Does my vote matter?

During a contentious election years ago, a good friend of mine asked me, half-jokingly, "What are the odds my vote matters?" We made some general assumptions:

- He would be the last person to cast his vote
- There would be an even number of voters before him (call this the population)
- Each voter was 50/50 likely to vote for either one of two candidates

Please answer the following:

1. What is the general formula for the probability he will cast the deciding vote based on the size of the population?
2. Some tools, for example Microsoft Excel, fail when the size of the population gets not all that big (in the version of Excel I have, at size 1,024), which isn't very convenient. Lots of populations are greater than this size. Can you find an alternative method to solving this problem that works on a population of size 10,000,000? What is that method, and what is the probability he would cast the deciding vote in a population of size 10,000,000?

D100 outcomes

A D100 is a roll of a die with 100 sides numbered 1, 2, 3, …, 98, 99, 100. For these questions please submit both a calculation showing how you derived the answer and a Monte Carlo simulation. The Monte Carlo simulation should prompt the user for how many trials to run and how many rolls of the die are to be performed, as well as output the answer observed so far through each 1% of the completed trials. Please create a stand-alone executable that I can run as well as provide your source code (.cpp file if using C++, .cs if using C#, .py if using Python, etc.). Often these types of file attachments are blocked from external sources, so please provide a link to these files on Google Drive, Dropbox, or similar for me to access them.

Please answer the following (EV is Expected Value, i.e. the average):

1. What is the EV[one D100 roll]?
2. What is the EV[maximum of 2 D100 rolls]?
3. What is the EV[maximum of 5 D100 rolls]?
4. What is the EV[minimum of 5 D100 rolls]?
   a. No Monte Carlo simulation for the minimum, only a calculation

Crazy 8s

This is a slot machine-like concept. Again, please submit both a calculation and a Monte Carlo simulation. The calculation may be done to an arbitrary level of precision, that is we know the game is unlikely to take more than X plays to complete (although it can be solved exactly as well). The Monte Carlo simulation should prompt the user for how many trials to run as well as output the answer observed so far through each 1% of the completed trials. As before, please submit a link to both an executable file as well as your source code for me to access.

Here is how the game works:

- There are two reels, one on the left and one on the right
- There are several symbols on each reel, one of which is an 8
- The 8 symbol lands on the left reel 1 in 9 plays of the game (aka spins)
- The 8 symbol lands on the right reel 1 in 10 plays
- The reels spin independently of each other, and each play is independent from previous plays
- Each play of the game spins both reels
- Each reel has a large 8 above it, split into 8 segments
- As the 8 symbol lands on the corresponding reel below it, one additional segment of the large 8 lights up
- Once both large 8s are fully lit (i.e. at least 8-8s have landed on each reel) a 50 credit bonus is paid and the game ends
- If one reel's large 8 is already fully lit, the game isn't over yet (i.e. do NOT pay the 10 credit bonus on a spin that completes the game), and a new 8 lands on its corresponding reel, a 10 credit bonus is paid
- Each play of the game costs 1 credit

Please answer the following:

1. What is the expected number of credits won or lost by playing the game?