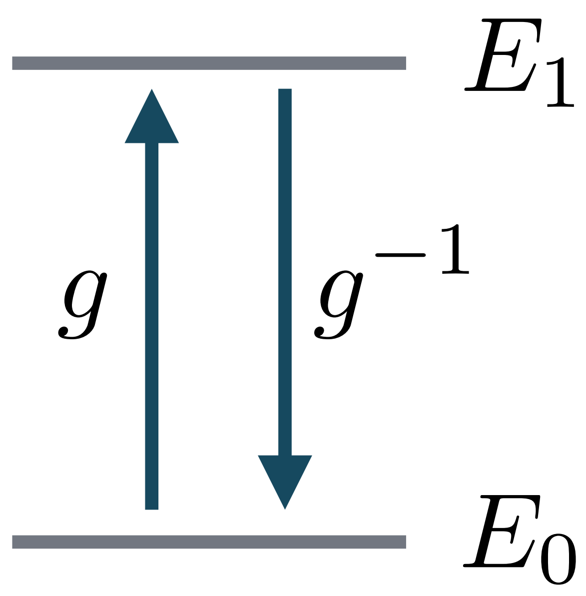
Developing fundamental framework and techniques for **rigorously studying energy consumption** of quantum and classical computation

Challenge in Bounding Energy Consumption

Idea: Computation as a thermodynamic **cycle**

Fact: Any (quantum & classical) computation can be implemented in a reversible way



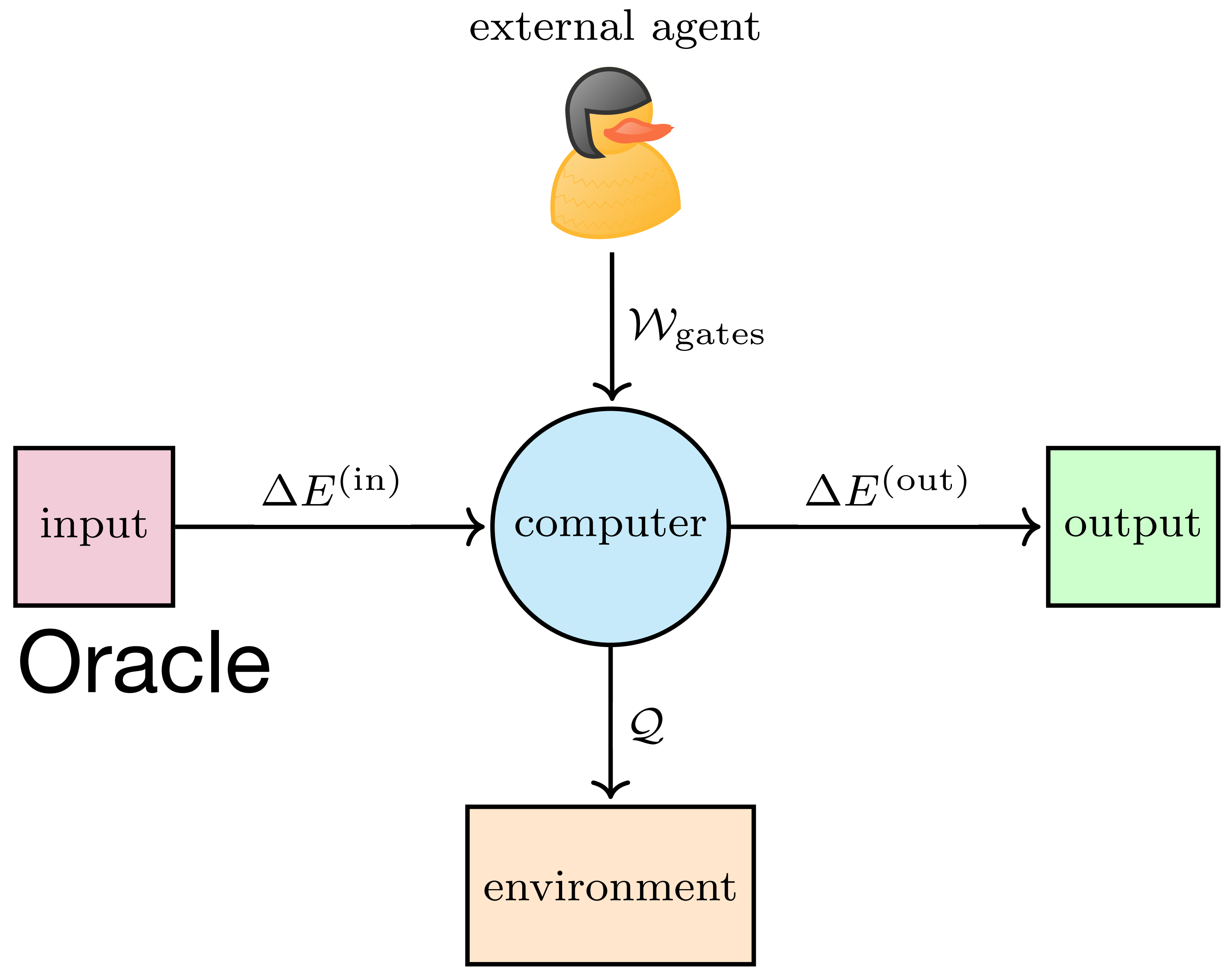


1. Invest energy to apply a gate g
2. **Returned** eventually with g^{-1}

Compute Output **Uncompute**: Invert all gates in the reverse order

Suppose the limit of ideal implementation (negligibly small friction or electrical resistance)

→ One can implement **any reversible computation with almost no energy consumption**



Observation: Nobody uses uncomputation in practice

→Theoretically impose **irreversibility** by introducing black-box **oracle**

Quantum access to an oracle $$O_f\left(\sum_x \alpha_x |x\rangle|0\rangle\right) = \sum_x \alpha_x |x\rangle|f(x)\rangle$$

Classical access to the same oracle $$|x\rangle|0\rangle \to |x\rangle|f(x)\rangle$$

Assumption: Oracle is irreversible → Uncomputation is prohibited

Computation with oracle = Learning a property of function f: Simon's problem

Given an N-bit function $f_N:\{0,1\}^N \to \{0,1\}^N$, decide if it is a one-to-one function

Bitwise XOR

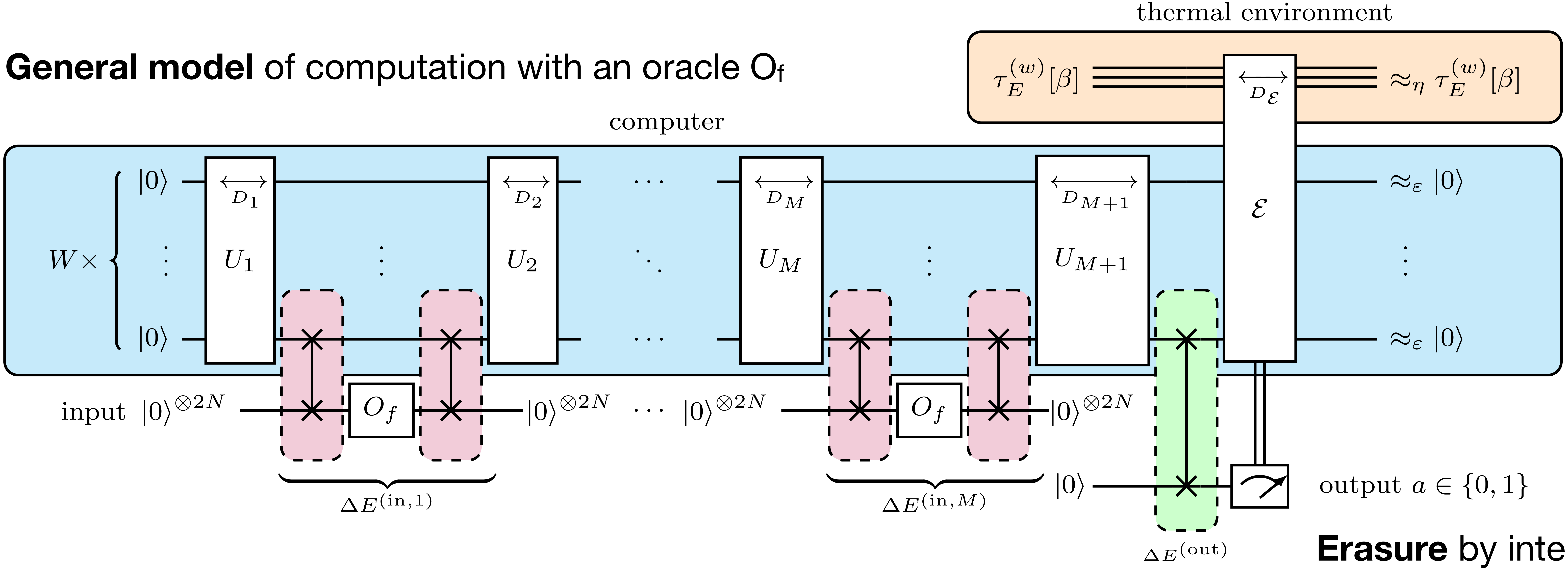
or a two-to-one function $f_N(x) = f_N(x \oplus s)$

Quantum: poly(N) queries ⟷ Classical: exp(N) queries

Quantum advantage in query complexity, but how about energy?

Formulation 2/3: Computational Model

General model of computation with an oracle O_f



Compute with input from O_f

Output

Erasure by interacting with a thermal bath

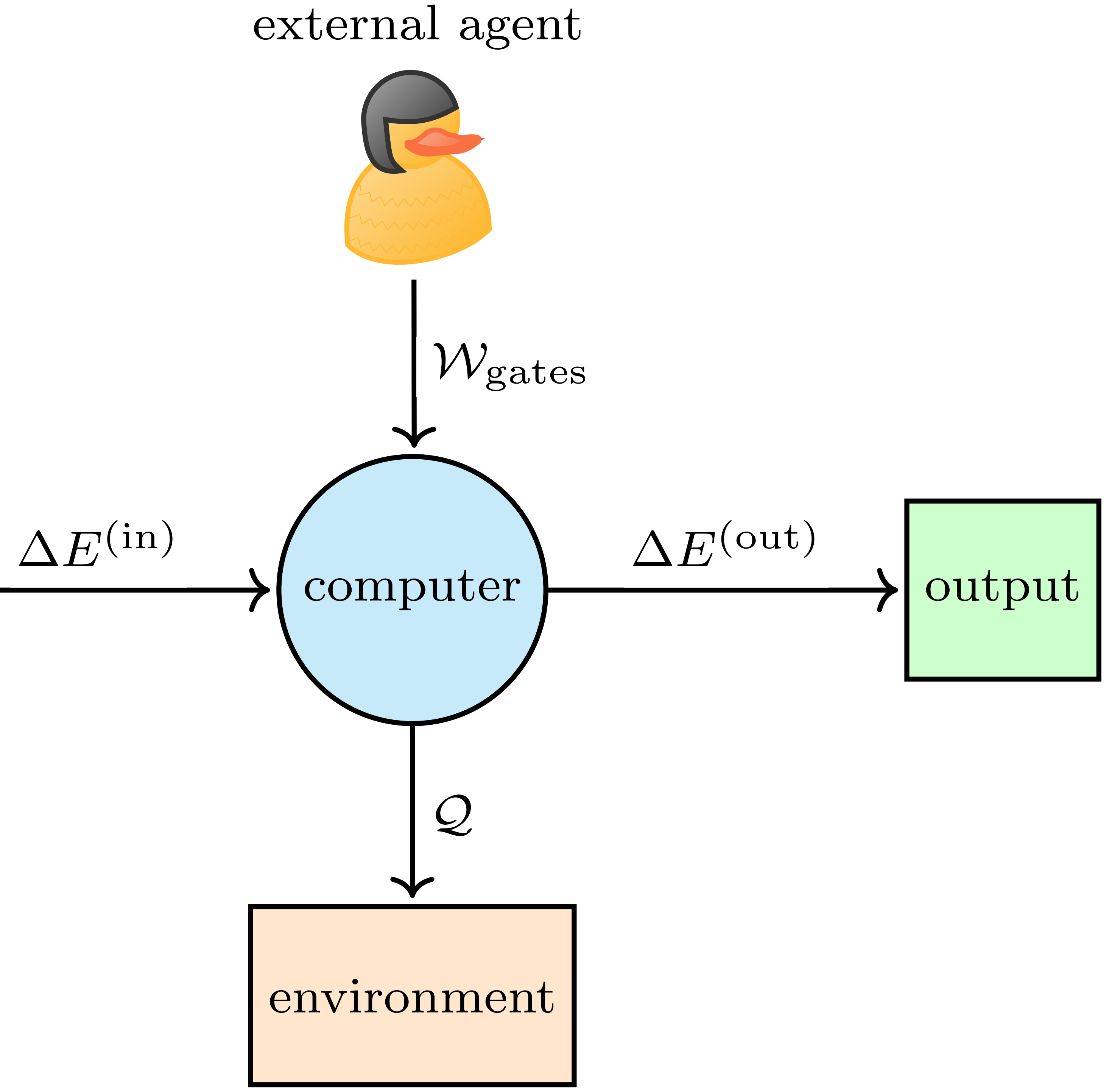
No measurement in computation

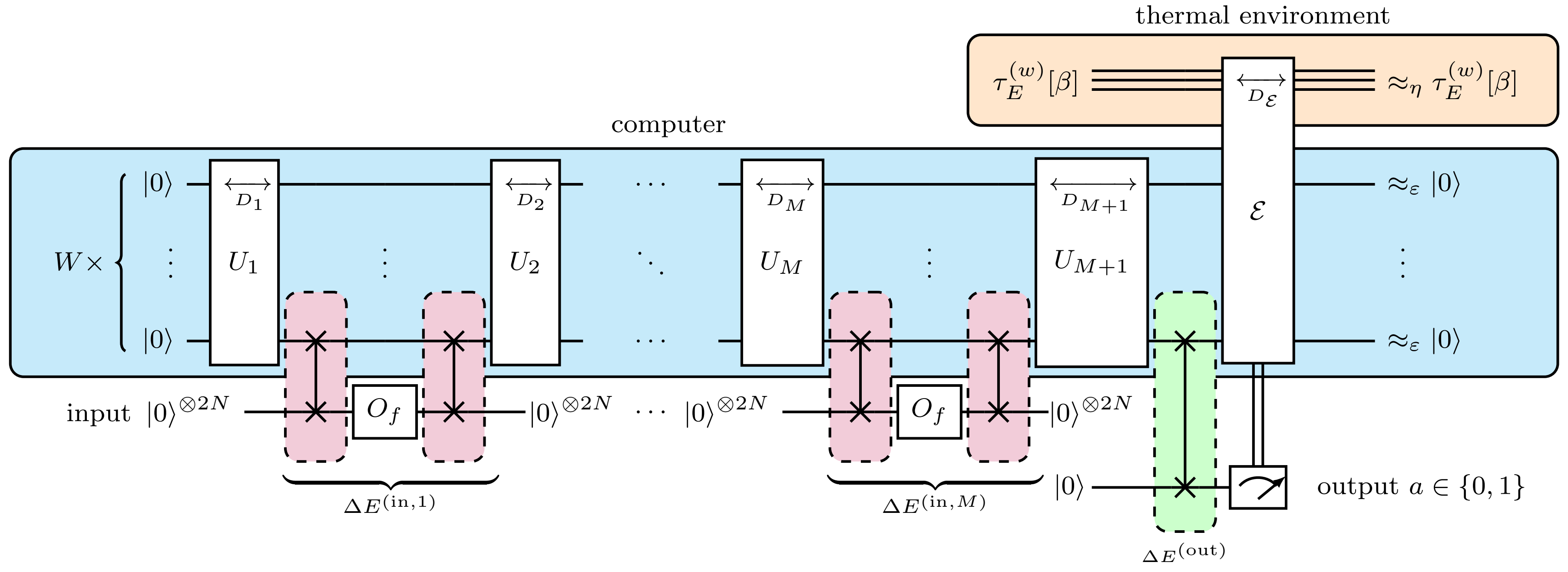
All information about function f input from the oracle remains in the computer

→ Erasure of the information requires the energy consumption = **Landaur's principle**

Formulation 3/3: Definition of Energy Consumption

Oracle





Def: Energy consumption

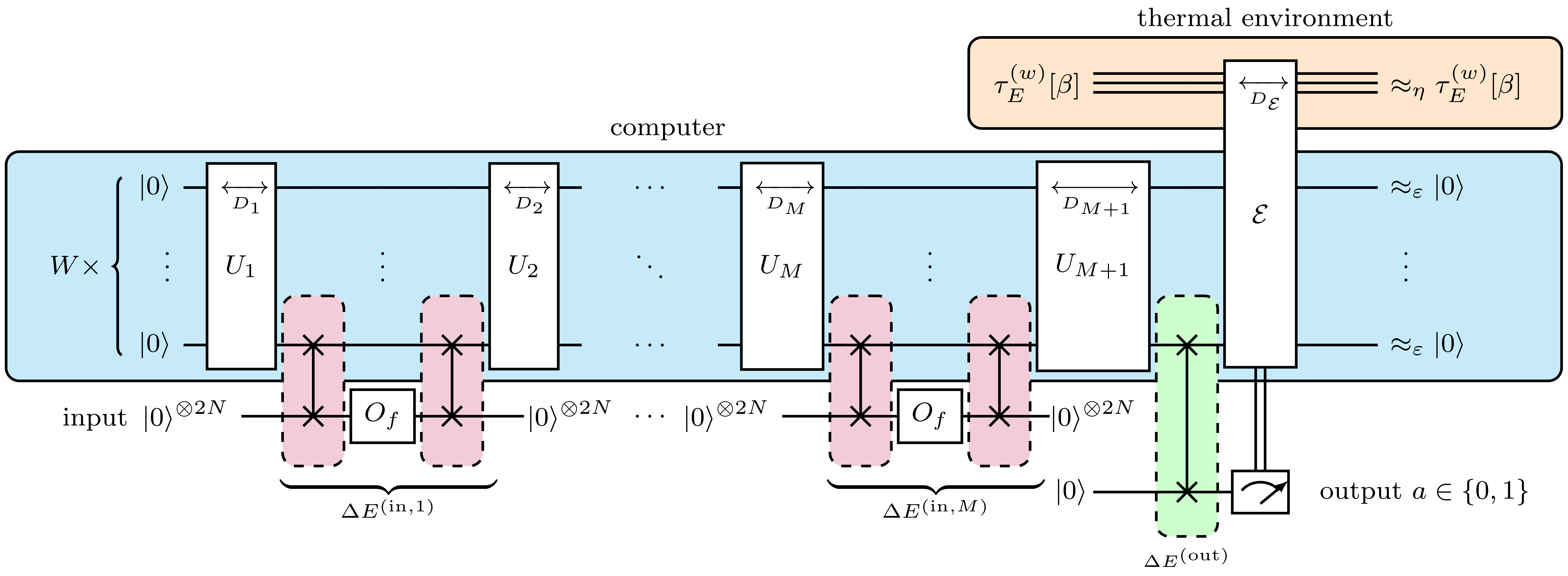
$$\mathcal{W} := \mathcal{W}_{\text{gate}} + \Delta E^{(\text{in})} - \Delta E^{(\text{out})}$$

Law of energy conservation

$$\mathcal{W} = \mathcal{Q}$$

Formulating the **thorough framework** to analyze the energy consumption in computation with oracle

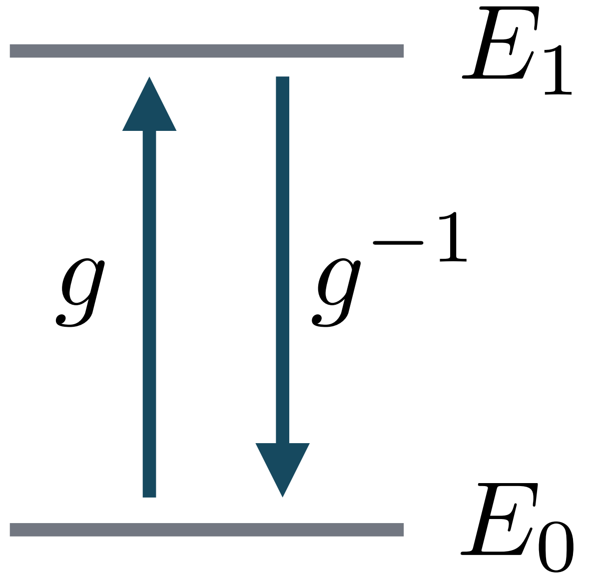
Result 1/4: Energy Consumption in Computation



Energy consumption

$$\mathcal{W} := \mathcal{W}_{\text{gate}} + \Delta E^{(\text{in})} - \Delta E^{(\text{out})}$$

$$\langle H \rangle_{\rho_{\text{out}}} - \langle H \rangle_{\rho_{\text{in}}}$$

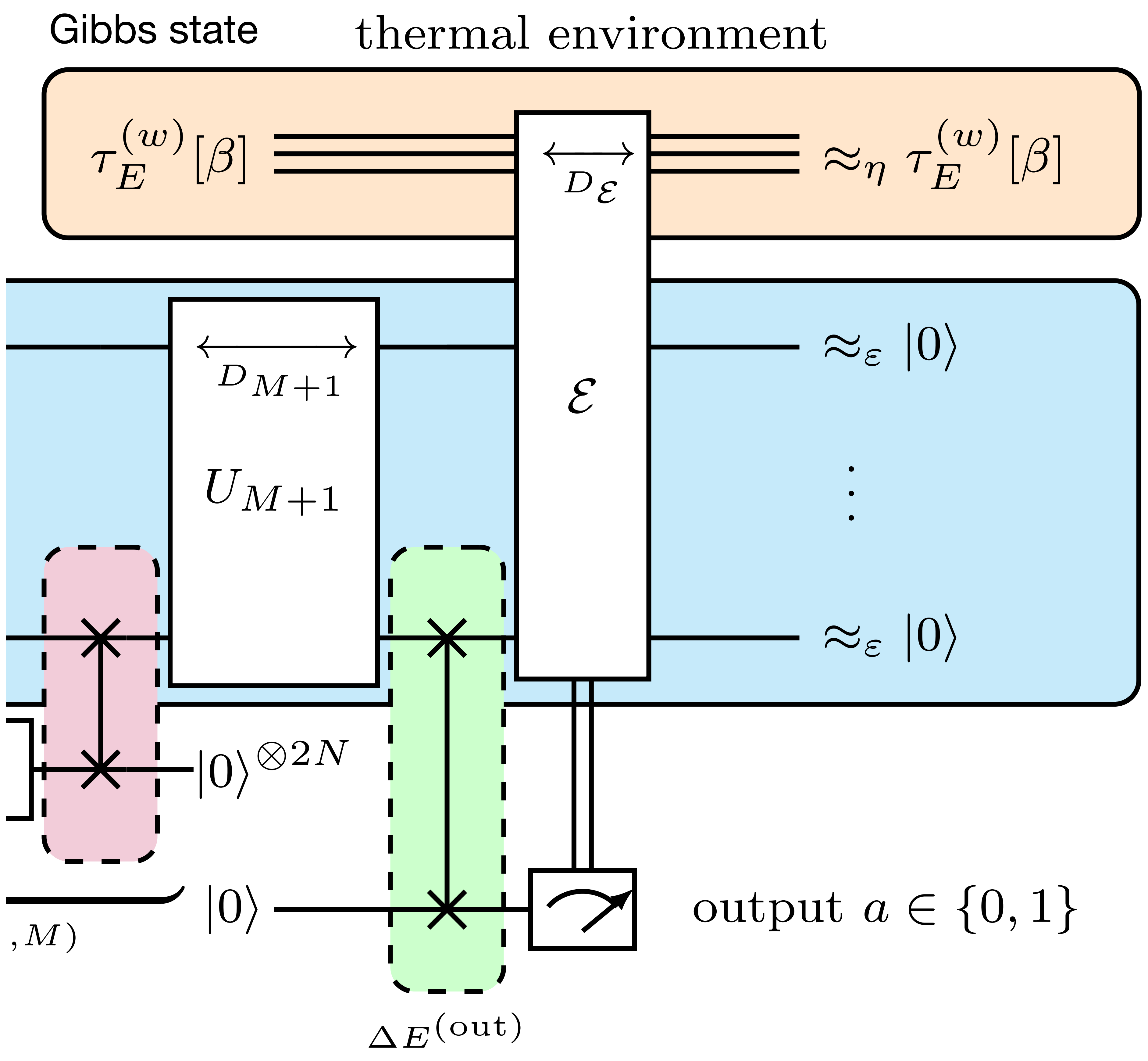


= **Energetic cost** + **Control cost** + **Initialization cost**

Energetic cost	Control cost	Initialization cost
Energy **change** of computer's state	Energy **loss** per gate	Dissipation in **initialization**
Can be positive or negative per gate	At most O(1) per gate	Nonzero due to Landauer
Sum up to zero for thermo cycle	Could be arbitrarily close to zero E.g., no friction or electrical resistence	We also need upper bound (Next slide)

Identifying all the factors that contribute to the energy consumption in our computatinal framework

Result 2/4: Finite-Step Landauer Erasure



Task: Given $\rho_S \in \mathcal{D}(\mathbb{C}^d), \epsilon \in (0, 1/2], \eta \in (0, 1]$, achieve

$$\rho_S \otimes \tau_E[\beta] \xrightarrow{U_{\mathcal{E}}} \rho'_S \otimes \tau_{E'}$$ Auxiliary Gibbs state for erasure

$$F(\rho'_S, |0\rangle) \geq 1 - \epsilon$$ High fidelity to the initial pure state

$$D(\tau_{E'} || \tau_E) \leq \eta$$ Negligible disturbance of thermal bath

→ **Finite precision** suffices for quantum error correction

Thm: We construct a T-step Landauer-erasure protocol

$$T = O\left(\frac{1}{\eta} \log\left(\frac{d^2}{\epsilon\eta}\right)\right)$$ Finitely bounded steps

$$\Delta S \leq \beta Q_E \leq \Delta S + \eta$$ Optimal up to finite tunable parameter

Landauer's lower bound on heat dissipation

Existing work: Infinite steps
Asymptotically optimal protocol

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Our work: Finite precision with finite steps
= Achieving bounded energy consumption

Reeb, Wolf, arXiv:1306.4352, Taranto et al., arXiv:2106.05151

Result 3/4: Energy-Consumption Bounds

Quantum upper bound $\mathcal{W} \leq \mathcal{W}^{(\mathrm{Q})}$

= Energetic cost + Control cost + Initialization cost

Energetic cost: Sum up to zero for thermo cycle

Control cost: O(1) per gate → O(#gates)

$$T = O\left(\frac{1}{\eta}\log\left(\frac{d^2}{\epsilon\eta}\right)\right)$$ for erasure

Initialization cost: ΔS+η: entropy to be erased

Our finite Landauer erasure

Classical lower bound $\mathcal{W} \geq \mathcal{W}^{(\mathrm{C})}$ = Initialization cost

ΔS: entropy to be erased

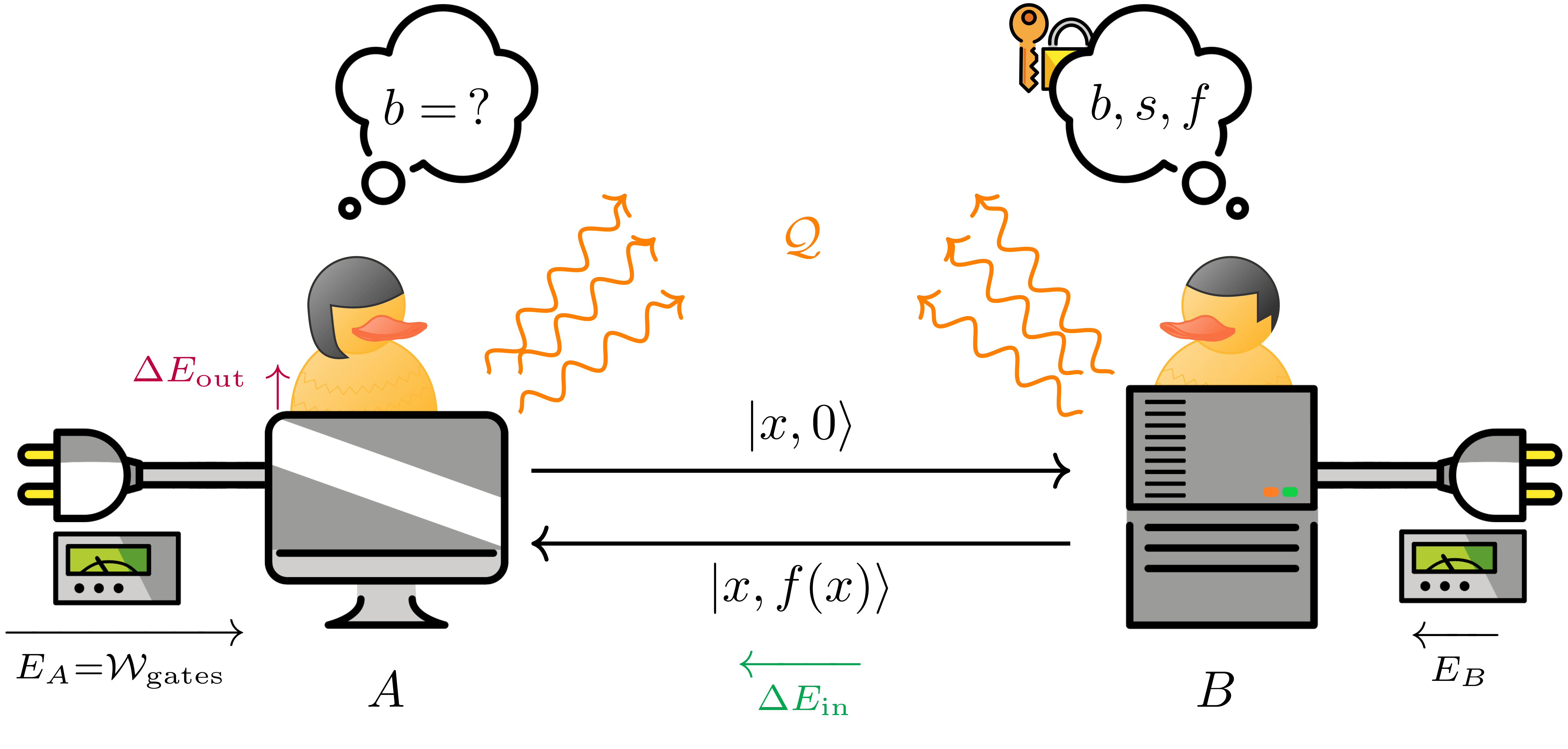
Landauer's principle

Implication: Exponential energy-consumption advantage in solving Simon's problem

Quantum: O(N) queries + O(N^3) gates for computation	⟺	Classical: Ω($2^{N/2}$) queries
Quantum: O(N^8) energy consumption Including overhead of quantum error correction	⟺	Classical: Ω($2^{N/2}$N) energy consumption

New quantum advantage

Result 4/4: Toward Experimental Demonstration



In charge of **computation**

Estimate b

In charge of **Simon's oracle**

b=0 → one-to-one N-bit random function

b=1 → two-to-one N-bit random function $f_N(x) = f_N(x \oplus s)$

To be implemented in poly(N) time with pseudo-random permutation

Classical lower bound to beat

- Extrapolation
- Fundamental lower bound

N	$\mathcal{W}^{(C)}$ (J)
50	2×10^{-13}
100	1×10^{-5}
150	7×10^{2}
200	3×10^{10}
250	1×10^{18}
300	5×10^{25}

Our analysis explicitly clarifies the constant factors of analytical lower bounds for Simon's problem

Opening way to demonstrate energy-consumption quantum advantage **over any possible classical**

Conclusion

- **Energy-consumption advantage of quantum computation** emerges in our framework of computation owing to the irreversibility of the oracle
- The energy consumption of quantum computation can be **upper bounded by the number of steps** (up to potentially large yet constant factors)
- The energy consumption of classical computation can be **lower bounded by Landauer's principle**, even in the limit of idealization with negligibly small friction or electrical resistance
- **Solid theoretical foundation for experimental demonstration** of this new advantage is now available

Thank you for your attention.

F.M. acknowledges funding by the European flagship on quantum technologies ('ASPECTS' consortium 101080167) and funding from the European Research Council (Consolidator grant 'Cocoquest' 101043705). H.Y. acknowledges JST PRESTO Grant Number JPMJPR201A, JPMJPR23FC, and MEXT Quantum Leap Flagship Program (MEXT QLEAP) JPMXS0118069605, JPMXS0120351339.