MTH 390 **Name:**_____
Quiz 1
Due: 2019-01-31 **Section: 21**

Instructions:

1. Read the directions carefully.
2. Write neatly in pencil and show all your work.
3. Use the appropriate notation.
4. Do not use decimals on any intermediate step.
5. If you have trouble during the quiz, feel free to ask me for help.

Score:_____

1. Suppose $E(X) = 2$, $Var(X) = 9$, $E(Y) = 0$, $Var(Y) = 4$, and $Corr(X, Y) = 0.25$. Find the following to 2 decimal places.

   a. $Var(X + Y)$

   b. $Cov(X, X + Y)$

   c. $Corr(X + Y, X - Y)$.

2. Show that the autocovariance function can be written as

   $\gamma_X(s, t) = E[(X_s - \mu_x(s))(X_t - \mu_x(t))] = E[X_s X_t] - \mu_x(s)\mu_x(t)$, where $\mu_x(t) = E[X_t]$.

3. The package "color" provides data for a time series from an industrial chemical process. There variable measured here is the color property from consecutive batches in the process. Provide the code and corresponding plot for this time series. Label the x-axis "Batch" and the y-axis "Color Property." You will need the TSA library.

4. The Excel file "stlfreeze1875-2018" contains the observed number of days below freezing in St. Louis, 1875-2018. Provide the code and corresponding plot for this time series. Label the x-axis "Years" and the y-axis "Number of Days."

5. Determine if the time series $X_t = a + bZ_t + Z_{t-2}$, $Z_t \sim WN(0, \sigma^2)$ is a stationary time series. If so, find the corresponding autocorrelation function. If not, state why.

6. Determine if the time series $X_t = Z_1 \sin(kt)$, $Z_1 \sim WN(0, \sigma^2)$ is a stationary time series. If so, find the corresponding autocorrelation function. If not, state why.

7. Determine if the time series $X_t = 1 + 3t^2 + 0.2Z_{t-1}$, $Z_t \sim WN(0, \sigma^2)$ is a stationary time series. If so, find the corresponding autocorrelation function. If not, state why.

8. Identify each time series, where $W_t \sim WN(0,1)$. Plot each time series and its corresponding autocorrelation function. Provide the code for each. Use 200 iterations.

a. $X_t = 0.3W_t + W_{t-1}$

b. $X_t = 2 + 0.1X_{t-1} + 0.4X_{t-2} + W_t$

c. $X_t = 4 + 3t^2 + 0.1W_t$

d. $X_t = 6\cos\left(\frac{\pi}{3}t\right) + W_t$