MTH 390
Quiz 5
Due: 2019-04-11

Name:________________________

Section: 21

Instructions:

1. Read the directions carefully.
2. Write <u>neatly</u> in pencil and <u>show all your work</u>.
3. Use the appropriate notation.
4. Do not use decimals on any intermediate step.
5. If you have trouble during the quiz, feel free to ask me for help.

Score:________

1. Simulate the following ARMA models using $n = 140$ iterations. Then forecast the time series next 10 steps. Plot the time series as well as the 10-step forecast.

a. $X_t = 0.75X_{t-1} - 0.125X_{t-2} + W_t$, $W_t \sim WN(0, \sigma^2)$

AR(2) model

b. $X_t = W_t - 0.55W_{t-1} - 0.3W_{t-2}$, $W_t \sim WN(0, \sigma^2)$

MA(2) model

c. $X_t = 0.6X_{t-1} - 0.2X_{t-2} + W_t - 0.9W_{t-1}$, $W_t \sim WN(0, \sigma^2)$.

ARMA(2,1) model

2. The package "color" provides data for a time series from an industrial chemical process. The variable measured here is the color property from consecutive batches in the process. Provide the code and corresponding plot for this time series. Label the x-axis "Batch" and the y-axis "Color Property." You will need the TSA library. Use auto.arima to forecast the color property for the next five batches. Then plot the forecast.

Online

3. Below are mystery zero-mean ARMA models. Use the autocorrelation estimates by lag to estimate the coefficients for each ARMA model, if possible. If not state why. Round your answers to 3 decimal places.

a. AR(1) $X_t = \phi X_{t-1} + W_t$

1	2	3	4	5
0.034	0.052	-0.159	-0.102	-0.052

$\gamma(h) = \phi\gamma(h-1)$

$\gamma(1) = \phi\gamma(0)$

$\rho(1) = \dfrac{\gamma(1)}{\gamma(0)}$

b. AR(2)

1	2	3	4	5
-0.117	0.173	-0.063	0.028	-0.023

c. MA(1)

1	2	3	4	5
-0.148	0.061	-0.054	-0.031	-0.072

d. MA(1)

1	2	3	4	5
0.639	0.438	0.056	0.147	-0.124

-0.125

4. Consider the AR(2) model $X_t = 0.55X_{t-1} - \cancel{1.25}X_{t-2} + W_t$, $W_t \sim WN(0, \sigma^2)$.

   a. Simulate the model the model for $n = 120$. Compare the MOM, LS, and ML estimators for the coefficients ϕ_1 and ϕ_2. What can you say about the results?

MOM : 0.7038, −0.1682 $\sigma^2 = 0.9261$

LS: 0.7187, −0.1792

ML: 0.7175, −0.1776 $\sigma^2 = 0.8903$

Mom is better

   b. Simulate the model the model for $n = 20$. Compare the MOM, LS, and ML estimators for the coefficients ϕ_1 and ϕ_2. What can you say about the results?

MoM : 0.4818, −0.3146 $\sigma^2 = 0.8159$

LS : 0.4726, −0.3193 $\sigma^2 = 0.7285$

ML: 0.4769, −0.3020 $\sigma^2 = 0.6874$

AR 1 coefficient is better

but AR 2 is getting worse

5. Consider the MA(2) model $X_t = W_t + 1.64W_{t-1} - 0.76W_{t-2}$, $W_t \sim WN(0, \sigma^2)$.

a. Simulate the model the model for $n = 120$. Compare the LS, and ML estimators for the coefficients θ_1 and θ_2. What can you say about the results?

LS: 1: 0.2400 Intercept: -0.2670

2: -0.1459 $\hat{\sigma}^2_w = 3.557$

ML: 1: 0.2383 Intercept: -0.2677

2: -0.1413 $\tilde{\sigma}^2_w = 3.556$

Not a big difference between these

but, ML is better estimation

b. Simulate the model the model for $n = 20$. Compare the LS, and ML estimators for the coefficients θ_1 and θ_2. What can you say about the results?

LS: 1, 0.5038 2, -1.0489, $\hat{\sigma}^2_w = 2.163$

ML: 1. 0.1923 2, -0.8096 $\tilde{\sigma}^2_w = 2.785$

For MA1, LS is better estimation

but for MA2, ML is better.

6. Consider the ARMA(1,1) model $X_t = 04.X_{t-1} + W_t + 1.27W_{t-1}$, $W_t \sim WN(0, \sigma^2)$.

a. Simulate the model the model for $n = 120$. Compare the LS, and ML estimators for the coefficients ϕ and θ. What can you say about the results?

LS: 0.5457 , 0.8497 $\hat{\sigma}^2 = 1.476$

0.0814 , 0.0619

ML: 0.5247 , 0.8645 $\hat{\sigma}^2 = 1.41$

0.0831 , 0.0604

ML is better estimation

b. Simulate the model the model for $n = 20$. Compare the LS, and ML estimators for the coefficients ϕ and θ. What can you say about the results?

LS: 0.3227 , 0.8463 $\hat{\sigma}^2 = 1.219$

0.2238 , 0.2314

ML: 0.1961 , 0.7663 $\hat{\sigma}^2 = 1.028$

0.2455 , 0.6054

ML is better estimation

7. The package "sunspots" provides the monthly number of sunspots since 1749. Fit the data to an AR(3) model. Then compare the MOM, LS, and ML estimators for the coefficients ϕ_1, ϕ_2, and ϕ_3. What can you say about the results?