Design XOR Gate

Jisen Fang
CS550 - Machine Learning and Business Intelligence
2023 Spring

Table of contents

Calculate the output Z for the given input (X,Y).

Adjust weights by 0.5

Using step activation function

Z := (W0 * C + W1 * X + W2 * Y >= T) where T := 1.0

if (W0 * C + W1 * X + W2 * Y >= T), then output is 1, else output = 0

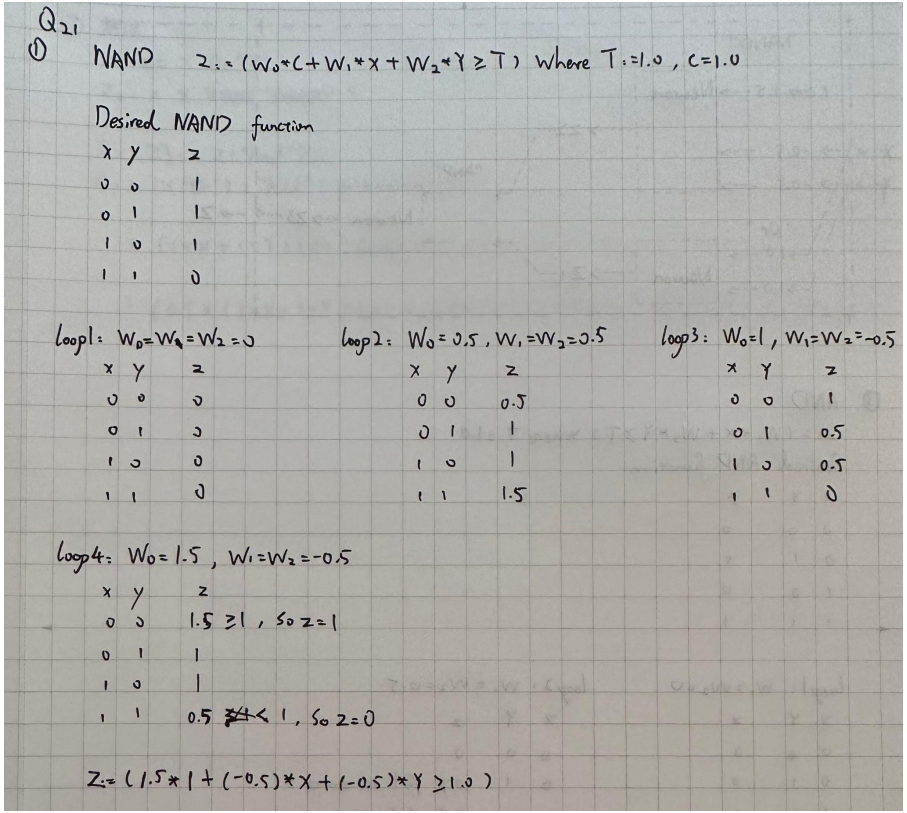
The bias C for NAND is 1.0

NAND gate

The formula for

Z2: = x “NAND” y

Z2: = (1.5*1+(-0.5)*x+(-0.5)*y>=1)

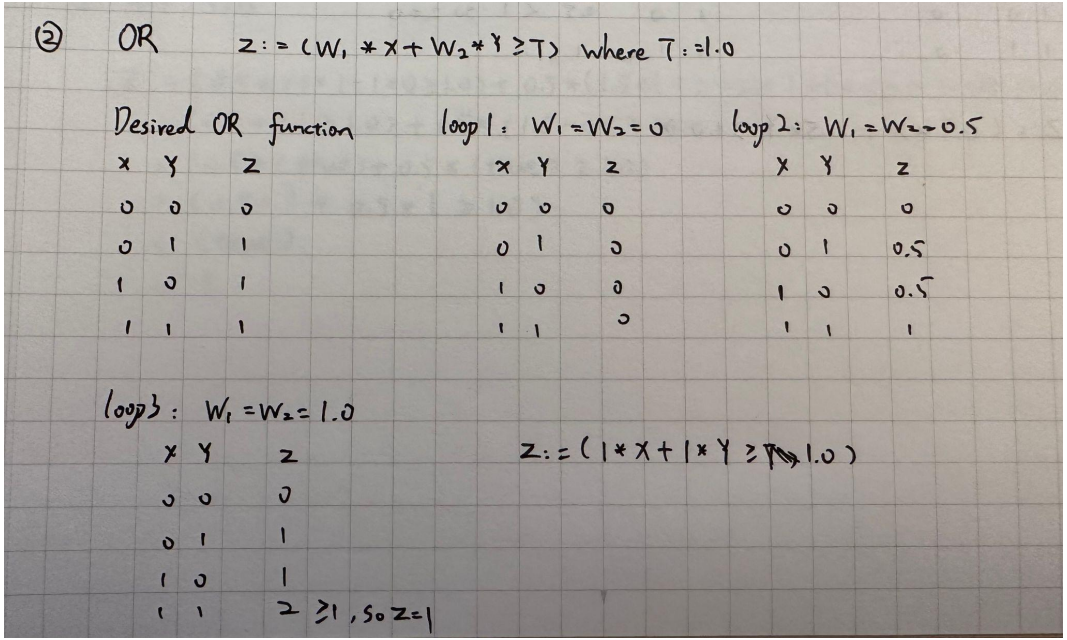


OR gate

The formula for

Z1: = x “OR” y

Z1: = (1*x+1*y>=1)

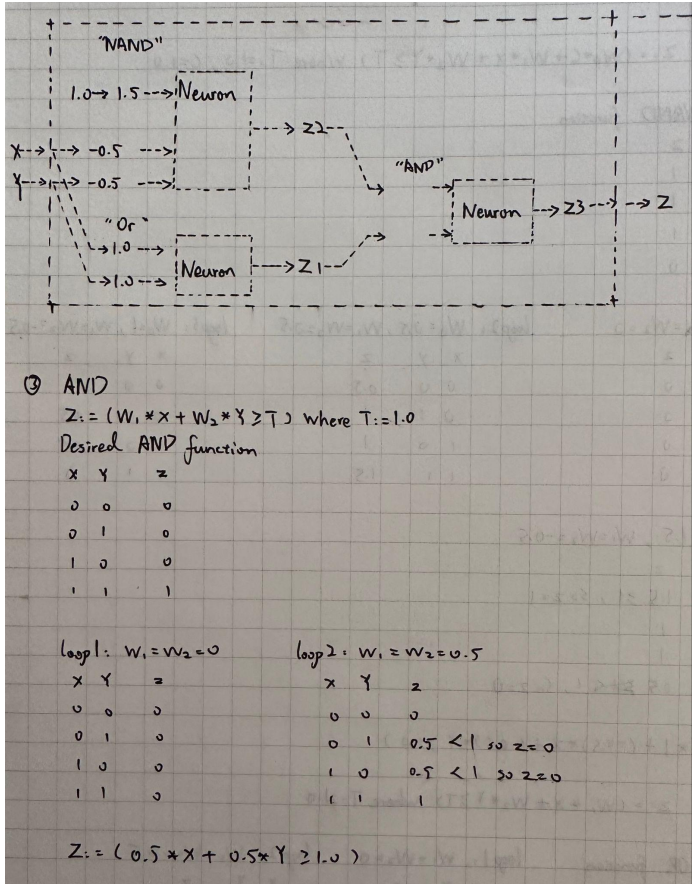


AND gate

The formula for

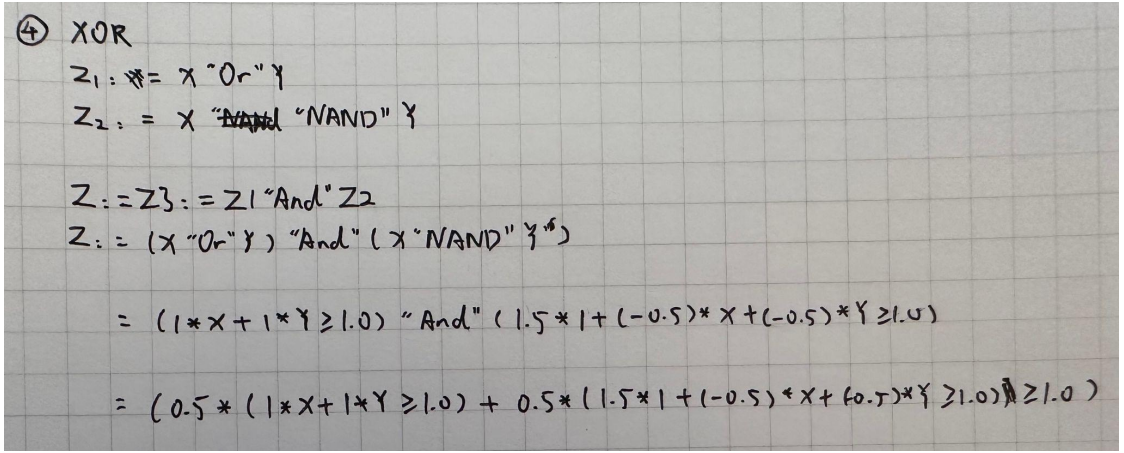
Z: = x “AND” y

Z: = (0.5*x+0.5*y>=1)

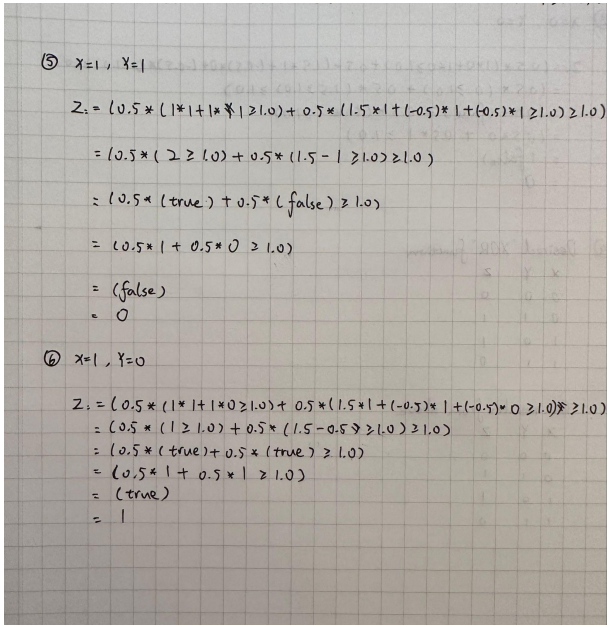


XOR gate

The formula for Z: = Z3: = Z1 “AND” Z2 = (0.5*(1*x+1*y>=1)+0.5*(1.5*1+(-0.5)*x+(-0.5)*y>=1)>=1)



Test



⑤ X=1, Y=1

$Z_1 = (0.5*(1*1+1*1)\geq 1.0) + 0.5*(1.5*1+(-0.5)*1+(-0.5)*1 \geq 1.0) \geq 1.0)$

$= (0.5*(2 \geq 1.0) + 0.5*(1.5-1 \geq 1.0) \geq 1.0)$

$= (0.5*(\text{true}) + 0.5*(\text{false}) \geq 1.0)$

$= (0.5*1 + 0.5*0 \geq 1.0)$

$= (\text{false})$

$= 0$

⑥ X=1, Y=0

$Z_1 = (0.5*(1*1+1*0 \geq 1.0) + 0.5*(1.5*1+(-0.5)*1+(-0.5)*0 \geq 1.0) \geq 1.0)$

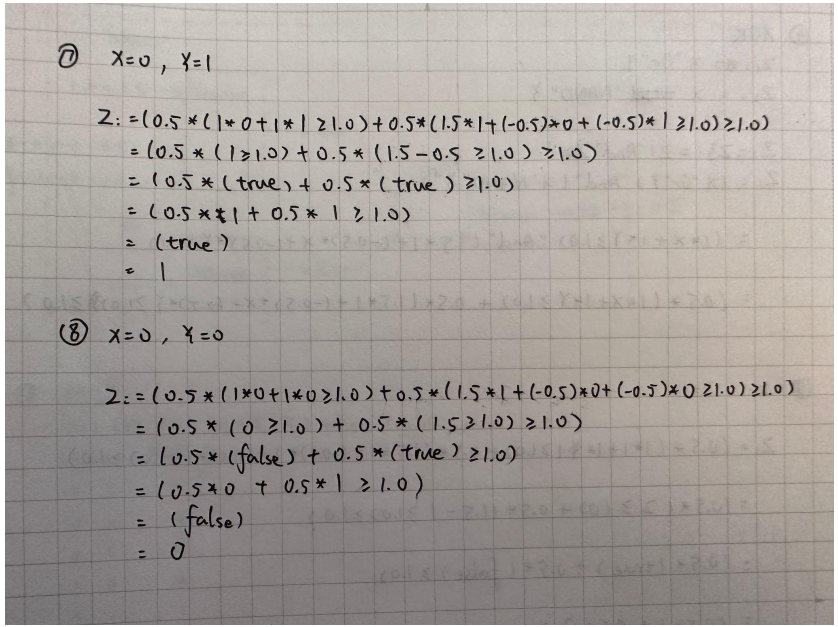
$= (0.5*(1 \geq 1.0) + 0.5*(1.5-0.5 \geq 1.0) \geq 1.0)$

$= (0.5*(\text{true}) + 0.5*(\text{true}) \geq 1.0)$

$= (0.5*1 + 0.5*1 \geq 1.0)$

$= (\text{true})$

$= 1$



⑦ X=0, Y=1

$Z_1 = (0.5*(1*0+1*1 \geq 1.0) + 0.5*(1.5*1+(-0.5)*0+(-0.5)*1 \geq 1.0) \geq 1.0)$

$= (0.5*(1 \geq 1.0) + 0.5*(1.5-0.5 \geq 1.0) \geq 1.0)$

$= (0.5*(\text{true}) + 0.5*(\text{true}) \geq 1.0)$

$= (0.5*1 + 0.5*1 \geq 1.0)$

$= (\text{true})$

$= 1$

⑧ X=0, Y=0

$Z_1 = (0.5*(1*0+1*0 \geq 1.0) + 0.5*(1.5*1+(-0.5)*0+(-0.5)*0 \geq 1.0) \geq 1.0)$

$= (0.5*(0 \geq 1.0) + 0.5*(1.5 \geq 1.0) \geq 1.0)$

$= (0.5*(\text{false}) + 0.5*(\text{true}) \geq 1.0)$

$= (0.5*0 + 0.5*1 \geq 1.0)$

$= (\text{false})$

$= 0$

Conclusion

Test results match the desired “XOR” function.