

John Fulgoni
 COMS 4721
 Homework 2
 3 March 2015

John Fulgoni - jsF2154

Homework 2

COMS 4721

3 March 2015

$$1) \sum_{n=1}^N \sum_{i=1}^K 1_{(j=i)} \left(\frac{e^{x_n^T w_i}}{\sum_{j=1}^K e^{x_n^T w_j}} \right)$$

$$\sum_{n=1}^N \sum_{i=1}^K \ln \left(\frac{e^{x_n^T w_i}}{\sum_{j=1}^K e^{x_n^T w_j}} \right) 1_{(j=i)}$$

$$\sum_{n=1}^N \sum_{i=1}^K 1_{(j=i)} \ln(e^{x_n^T w_i}) - \ln \left(\sum_{j=1}^K e^{x_n^T w_j} \right)$$

$$\sum_{n=1}^N \sum_{i=1}^K 1_{(j=i)} x_n^T w_i - \ln \left(\sum_{j=1}^K e^{x_n^T w_j} \right) = L$$

For coding

$$\nabla_w L = \sum_{n=1}^N \sum_{i=1}^K 1_{(j=i)} x_n^T - \frac{x_n^T e^{x_n^T w_i}}{\sum_{j=1}^K e^{x_n^T w_j}} = \sum_{n=1}^N \sum_{i=1}^K x_n^T \left(1 - \frac{e^{x_n^T w_i}}{\sum_{j=1}^K e^{x_n^T w_j}} \right)$$

$$\nabla^2 L = \sum_{n=1}^N \sum_{i=1}^K 1_{(j=i)} - \frac{\left(x_n^T \right)^2 e^{x_n^T w_i} \left(\sum_{j=1}^K e^{x_n^T w_j} \right) - \left(x_n^T \right) \left(x_n^T e^{x_n^T w_i} \right)}{\left(\sum_{j=1}^K e^{x_n^T w_j} \right)^2}$$

$$\nabla^2 L = \sum_{n=1}^N \left(\frac{(x_n^\top e^{x_n^\top w_i})^2}{\sum_{j=1}^K e^{x_j^\top w_i}} - \frac{(x_n^\top e^{x_n^\top w_i})^2}{(\sum_{j=1}^K e^{x_j^\top w_i})^2} \right)$$

2) $p(a|b) = \int p(a|c) p(c|b) + c$ example given

$$\phi(u) = \frac{1}{(2\pi\beta)^{d/2}} \exp\left\{-\frac{\|u\|^2}{2\beta}\right\}$$

$$\phi(u) \cdot \phi(v)$$

$$\int_{\mathbb{R}^d} \frac{1}{(2\pi\beta)^{d/2}} \exp\left\{-\frac{\|u\|^2}{2\beta}\right\} \frac{1}{(2\pi\beta)^{d/2}} \exp\left\{-\frac{\|v\|^2}{2\beta}\right\} dt$$

$$\left(\frac{1}{(2\pi\beta)^{d/2}}\right)^2 \int_{\mathbb{R}^d} \exp\left\{-\frac{\|u\|^2}{2\beta}\right\} \exp\left\{-\frac{\|v\|^2}{2\beta}\right\} dt$$

$$\frac{1}{(2\pi\beta)^d} \int_{\mathbb{R}^d} \exp\left\{-\frac{\|u\|^2 + \|v\|^2}{2\beta}\right\} dt$$

$$\frac{1}{(2\pi\beta)^d} \int_{\mathbb{R}^d} \exp \left\{ - \frac{\|u\|^2 - 2u^\top t + \|t\|^2 + \|v\|^2 - 2v^\top t + \|t\|^2}{2\beta^1} \right\} dt$$

$$\frac{1}{(2\pi\beta^1)^d} \int_{\mathbb{R}^d} \exp \left\{ - \frac{\|u\|^2 - 2(u+v)^\top t + \|v\|^2 + 2\|t\|^2}{2\beta^1} \right\} dt$$

$$\frac{1}{(2\pi\beta^1)^d} \int_{\mathbb{R}^d} \exp \left\{ - \frac{\|t - \frac{u+v}{2}\|^2 - \|\frac{u+v}{2}\|^2 + \frac{\|u\|^2}{2} + \frac{\|v\|^2}{2}}{\beta^1} \right\} dt$$

$$\frac{1}{(2\pi\beta^1)^d} \int_{\mathbb{R}^d} \exp \left\{ - \frac{\|t - \frac{u+v}{2}\|^2}{\beta^1} \right\} \exp \left\{ - \frac{-\|\frac{u+v}{2}\|^2 + \frac{\|u\|^2}{2} + \frac{\|v\|^2}{2}}{\beta^1} \right\} dt$$

$$\frac{1}{(2\pi\beta^1)^d} \exp \left\{ - \frac{-\|\frac{u+v}{2}\|^2 + \frac{\|u\|^2}{2} + \frac{\|v\|^2}{2}}{\beta^1} \right\} \int_{\mathbb{R}^d} \exp \left\{ - \frac{\|t - \frac{u+v}{2}\|^2}{\beta^1} \right\} dt$$

$$\frac{1}{(2\pi\beta^1)^d} \exp \left\{ - \frac{\|u\|^2 + \|v\|^2 - \|\frac{u+v}{2}\|^2}{2\beta^1} \right\} \quad \text{Integrated out to 1}$$

$$\frac{1}{(2\pi\beta^1)^d} \exp \left\{ - \frac{\|u\|^2 + \|v\|^2 - \|\frac{u+v}{2}\|^2 - \|u^2 + 2uv + v^2\|}{2\beta^1} \right\}$$

$$\frac{1}{(2\pi\beta^1)^d} \exp \left\{ - \frac{\|u-v\|^2}{2\beta^1} \right\} = \alpha \exp \left\{ - \frac{\|u-v\|^2}{\beta} \right\}$$

$$\frac{1}{(2\pi\beta^1)^d} = \alpha$$

$$2\beta^1 = \beta$$

Problem 3A – Nearest Neighbor

Accuracy for Each K

Accuracy

K = 1 0.948

K = 2 0.93

K = 3 0.938

K = 4 0.946

K = 5 0.946

Misclassified Examples for K = 1

Test - Class:0



Nearest Neighbor - Class:3



Test - Class:2



Nearest Neighbor - Class:6



Test - Class:8



Nearest Neighbor - Class:2



Misclassified Examples for K = 3

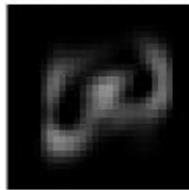
Test - Class:7



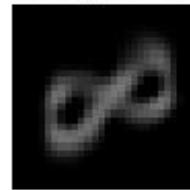
Nearest Neighbor - Class:9



Test - Class:3



Nearest Neighbor - Class:8



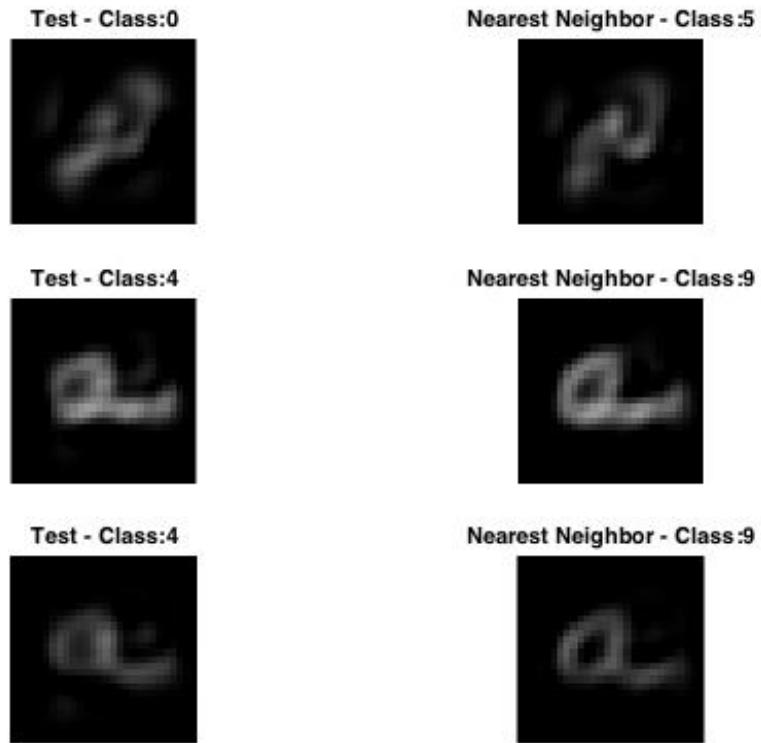
Test - Class:9



Nearest Neighbor - Class:6



Misclassified Examples for K = 5



Problem 3B - Bayes Classifier

Gaussian Mean for Class 2 (Index 3 in MATLAB):

-1.7275
-0.6510
0.8067
0.1514
-2.3694
-0.6690
1.2293
0.1673
-0.4831
-0.4152
0.3724
0.3473
-0.1608
0.0482
-0.0016
0.1905
0.3605
0.2868

0.1755
-0.1066

Covariance for Class 2 (Index 3 in MATLAB)

Columns 1 through 9

0.6327	-0.0674	-0.3085	-0.0855	0.1087	0.1076	0.0714	-0.1319	-0.1730
-0.0674	0.5280	0.1827	-0.0030	0.1033	0.0219	-0.1021	0.0476	-0.0051
-0.3085	0.1827	0.6964	-0.0438	-0.0247	-0.0476	-0.2826	0.1192	0.1249
-0.0855	-0.0030	-0.0438	0.3469	0.0226	-0.0975	0.0108	-0.0544	0.0798
0.1087	0.1033	-0.0247	0.0226	0.4303	0.1467	-0.1112	-0.0094	0.0286
0.1076	0.0219	-0.0476	-0.0975	0.1467	0.3355	0.0107	-0.0072	-0.1672
0.0714	-0.1021	-0.2826	0.0108	-0.1112	0.0107	0.5913	-0.0161	-0.1333
-0.1319	0.0476	0.1192	-0.0544	-0.0094	-0.0072	-0.0161	0.2929	0.1455
-0.1730	-0.0051	0.1249	0.0798	0.0286	-0.1672	-0.1333	0.1455	0.4684
0.0374	0.0021	-0.1319	0.0219	0.0435	-0.0084	0.0029	0.0816	0.0968
-0.1793	0.1791	0.1546	-0.0291	0.0603	0.0598	-0.0975	0.0198	-0.0110
-0.0199	-0.1218	-0.0078	-0.0600	-0.0255	0.0591	-0.0822	0.0463	-0.0036
0.0454	-0.0451	-0.0607	0.0497	0.0776	0.0039	0.0211	-0.1151	-0.0132
-0.0765	0.0663	0.0933	0.0518	0.0606	-0.0004	0.0193	-0.0116	0.0283
-0.0127	-0.0559	-0.0777	0.0586	0.0573	0.0215	-0.0264	0.0165	0.0622
0.0063	-0.0824	-0.1118	-0.0073	0.0286	0.0066	0.1082	0.0837	0.1013
-0.0419	0.0762	0.1523	-0.0580	0.0146	-0.0201	-0.0680	0.0059	0.0146
-0.0210	-0.0321	0.0902	0.1036	-0.0672	-0.1439	-0.1111	0.0022	0.1372
0.0539	-0.0817	-0.1392	0.0191	-0.0081	-0.0112	0.0990	-0.0261	-0.0050
-0.0028	0.0505	0.0309	0.0050	0.0448	-0.0228	0.0105	-0.0327	0.0040

Columns 10 through 18

0.0374	-0.1793	-0.0199	0.0454	-0.0765	-0.0127	0.0063	-0.0419	-0.0210
0.0021	0.1791	-0.1218	-0.0451	0.0663	-0.0559	-0.0824	0.0762	-0.0321
-0.1319	0.1546	-0.0078	-0.0607	0.0933	-0.0777	-0.1118	0.1523	0.0902
0.0219	-0.0291	-0.0600	0.0497	0.0518	0.0586	-0.0073	-0.0580	0.1036
0.0435	0.0603	-0.0255	0.0776	0.0606	0.0573	0.0286	0.0146	-0.0672
-0.0084	0.0598	0.0591	0.0039	-0.0004	0.0215	0.0066	-0.0201	-0.1439
0.0029	-0.0975	-0.0822	0.0211	0.0193	-0.0264	0.1082	-0.0680	-0.1111
0.0816	0.0198	0.0463	-0.1151	-0.0116	0.0165	0.0837	0.0059	0.0022
0.0968	-0.0110	-0.0036	-0.0132	0.0283	0.0622	0.1013	0.0146	0.1372
0.2342	-0.0108	0.0514	-0.0309	-0.0013	0.0473	0.1072	-0.0510	0.0289
-0.0108	0.3830	-0.0171	-0.0087	0.0630	-0.0104	-0.0319	0.0872	-0.0738
0.0514	-0.0171	0.2155	-0.0394	-0.0077	0.0198	0.0250	-0.0319	0.0172
-0.0309	-0.0087	-0.0394	0.2541	0.0309	0.0221	-0.0222	-0.0355	0.0087
-0.0013	0.0630	-0.0077	0.0309	0.2108	-0.0236	-0.0225	0.0277	-0.0162
0.0473	-0.0104	0.0198	0.0221	-0.0236	0.2083	0.0558	-0.0469	-0.0164
0.1072	-0.0319	0.0250	-0.0222	-0.0225	0.0558	0.2198	-0.0287	0.0271

-0.0510	0.0872	-0.0319	-0.0355	0.0277	-0.0469	-0.0287	0.1774	0.0016
0.0289	-0.0738	0.0172	0.0087	-0.0162	-0.0164	0.0271	0.0016	0.2896
-0.0008	-0.0524	0.0043	0.0438	-0.0229	0.0250	0.0168	-0.0304	0.0036
-0.0248	0.0105	-0.0642	0.0717	0.0274	-0.0146	-0.0214	0.0225	-0.0288

Columns 19 through 20

0.0539	-0.0028
-0.0817	0.0505
-0.1392	0.0309
0.0191	0.0050
-0.0081	0.0448
-0.0112	-0.0228
0.0990	0.0105
-0.0261	-0.0327
-0.0050	0.0040
-0.0008	-0.0248
-0.0524	0.0105
0.0043	-0.0642
0.0438	0.0717
-0.0229	0.0274
0.0250	-0.0146
0.0168	-0.0214
-0.0304	0.0225
0.0036	-0.0288
0.1761	-0.0087
-0.0087	0.1288

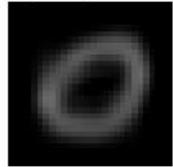
Confusion Matrix for Bayes Classifier

	Zero	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
Zero	48	0	0	1	0	1	0	0	0	0
One	0	49	0	0	0	0	0	0	1	0
Two	0	0	48	0	1	0	1	0	0	0
Three	0	0	1	47	0	0	0	0	2	0
Four	0	0	0	0	48	0	0	0	1	1
Five	0	0	0	1	0	45	2	0	1	1
Six	0	0	0	0	1	5	43	0	0	1
Seven	0	0	2	0	2	0	0	46	0	0
Eight	0	0	1	0	0	1	0	0	47	1
Nine	1	0	0	0	2	0	0	0	0	47

Accuracy = 0.9360

Average Image for Each Class:

Average Image for Class=0



Average Image for Class=1



Average Image for Class=2



Average Image for Class=3



Average Image for Class=4



Average Image for Class=5



Average Image for Class=6



Average Image for Class=7



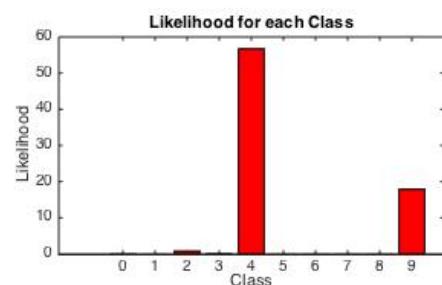
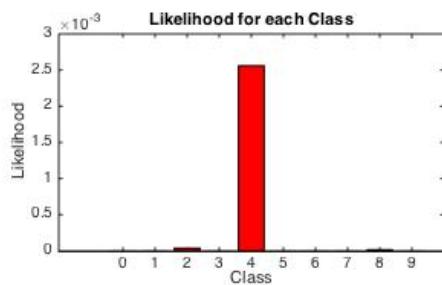
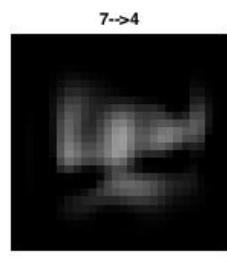
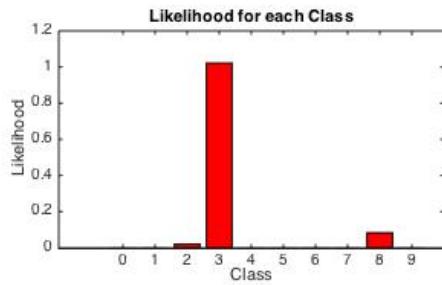
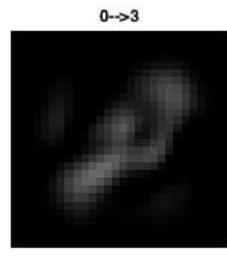
Average Image for Class=8



Average Image for Class=9

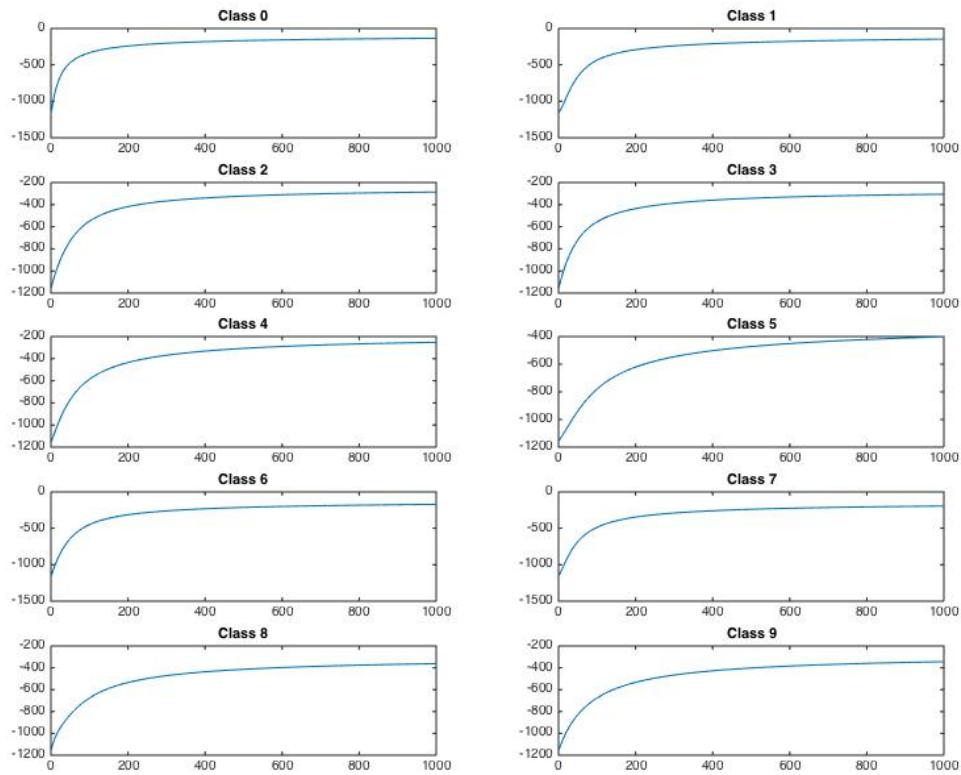


Three Misclassified Examples and their Distributions:



Problem 3C – Multiclass Softmax Logistic Regression

Likelihood as a function of Iteration:



Confusion Matrix for Softmax Classifier

	Zero	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
Zero	46	0	1	1	0	0	2	0	0	0
One	0	49	0	0	0	0	0	0	1	0
Two	0	0	38	2	1	0	4	0	5	0
Three	1	0	2	39	0	2	0	1	5	0
Four	0	0	1	0	42	1	0	0	1	5
Five	1	1	0	4	2	39	1	0	0	2
Six	0	0	1	0	4	3	42	0	0	0
Seven	0	0	3	0	1	0	0	44	1	1
Eight	0	0	0	0	0	2	1	0	46	1
Nine	0	1	1	0	3	0	0	1	0	44

Accuracy = 0.8580

Three Misclassified Examples and their Distributions:

