Calculus and Probability

Assignment 5

Note:

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
Consider the function $f(x) = (x^2 - 2)^3$.

a) What is the domain and range of $f(x)$?

b) What are the roots of $f(x)$

c) Find the local minima and maxima of $f(x)$

d) Which parts of the function are convex and concave? Does function f have points of inflection?

Exercise 2
For which values of c has the equation $\ln x = cx^2$ precisely one solution? (Hint: There is a value $0.1 < c < 0.2$ for which the curves just touch each other. What do these curves also have in common, besides the point of intersection?)

Exercise 3
Find the limits of the following functions.

a) $\lim_{x \to 1} \frac{x^a - ax + a - 1}{(x-1)^2}$;

b) $\lim_{x \to \infty} \frac{\ln(2015x)}{x^3}$;

Exercise 4
Find primitives of the following functions f. That is, find F such that $F'(x) = f(x)$.

a) $f(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$;

b) $f(x) = 2x + 5(1 - x^2)^{-\frac{1}{2}}$

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 5
Consider the function: $f(x) = e^{x^3 - x} - 1$.

a) What is the domain and range of $f(x)$? 1 pt

b) What are the roots of $f(x)$? 1 pt

c) Find the local minima and maxima of $f(x)$. 1 pt

d) Find the limits of the function for the boundaries of the domain of the function. 1 pt

Exercise 6

a) $f(x) = x^x$, find $f'(x)$; 2 pt

b) $f^{-1}(x) = \sin(x^2)$, find $f'(x)$; 2 pt

Exercise 7
Find the limits of the following functions.

a) $\lim_{a \to -3} \frac{\sin(a \cdot \pi)}{a^2 - 9}$; 2 pt

b) $\lim_{x \to -\infty} \frac{e^{3-x}}{7x^2}$. 2 pt

Exercise 8
Find primitives of the following functions f. That is, find F such that $F'(x) = f(x)$.

a) $f(x) = 2\sin(x)\cos(x)$; 2 pt

b) $f(x) = \frac{2}{1+4x^2}$; 2 pt

Exercise 9

a) Given three lines, $y = x + 2$, $y = -x + 6$ and $y = 2x - 3$, enclosing a triangle. Determine the *coordinates* of the three vertices and the *area* of the triangle. 2 pt

b) Suppose the triangular area calculated in (a) represents a piece of land and that we want to enlarge the area by placing a fence at *exactly* a meters from the boundary of the triangular area. Compute both the area A and length of the boundary B of this new area. 2 pt

c) **(Bonus 2 pt)** If we want the values A and B from (b) to be equal, i.e., $A = B$, does the distance a has to be smaller or larger than 1?

Your final grade is the sum of your scores divided by 2.0.