



Design XOR Gate

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Calculate the output Z for the given input (X,Y) .

Adjust weights by 0.5

Using step activation function

$Z := (W_0 * C + W_1 * X + W_2 * Y \geq T)$ where $T := 1.0$

if $(W_0 * C + W_1 * X + W_2 * Y \geq T)$, then output is 1, else output = 0

The bias C for NAND is 1.0

NAND gate

The formula for

$$Z2 := x \text{ "NAND" } y$$

$$Z2 := (1.5 * 1 + (-0.5) * x + (-0.5) * y \geq 1)$$

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① NAND $Z := (W_0 * C + W_1 * x + W_2 * y \geq T)$ where $T := 1.0$, $C := 1.0$

Desired NAND function

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

loop1: $W_0 = W_1 = W_2 = 0$

x	y	z
0	0	0
0	1	0
1	0	0
1	1	0

loop2: $W_0 = 0.5, W_1 = W_2 = 0.5$

x	y	z
0	0	0.5
0	1	1
1	0	1
1	1	1.5

loop3: $W_0 = 1, W_1 = W_2 = -0.5$

x	y	z
0	0	1
0	1	0.5
1	0	0.5
1	1	0

loop4: $W_0 = 1.5, W_1 = W_2 = -0.5$

x	y	z
0	0	1.5 ≥ 1 , so $z = 1$
0	1	1
1	0	1
1	1	0.5 < 1 , so $z = 0$

$Z := (1.5 * 1 + (-0.5) * x + (-0.5) * y \geq 1.0)$

OR gate

The formula for

$$Z1: = x \text{ "OR" } y$$

$$Z1: = (1*x + 1*y \geq 1)$$

② OR $z := (w_1 * x + w_2 * y \geq T)$ where $T := 1.0$

Desired OR function

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

loop 1: $w_1 = w_2 = 0$

x	y	z
0	0	0
0	1	0
1	0	0
1	1	0

loop 2: $w_1 = w_2 = 0.5$

x	y	z
0	0	0
0	1	0.5
1	0	0.5
1	1	1

loop 3: $w_1 = w_2 = 1.0$

x	y	z
0	0	0
0	1	1
1	0	1
1	1	2

$$z := (1 * x + 1 * y \geq 1.0)$$

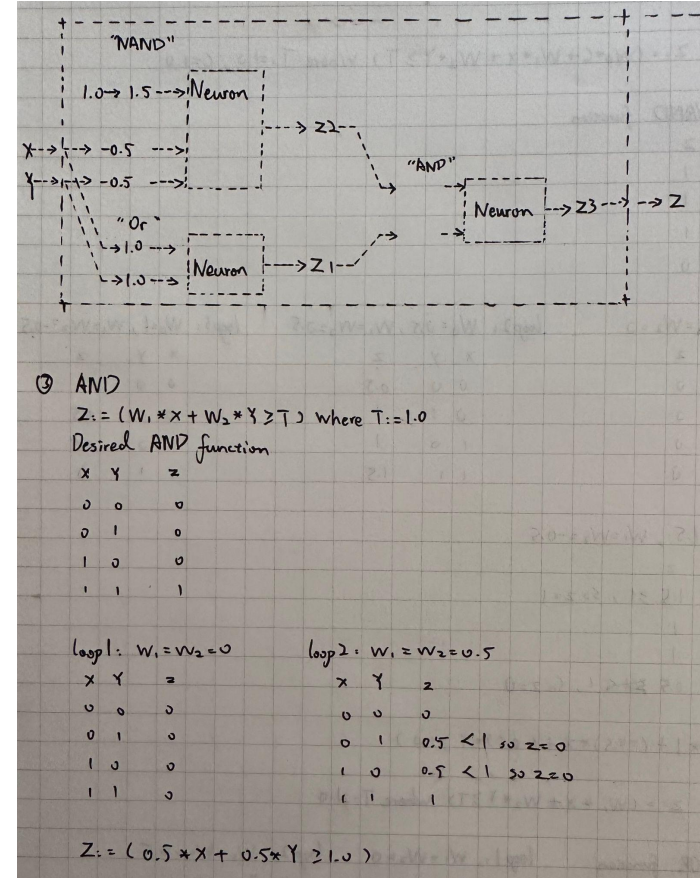
$z \geq 1$, so $z = 1$

AND gate

The formula for

$$Z = x \text{ "AND" } y$$

$$Z = (0.5 * x + 0.5 * y \geq 1)$$



XOR gate

The formula for Z: $Z = Z_1 \text{ "AND" } Z_2 = (0.5 * (1 * x + 1 * y \geq 1) + 0.5 * (1.5 * 1 + (-0.5) * x + (-0.5) * y \geq 1)) \geq 1$

④ XOR

$$Z_1: = x \text{ "Or" } y$$

$$Z_2: = x \text{ "NAND" } y$$

$$Z: = Z_1 \text{ "And" } Z_2$$

$$Z: = (x \text{ "Or" } y) \text{ "And" } (x \text{ "NAND" } y)$$

$$= (1 * x + 1 * y \geq 1.0) \text{ "And" } (1.5 * 1 + (-0.5) * x + (-0.5) * y \geq 1.0)$$

$$= (0.5 * (1 * x + 1 * y \geq 1.0) + 0.5 * (1.5 * 1 + (-0.5) * x + (-0.5) * y \geq 1.0)) \geq 1.0$$

Test

⑤ $X=1, Y=1$

$$\begin{aligned} Z &= (0.5 * (1 * 1 + 1 * 1 \geq 1.0) + 0.5 * (1.5 * 1 + (-0.5) * 1 + (0.5) * 1 \geq 1.0) \geq 1.0) \\ &= (0.5 * (2 \geq 1.0) + 0.5 * (1.5 - 1 \geq 1.0) \geq 1.0) \\ &= (0.5 * (\text{true}) + 0.5 * (\text{false}) \geq 1.0) \\ &= (0.5 * 1 + 0.5 * 0 \geq 1.0) \\ &= (\text{false}) \\ &= 0 \end{aligned}$$

⑥ $X=1, Y=0$

$$\begin{aligned} Z &= (0.5 * (1 * 1 + 1 * 0 \geq 1.0) + 0.5 * (1.5 * 1 + (-0.5) * 1 + (-0.5) * 0 \geq 1.0) \geq 1.0) \\ &= (0.5 * (1 \geq 1.0) + 0.5 * (1.5 - 0.5 \geq 1.0) \geq 1.0) \\ &= (0.5 * (\text{true}) + 0.5 * (\text{true}) \geq 1.0) \\ &= (0.5 * 1 + 0.5 * 1 \geq 1.0) \\ &= (\text{true}) \\ &= 1 \end{aligned}$$

⑦ $X=0, Y=1$

$$\begin{aligned} Z &= (0.5 * (1 * 0 + 1 * 1 \geq 1.0) + 0.5 * (1.5 * 1 + (-0.5) * 0 + (-0.5) * 1 \geq 1.0) \geq 1.0) \\ &= (0.5 * (1 \geq 1.0) + 0.5 * (1.5 - 0.5 \geq 1.0) \geq 1.0) \\ &= (0.5 * (\text{true}) + 0.5 * (\text{true}) \geq 1.0) \\ &= (0.5 * 1 + 0.5 * 1 \geq 1.0) \\ &= (\text{true}) \\ &= 1 \end{aligned}$$

⑧ $X=0, Y=0$

$$\begin{aligned} Z &= (0.5 * (1 * 0 + 1 * 0 \geq 1.0) + 0.5 * (1.5 * 1 + (-0.5) * 0 + (-0.5) * 0 \geq 1.0) \geq 1.0) \\ &= (0.5 * (0 \geq 1.0) + 0.5 * (1.5 \geq 1.0) \geq 1.0) \\ &= (0.5 * (\text{false}) + 0.5 * (\text{true}) \geq 1.0) \\ &= (0.5 * 0 + 0.5 * 1 \geq 1.0) \\ &= (\text{false}) \\ &= 0 \end{aligned}$$

Conclusion

Test results match the desired "XOR" function.

② Desired "XOR" function

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

The results from above

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0