3.) 2D DFT Separability

Analysis →

$$F'(k,l) = \frac{1}{\sqrt{MN}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} F(m,n)\, e^{-i2\pi\left(k\frac{m}{M} + l\frac{n}{N}\right)}$$

Synthesis →

$$F(m,n) = \frac{1}{\sqrt{MN}} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F'(k,l)\, e^{i2\pi\left(k\frac{m}{M} + l\frac{n}{N}\right)}$$

Separability

$$F'(k,l) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left[\frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} F(m,n)\, e^{-i2\pi\left(k\frac{m}{M}\right)} \right] e^{-i2\pi\left(l\frac{n}{N}\right)}$$

2D Forward DFT: $F' = (W' \cdot F)\, W'$,

$$W'_{mn} = \frac{1}{\sqrt{N}} e^{-i2\pi m \frac{n}{N}}$$

$$F(m,n) = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \left[\frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} F'(k,l)\, e^{i2\pi\left(k\frac{m}{M}\right)} \right] e^{i2\pi\left(l\frac{n}{N}\right)}$$

2D inverse DFT: $F = (W \cdot F')\, W$,

$$W_{mn} = \frac{1}{\sqrt{N}} e^{i2\pi\left(m \cdot \frac{n}{N}\right)}$$