

# Advanced Lane Lines

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## Introduction

## Methods

The goals / steps of this project are the following:

- Compute the camera calibration matrix and distortion coefficients given a set of chessboard images.
- Apply a distortion correction to raw images.
- Use color transforms, gradients, etc., to create a thresholded binary image.
- Apply a perspective transform to rectify binary image ("birds-eye view").
- Detect lane pixels and fit to find the lane boundary.
- Determine the curvature of the lane and vehicle position with respect to center.
- Warp the detected lane boundaries back onto the original image.
- Output visual display of the lane boundaries and numerical estimation of lane curvature and vehicle position.

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## Setup

The initial setup includes creating the Python environment with the packages that the project needs and uses.

**matplotlib** plotting and image processing tools

**NumPy** foundational scientific computing library

**MoviePy** video processing tools

**OpenCV** computer vision library

The GitHub repository for this project contains an environment.yml file that can be used to create and activate a Conda environment with these commands.

```
conda env create --file environment.yml --name CarND-Advanced-Lane-Lines
source activate CarND-Advanced-Lane-Lines
```

Once activated this environment is used to launch Python in whatever way one likes, such as a Python shell, a IPython shell, or a jupyter notebook. Having done that, the usual first step is to import the packages that are used.

```
from collections import deque
from itertools import groupby, islice, zip_longest, cycle, filterfalse
from moviepy.editor import VideoFileClip
import cProfile
import cv2
import glob
import matplotlib
import matplotlib.image as mpimg
import matplotlib.pyplot as plt
import numpy as np
import pdb
```

Besides the third-party packages listed above, the project also makes use of these standard-library library packages.

**deque** ring buffers for moving averages

**itertools** handy for Python generators

**cProfile** run-time optimization

**pdb** Python debugger

## Processing Pipeline

In order to detect lane lines in a video of a car driving on a road, and then generate an annotated video with the detected lane overlaid, we need an image processor that performs these two tasks—detection and annotation—on every frame of the video. That image processor encompasses a "processing pipeline."

The pipeline depends on these preliminary tasks.

1. Camera Calibration
2. Perspective Measurement

Then, the pipeline applies these stages.

1. Distortion Correction
2. Gradient and Color Thresholds
3. Perspective Transform
4. Lane-line Detection

Let us examine these preliminary tasks and pipeline stages in greater detail.

### Camera Calibration

Camera calibration measures the distortion inherent in cameras that utilize lenses so that the images taken with the camera can be corrected by removing the distortion. A standard way to do this is to measure the distortion the camera imposes on standard images of known geometry. Checkerboard patterns are useful for this tasks because of their high contrast, known geometry, and regular pattern.

The `measure_distortion` function takes a Python sequence of checkerboard image filenames taken at different distances, center-offsets, and orientations and applies the OpenCV functions `findChessboardCorners` and `drawChessboardCorners` to identify corners in the images and highlight the corners. Then, the `calibrateCamera` function measures the distortion. This function returns the distortion parameters and matrix, along with a sequence of tuples with the original filenames and the annotated images.

```

def measure_distortion(calibration_files):
    files = calibration_files
    objp = np.zeros((9*6,3), np.float32)
    objp[:, :2] = np.mgrid[0:9, 0:6].T.reshape(-1, 2)
    stage1 = map(lambda x: (x,), cycle(files))
    stage2 = map(lambda x: x + (mpimg.imread(x[0]),), stage1)
    stage3 = map(lambda x: x + (cv2.findChessboardCorners(cv2.cvtColor(x[1], cv2.COLOR_BGR2GRAY), (9, 6)),), stage2)
    stage4 = map(lambda x: x + (cv2.drawChessboardCorners(np.copy(x[1]), (9, 6), *(x[2])),), stage3)
    filenames, images, corners, annotated_images = zip(*filter(lambda x: x[2][0], stage4))
    imgpoints = zip(*corners)
    objpoints = [objp for i in range(len(imgpoints))]
    ret, mtx, dist, rvecs, tvecs = cv2.calibrateCamera(objpoints, imgpoints, list(islice(filenames, len(imgpoints))), None)
    return mtx, dist, zip(filenames, annotated_images)

```

This function is used in subsequent distortion corrections.

### Distortion Correction

The `get_undistorter` function takes a sequence of calibration checkerboard image filenames, applies the `measure_distortion` function, and returns a new function. The new function uses the OpenCV `undistort` function to remove distortion from images taken with the same camera.

```

def get_undistorter(calibration_files):
    mtx, dist, annotated_images = measure_distortion(calibration_files)
    return lambda x: cv2.undistort(x, mtx, dist, None, mtx), annotated_images

```

In the example shown below, we get an "image undistorter" function for a set of calibration images.

```

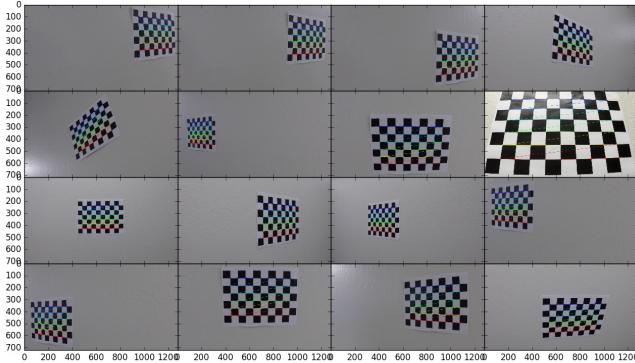
undistort, annotated_images = get_undistorter(glob.glob("camera_cal/*.jpg"))
fig = plt.figure()
grid = ImageGrid(fig, 111, nrows_ncols=(4, 4), axes_pad=0.0)

for p in zip(annotated_images, grid):
    p[1].imshow(p[0][1])

fig.savefig("output_images/annotated_calibration_images.jpg")

```

The annotated calibration images are shown in the figure below.



We show the effects of applying the image undistorter to a sequence of 6 road images taken with this same camera. These 6 images are a test sequence that will reappear many times through the remainder of this discussion as other image processing steps are taken up.

The `visualize` function helps us view a gallery of test images in "ganged up" layout, and this is helpful as we develop the processing pipeline stages.

```
def visualize(filename, a):
    fig, axes = plt.subplots(2,3,figsize=(24,12),subplot_kw={'xticks':[],'yticks':[]})
    fig.subplots_adjust(hspace=0.03, wspace=0.05)
    for p in zip(sum(axes.tolist(),[]), a):
        p[0].imshow(p[1],cmap='gray')
    plt.tight_layout()
    fig.savefig(filename)
    plt.close()
```

The 6 test images that we use repeatedly are shown in the figure below, without any image processing at all.

```
visualize("output_images/test_images.jpg",
          (mpimg.imread(f) for f in cycle(glob.glob("test_images/test*.jpg"))))
```



These test images are shown again, only this time the image undistorter that we acquired above now is used to remove distortion introduced by the camera. The effect is subtle and difficult to notice, but close inspection shows that at least a small amount of radial distortion is removed by this process.

```
visualize("output_images/undistorted_test_images.jpg",
          (undistort(mpimg.imread(f)) for f in cycle(glob.glob("test_images/test*.jpg"))))
```



Next, we move on to perspective measurement.

### Perspective Measurement

Perspective measurement applies to two-dimensional images taken of three-dimensional scenes wherein objects of interest—typically planar objects like roads—are oriented such that their normal vector is not parallel with the camera’s line of site. Another way to put it is that the planar object is

not parallel with the image plane. While there undoubtedly are more sophisticated, perhaps automated or semi-automated ways of doing this, a tried-and-true method is to identify a non-rectilinear region in the image that corresponds to the planar object of interest (the road) and then map those to a corresponding rectilinear region on the image plane.

The `measure_warp` function helps measure perspective. It takes an image as a NumPy array and displays the image to the user in an interactive window. The user only has to click four corners in sequence for the source region and then close the interactive window. The destination region on the image plane for now is hard-code to a bounding box between the top and bottom of the image and 300 pixels from the left edge and 300 pixels from the right edge. These values were obtained through experimentation, and while they are not as sophisticated as giving the user interactive control, they do have the virtue of being perfectly rectilinear. This is something that is difficult to achieve manually. Setting the src region coordinates, along with drawing guidelines to aid the eye, is accomplished in an event handler function for mouse-click events. The function returns the transformation matrix  $M$  and the inverse transformation matrix  $M_{inv}$ .

```
def measure_warp(img):
    top = 0
    bottom = img.shape[0]
    def handler(e):
        if len(src)<4:
            plt.axhline(int(e.ydata), linewidth=2, color='r')
            plt.axvline(int(e.xdata), linewidth=2, color='r')
            src.append((int(e.xdata),int(e.ydata)))
        if len(src)==4:
            dst.extend([(300,bottom),(300,top),(980,top),(980,bottom)])
            was_interactive = matplotlib.is_interactive()
            if not matplotlib.is_interactive():
                plt.ioff()
            fig = plt.figure()
            plt.imshow(img)
            global src
            global dst
            src = []
            dst = []
            cid1 = fig.canvas.mpl_connect('button_press_event', handler)
            cid2 = fig.canvas.mpl_connect('close_event', lambda e: e.canvas.stop_event_loop())
```

```

fig.canvas.start_event_loop(timeout=-1)
M = cv2.getPerspectiveTransform(np.asarray(src, np.float32), np.asarray(dst, np.
Minv = cv2.getPerspectiveTransform(np.asarray(dst, np.float32), np.asarray(src, np.
matplotlib.interactive(was_interactive)
return M, Minv

```

Like with the `get_undistorter` function described above, we use Python closures to create a function generator called `get_warpers`, which measures the perspective, remembers the transformation matrices, and then generate a new function that uses OpenCV `warpPerspective` to transform a target image. Note that it actually generates two functions, both to "warp" and "unwarp" images.

```

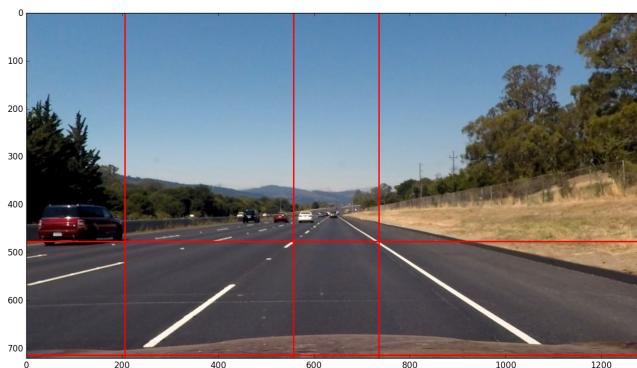
def get_warpers(corrected_image):
    M, Minv = measure_warp(corrected_image)
    return lambda x: cv2.warpPerspective(x,
M,
x.shape[:2][::-1],
flags=cv2.INTER_LINEAR), \
    lambda x: cv2.warpPerspective(x,
Minv,
x.shape[:2][::-1],
flags=cv2.INTER_LINEAR), M, Minv

```

The following code illustrates how this is put into practice. We get an image with the `matplotlib imread` function, correct for camera distortion using the `undistort` function we generated with the `undistorter` function created above (after camera calibration on checkerboard images), then use `get_warpers` to generate both the `warp` and `unwarp` functions. It also returns the  $M$  and  $M_{inv}$  matrices as `M` and `Minv` for good measure.

```
warp,unwarp,M,Minv = get_warpers(undistort(mpimg.imread("test_images/straight_lines2.j
```

The next sequence of four figures illustrates the interactive experience the user has in this operation, showing step-by-step the orthogonal guidelines that appear. The trapezoidal area formed about the outside bottom two corners and the inside top two corners of the last figure defines the source region that is then mapped to the target region. Again, as discussed above the target region is a rectangle running from the bottom of the image to the top, 300 pixels in from the left edge and 300 pixels in from the right edge.

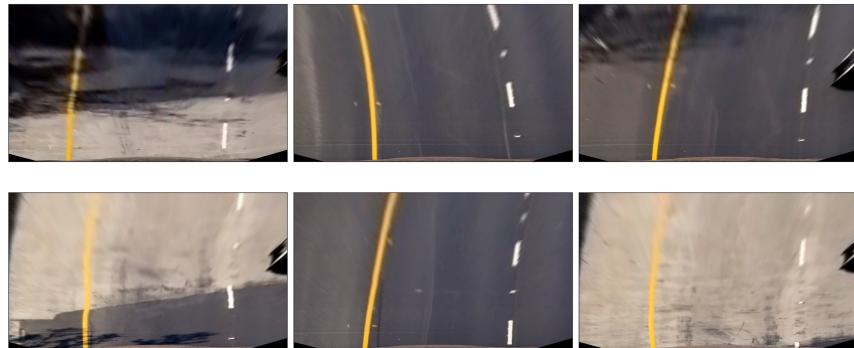




Equipped not just with an `undistort` function (obtained via camera calibration) but also a `warp` (obtained via perspective measurement) function, we can compose both functions in the proper sequence (`undistort` then `warp`) and apply it to our 6 test images.

```
visualize("output_images/warped_undistorted_test_images.jpg",
(warp(undistort(mpimg.imread(f))) for f in cycle(glob.glob("test_images/test*.jpg")))
```

As you can see in the following gallery we now have a "birds-eye" (i.e. top-down) view of the road for these 6 test images. Note also that the perspective transform has also had the effect of shoving out of the frame much of the extraneous details (sky, trees, guardrails, other cars). This is serendipitous as it saves us from having to apply a mask just to the lane region.



Camera calibration and perspective measurement are preliminary steps that occur before applying the processing pipeline to images taken from the

video stream. However, they are essential and they enable the distortion correction and perspective transformation steps which *are* part of the processing pipeline. Another set of essential pipeline steps involve gradient and color thresholds, discussed in the next sections.

## Gradient and Color Thresholds

Next we develop a set of useful utility functions for scaling images, taking gradients across them, isolating different color channels, and generating binary images.

The `scale` function scales the values of NumPy image arrays to arbitrary ranges (e.g., [0,1] or [0,255]). The default range is [0,255], and this is useful in order to give all images the same scale. Different operations (e.g., taking gradients, producing binary images) can introduce different scales and it eases combining and comparing images when they have the same scale.

```
def scale(img, factor=255.0):
    scale_factor = np.max(img)/factor
    return (img/scale_factor).astype(np.uint8)
```

The `derivative` function uses the OpenCV `sobel` function to apply the Sobel operator in order to estimate derivatives in the  $x$  and  $y$  directions across the image. For good measure, it also returns both the *magnitude* and the *direction* of the gradient computed from these derivative estimates.

```
def derivative(img, sobel_kernel=3):
    derivx = np.absolute(cv2.Sobel(img, cv2.CV_64F, 1, 0, ksize=sobel_kernel))
    derivy = np.absolute(cv2.Sobel(img, cv2.CV_64F, 0, 1, ksize=sobel_kernel))
    gradmag = np.sqrt(derivx**2 + derivy**2)
    absgraddir = np.arctan2(derivy, derivx)
    return scale(derivx), scale(derivy), scale(gradmag), absgraddir
```

The `grad` function adapts the `derivative` function to return both the gradient *magnitude* and *direction*. You might wonder what this function adds to the `derivative` function, and that is a valid consideration. Largely it exists because the lecture notes seemed to suggest that it's worthwhile to use different kernel sizes for the Sobel operator when computing the gradient direction. In hindsight it's not clear this function really is adding value and it may be removed in future versions.

```
def grad(img, k1=3, k2=15):
```

```

    _,_,g,_ = derivative(img, sobel_kernel=k1)
    _,_,_,p = derivative(img, sobel_kernel=k2)
    return g,p

```

The `hls_select` function is a convenience that fans out the three channels of the HLS color-space into separate NumPy arrays.

```

def hls_select(img):
    hsv = cv2.cvtColor(img, cv2.COLOR_RGB2HLS).astype(np.float)
    h = hsv[:, :, 0]
    l = hsv[:, :, 1]
    s = hsv[:, :, 2]
    return h, l, s

```

The `rgb_select` function is another convenience that returns the three channels of the RGB color-space.

```

def rgb_select(img):
    rgb = img
    r = rgb[:, :, 0]
    g = rgb[:, :, 1]
    b = rgb[:, :, 2]
    return r, g, b

```

The `threshold` function is a convenience that applies `thresh_min` and `thresh_max` *min-max* values and logical operations in order to obtain "binary" images. Binary images have activated pixels (non-zero values) for desired features.

```

def threshold(img, thresh_min=0, thresh_max=255):
    binary_output = np.zeros_like(img)
    binary_output[(img >= thresh_min) & (img <= thresh_max)] = 1
    return binary_output

```

The `land` and `lor` functions are conveniences for combining binary images, either with logical conjunction or disjunction, respectively.

```

land = lambda *x: np.logical_and.reduce(x)
lor = lambda *x: np.logical_or.reduce(x)

```

There are various ways of doing this. Another way is to stack binary image arrays using the NumPy `stack` function and then interleave various combinations of such interleavings along with the NumPy `any` function and `all` function. It's a clever approach, but I find that applying the NumPy `logical_and` and `logical_or` functions as above leads to less typing.

The `highlight` function composes the color channel selection, gradient estimation, binary threshold, logical composition, and scaling operations to an input image in order to "highlight" the desired features, such as lane lines. Note that distortion correction and perspective transformation are considered outside the scope of this function. In a real pipeline, those two operations almost certainly should be applied to an image before presenting it to the `highlight` function. In general, they need not be, which can be useful during the exploratory phase of pipeline development.

```
def highlight(img):
    r,g,b = rgb_select(img)
    h,l,s = hls_select(img)
    o01 = threshold(r, 200, 255)
    o02 = threshold(g, 200, 255)
    o03 = threshold(s, 200, 255)
    return scale(lor(land(o01,o02),o03))
```

In fact, the `highlight` and `undistort` operations are combined *without* perspective transform in the next gallery of 6 test images. This is an example of a common iteration pattern while exploring pipeline options.

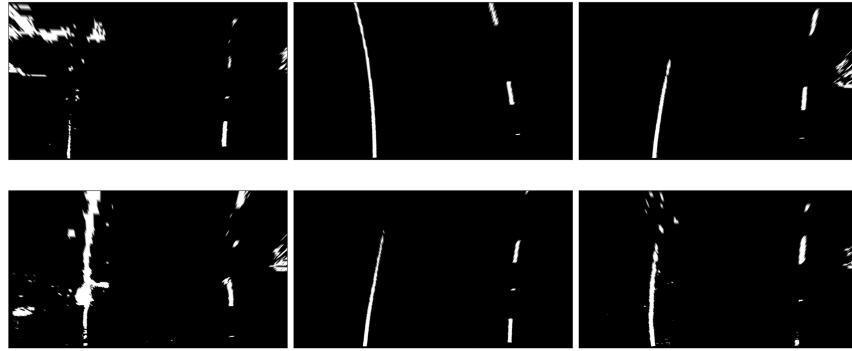
```
visualize("output_images/binary_undistorted_test_images.jpg",
          (highlight(undistort(mpimg.imread(f)))) for f in cycle(glob.glob("test_images/test*.jpg")))
```



## Perspective Transform

Armed with a pipeline which, based on the 6 test images, we believe may be a good candidate for detecting lane lines, we then see what the pipeline-processed test images look like after transforming them to a "bird's-eye" view.

```
visualize("output_images/warped_binary_undistorted_images.jpg",
(warp(highlight(undistort(mpimg.imread(f)))) for f in cycle(glob.glob("test_images/t
```



## Lane-Finding

Lane-line detection can be done somewhat laboriously—but perhaps more accurately—using a "sliding window" technique. Roughly, the algorithm implemented in `detect_lines_sliding_window` below has these steps, also discussed in the code comments.

1. Take a histogram across the bottom of the image.
2. Find the histogram peaks to identify the lane lines at the bottom of the image.
3. Divide the image into a vertical stack of narrow horizontal slices.
4. Select activated pixels (remember, the input is a binary image) only in a "neighborhood" of our current estimate of the lane position. This neighborhood is the "sliding window." To bootstrap the process, our initial estimate of the lane line location is taken from the histogram peak steps listed above. Essentially, we are removing "outliers"

5. Estimate the new lane-line location for this window from the mean of the pixels falling within the sliding window.
6. March vertically up through the stack, repeating this process.
7. Select all activated pixels within all of our sliding windows.
8. Fit a quadratic function to these selected pixels, obtaining model parameters.

The model parameters essentially represent the detected lane-line. We do this both for the left and right lines. Moreover, we also perform a few somewhat ancillary operations while we're at it.

1. Draw the sliding windows, the selected pixels, and the modeled quadratic curve onto a copy of the image.
2. Recompute the function fit after scaling the pixel locations to real world values, then use these model fit parameters to compute a real-world radius of curvature for both lanes.

The function `detect_lines_sliding_window` returns quite a few values:

1. left lane fit parameters
2. right lane fit parameters
3. left lane fit residuals
4. right lane fit residuals
5. left lane real-world radius (in meters)
6. right lane real-world radius (in meters)
7. annotated image, with sliding windows, selected pixels, and modeled curves

```
def detect_lines_sliding_window(warped_binary):
    # Assuming you have created a warped binary image called "warped_binary"
    # Take a histogram of the bottom half of the image
    histogram = np.sum(warped_binary[warped_binary.shape[0]/2:,:,:], axis=0)
    # Create an output image to draw on and visualize the result
    out_img = np.dstack((warped_binary, warped_binary, warped_binary))*255
```

```

# Find the peak of the left and right halves of the histogram
# These will be the starting point for the left and right lines
midpoint = np.int(histogram.shape[0]/2)
leftx_base = np.argmax(histogram[:midpoint])
rightx_base = np.argmax(histogram[midpoint:]) + midpoint
# Choose the number of sliding windows
nwindows = 9
# Set height of windows
window_height = np.int(warped_binary.shape[0]/nwindows)
# Identify the x and y positions of all nonzero pixels in the image
nonzero = warped_binary.nonzero()
nonzeroy = np.array(nonzero[0])
nonzerox = np.array(nonzero[1])
# Current positions to be updated for each window
leftx_current = leftx_base
rightx_current = rightx_base
# Set the width of the windows +/- margin
margin = 100
# Set minimum number of pixels found to recenter window
minpix = 50
# Create empty lists to receive left and right lane pixel indices
left_lane_inds = []
right_lane_inds = []
# Step through the windows one by one
for window in range(nwindows):
    # Identify window boundaries in x and y (and right and left)
    win_y_low = warped_binary.shape[0] - (window+1)*window_height
    win_y_high = warped_binary.shape[0] - window*window_height
    win_xleft_low = leftx_current - margin
    win_xleft_high = leftx_current + margin
    win_xright_low = rightx_current - margin
    win_xright_high = rightx_current + margin
    # Draw the windows on the visualization image
    cv2.rectangle(out_img,(win_xleft_low,win_y_low),(win_xleft_high,win_y_high),(0,255,0),
    cv2.rectangle(out_img,(win_xright_low,win_y_low),(win_xright_high,win_y_high),(0,255,0)
    # Identify the nonzero pixels in x and y within the window
    good_left_inds = ((nonzeroy >= win_y_low) & (nonzeroy < win_y_high) & (nonzerox >= win_xleft_low) & (nonzerox <= win_xleft_high))
    good_right_inds = ((nonzeroy >= win_y_low) & (nonzeroy < win_y_high) & (nonzerox >= win_xright_low) & (nonzerox <= win_xright_high))
    # Append these indices to the lists
    left_lane_inds.append(good_left_inds)

```

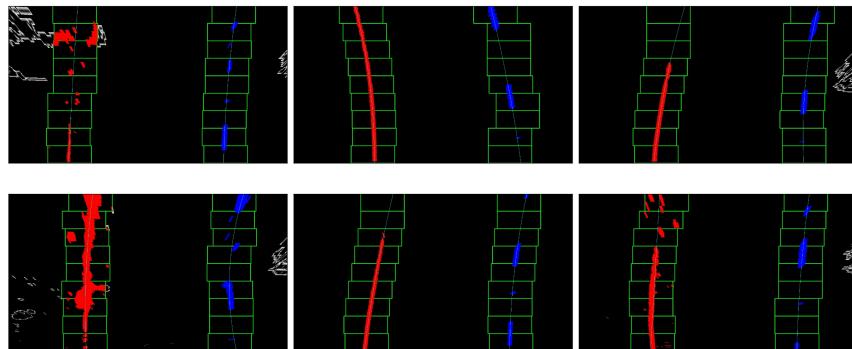
```

right_lane_inds.append(good_right_inds)
# If you found > minpix pixels, recenter next window on their mean position
if len(good_left_inds) > minpix:
    leftx_current = np.int(np.mean(nonzerox[good_left_inds]))
if len(good_right_inds) > minpix:
    rightx_current = np.int(np.mean(nonzerox[good_right_inds]))
# Concatenate the arrays of indices
left_lane_inds = np.concatenate(left_lane_inds)
right_lane_inds = np.concatenate(right_lane_inds)
# Extract left and right line pixel positions
leftx = nonzerox[left_lane_inds]
lefty = nonzeroy[left_lane_inds]
rightx = nonzerox[right_lane_inds]
righty = nonzeroy[right_lane_inds]
# Fit a second order polynomial to each
left_fit, left_res, _, _, _ = np.polyfit(lefty, leftx, 2, full=True)
right_fit, right_res, _, _, _ = np.polyfit(righty, rightx, 2, full=True)
# Generate x and y values for plotting
ploty = np.linspace(0, warped_binary.shape[0]-1, warped_binary.shape[0] )
left_fitx = left_fit[0]*ploty**2 + left_fit[1]*ploty + left_fit[2]
right_fitx = right_fit[0]*ploty**2 + right_fit[1]*ploty + right_fit[2]
out_img[nonzeroy[left_lane_inds], nonzerox[left_lane_inds]] = [255, 0, 0]
out_img[nonzeroy[right_lane_inds], nonzerox[right_lane_inds]] = [0, 0, 255]
out_img[ploty.astype('int'), left_fitx.astype('int')] = [0, 255, 255]
out_img[ploty.astype('int'), right_fitx.astype('int')] = [0, 255, 255]
y_eval = warped_binary.shape[0]
# Define conversions in x and y from pixels space to meters
ym_per_pix = 30/720 # meters per pixel in y dimension
xm_per_pix = 3.7/700 # meters per pixel in x dimension
# Fit new polynomials to x,y in world space
left_fit_cr = np.polyfit(lefty*ym_per_pix, leftx*xm_per_pix, 2)
right_fit_cr = np.polyfit(righty*ym_per_pix, rightx*xm_per_pix, 2)
# Calculate the new radii of curvature
left_curverad = ((1 + (2*left_fit_cr[0]*y_eval*ym_per_pix + left_fit_cr[1])**2)**1.5)
right_curverad = ((1 + (2*right_fit_cr[0]*y_eval*ym_per_pix + right_fit_cr[1])**2)**1.5)
return left_fit, right_fit, np.sqrt(left_fit[1]/len(leftx)), np.sqrt(right_fit[1]/len(rightx))

```

The following figures shows the annotated image resulting from applying this particular lane-finding algorithm to our 6 test images, after distortion correction, highlighting, and perspective transformation.

```
visualize("output_images/detected_lines_test_images.jpg",
(detect_lines_sliding_window(warp(highlight(undistort(mpimg.imread(f)))))[6] for f in
```



Armed with a good estimate for the current lane-line locations and with the observation that the lanes do not change dramatically from one frame to the next, we can implement an optimization. Recall that the *only reason* for the sliding window algorithm is to remove outliers. If we were content just to fit all of the pixels, good or bad, we would only need to divide the frame into a left half and a right half and then fit the quadratic curves straight away. However, guided by the lecture we chose to remove outliers. That requires a good guess for where the lane line is, which almost inevitably leads us to the sliding window technique.

The `detect_lines` function takes `left_fit` and `right_fit` arguments, which are good estimates of the model fit parameters obtained from the previous video frame. It then selects pixels in the neighborhood of the curve computed for these parameters, and fits new parameters for the current frame from the selected pixels. Thus, it avoids the labor of the sliding window technique so long as one already has a good estimate of the model fit parameters. Note that, because this function does *not* apply the sliding window technique, it cannot draw the sliding windows. Therefore, the last parameter returned is `None`.

```
def detect_lines(warped_binary, left_fit, right_fit):
    # from the next frame of video (also called "binary_warped")
    # It's now much easier to find line pixels!
    nonzero = warped_binary.nonzero()
    nonzeroy = np.array(nonzero[0])
    nonzerox = np.array(nonzero[1])
```

```

margin = 100
left_lane_inds = ((nonzerox > (left_fit[0]*(nonzerooy**2) + left_fit[1]*nonzerooy +
right_lane_inds = ((nonzerox > (right_fit[0]*(nonzerooy**2) + right_fit[1]*nonzerooy
# Again, extract left and right line pixel positions
leftx = nonzerox[left_lane_inds]
lefty = nonzeroy[left_lane_inds]
rightx = nonzerox[right_lane_inds]
righty = nonzeroy[right_lane_inds]
# Fit a second order polynomial to each
left_fit, left_res, _, _, _ = np.polyfit(lefty, leftx, 2, full=True)
right_fit, right_res, _, _, _ = np.polyfit(righty, rightx, 2, full=True)
# Generate x and y values for plotting
ploty = np.linspace(0, warped_binary.shape[0]-1, warped_binary.shape[0] )
left_fitx = left_fit[0]*ploty**2 + left_fit[1]*ploty + left_fit[2]
right_fitx = right_fit[0]*ploty**2 + right_fit[1]*ploty + right_fit[2]
y_eval = warped_binary.shape[0]
# Define conversions in x and y from pixels space to meters
ym_per_pix = 30/720 # meters per pixel in y dimension
xm_per_pix = 3.7/700 # meters per pixel in x dimension
# Fit new polynomials to x,y in world space
left_fit_cr = np.polyfit(lefty*ym_per_pix, leftx*xm_per_pix, 2)
right_fit_cr = np.polyfit(righty*ym_per_pix, rightx*xm_per_pix, 2)
# Calculate the new radii of curvature
left_curverad = ((1 + (2*left_fit_cr[0]*y_eval*ym_per_pix + left_fit_cr[1])**2)**1.5)
right_curverad = ((1 + (2*right_fit_cr[0]*y_eval*ym_per_pix + right_fit_cr[1])**2)**1.5)
return left_fit, right_fit, np.sqrt(left_fit[1]/len(leftx)), np.sqrt(right_fit[1]/len(rightx)))

```

Note in the function above how the radius of curvature is calculated for the two lanes. First, constants establish a conversion between pixel coordinates in the  $x$  and  $y$  directions and corresponding real-world coordinates (in meters) in the  $x$  and  $z$  direction. By  $z$  direction I mean depth into the frame. This is an important point, because we must account for the fact that the three-dimensional real-world image has been warped by the perspective transform into a two-dimensional pixel-space image. Second, we fit our model again, this time after converting our pixel coordinates into real-world values. This is important! A simple conversion of radius-of-curvature estimates taken from our original fit would not be correct, because that fit does not account for the warping between the three-dimensional real world and the two-dimensional pixel-space of the image plane. Third, for the left and right lanes we calculate the radius of curvature using the model fit pa-

rameters, according to this formula.

$$R_{curve} = \frac{(1 + (2Ay + B)^2)^{3/2}}{|2A|}$$

The `draw_lane` function takes a distortion-corrected unwarped image, a warped binary image like, model fit parameters, real-world lane-curvature estimates in meters, and an image unwarping function. It uses these to annotate the undistorted image with a depiction of the lane, along with vital statistics on the left and right lane curvature, and the position of the camera with respect to the center of the lane (taken as the mean of the two lane locations).

```
def draw_lane(undistorted, warped_binary, l_fit, r_fit, l_rad, r_rad, un warp):
    # Create an image to draw the lines on
    warp_zero = np.zeros_like(warped_binary).astype(np.uint8)
    color_warp = np.dstack((warp_zero, warp_zero, warp_zero))
    # Generate x and y values for plotting
    ploty = np.linspace(0, warped_binary.shape[0]-1, warped_binary.shape[0])
    l_fitx = l_fit[0]*ploty**2 + l_fit[1]*ploty + l_fit[2]
    r_fitx = r_fit[0]*ploty**2 + r_fit[1]*ploty + r_fit[2]
    # Recast the x and y points into usable format for cv2.fillPoly()
    pts_left = np.array([np.transpose(np.vstack([l_fitx, ploty]))])
    pts_right = np.array([np.flipud(np.transpose(np.vstack([r_fitx, ploty])))])
    pts = np.hstack((pts_left, pts_right))
    # Draw the lane onto the warped_binary blank image
    cv2.fillPoly(color_warp, np.int_(pts), (0,255, 0))
    # Warp the blank back to original image space using inverse perspective matrix (Mi
    # newwarp = cv2.warpPerspective(color_warp, Minv, (image.shape[1], image.shape[0]))
    newwarp = un warp(color_warp)
    # Combine the result with the original image
    result = cv2.addWeighted(undistorted, 1, newwarp, 0.3, 0)
    # Annotate image with lane curvature estimates
    cv2.putText(result, "L. Curvature: %.2f km" % (l_rad/1000), (50,50), cv2.FONT_HERSHEY_SIMPLIFIED_ARIAL_9)
    cv2.putText(result, "R. Curvature: %.2f km" % (r_rad/1000), (50,80), cv2.FONT_HERSHEY_SIMPLIFIED_ARIAL_9)
    # Annotate image with position estimate
    cv2.putText(result, "C. Position: %.2f m" % ((np.average((l_fitx + r_fitx)/2) - w
    return result
```

Note in the function above how we annotate the image with an estimate of the position of the car with respect to the center of the road. It is a

simple average of the pixel coordinates of the two lanes at the bottom of the image, minus the pixel coordinate of the image center, then scaled to a real-world value (meters). Note that we do *not* need the second curve fit in real-world coordinates that was done in the two lane-detecting functions to do this. Because we are estimating the position at the *bottom* of the image frame, the horizontal direction only comes into play and we only need account for  $x$  coordinates. We had to perform the second fit for the radius of curvature calculation to compensate for the warping of the image, but that warping *only* relates the  $z$  direction in the three-dimensional world and the  $y$  direction in the image plane. It plays no role in calculating the car position, but *only* if we assume that position is to be taken at the bottom of the image.

With that note, finally we can move on to the full processing pipeline.

The `get_processor` function returns a "processor" function. A processor function embodies *all* of the steps of the pipeline outlined above:

1. Distortion Correction
2. Perspective Transformation
3. Lane-line detection *with* bootstrapping
4. Radius of curvature and vehicle position calculations
5. Image annotation with drawn lane lines and vital statistics

One other thing that this function does is this. It takes a weighted average of some number of recent frames, along with the current frame. This removes "jitter" from the lanes and values on the video streams, and adds robustness against bad detections on individual frames. It uses `deque` to create "ring buffers" for the left lane parameters, right lane parameters, left lane radius, and right lane radius. The buffers can be of any size, though the default has 10 slots. Note that a buffer size of 1 essentially computes no average at all. Weighted averages are taken across these buffers. The weights could be taken from any function, simple or complex, that is appropriate for the situation. In practice I did not try for anything complicated, and used a simple linear weighting scheme: older frames have strictly linearly less weight.

```
def get_processor(nbins=10):
    bins = nbins
    l_params = deque(maxlen=bins)
    r_params = deque(maxlen=bins)
```

```

l_radius = deque(maxlen=bins)
r_radius = deque(maxlen=bins)
weights = np.arange(1,bins+1)/bins
def process_image(img0):
    undistorted = undistort(img0)
    warped_binary = warp(highlight(undistorted))
    l_fit, r_fit, l_res, r_res, l_curverad, r_curverad, _ = detect_lines_sliding_window(warped_binary)
    l_params.append(l_fit)
    r_params.append(r_fit)
    l_radius.append(l_curverad)
    r_radius.append(r_curverad)
    annotated_image = draw_lane(undistorted,
        warped_binary,
        np.average(l_params,0,weights[-len(l_params):]),
        np.average(r_params,0,weights[-len(l_params):]),
        np.average(l_radius,0,weights[-len(l_params):]),
        np.average(r_radius,0,weights[-len(l_params):]),
        unwarped)
    return annotated_image
return process_image

```

Equipped with a bona-fide image processor, the very one we use on the video stream we can examine its effect on our 6 test images.

```

visualize("output_images/drawn_lanes_test_images.jpg",
(get_processor(1)(mpimg.imread(f)) for f in cycle(glob.glob("test_images/test*.jpg"))

```



Finally, generate a new processor and apply it to the video stream. We generate a new processor in order to give it a different buffer size for the

ring buffers supporting the weighted averages. For the video stream, the ring buffers have 50 slots, not 10. Since the video stream is at 25 frames per second, this constitutes a full 2 second window for the weighted average. That may seem like a lot, and we *do* have to be careful not to push it too far. There is a trade-off between the smoothness and robustness added by the weighted average, and a stiffness to the model that may cause it to lag on sharp turns. In practice, however, the weighted average quickly deweights older frames, and in experimentation no deleterious effects were noticed with a set of 50-slot ring buffers.

```
in_clip = VideoFileClip("project_video.mp4")
out_clip = in_clip.fl_image(get_processor(50))
cProfile.run('out_clip.write_videofile("output_images/project_output.mp4", audio=False)
```

We can see the result for the project video in the following video clip.

## Discussion

### What Worked Well

### What Could Be Improved