EE 5111: Estimation

Jan - May 2025

Mini Project 1

March 23, 2025

1 Performance of MLE

The aim of this exercise is to study the variation in the performance of MLE with increase in the number of samples. Consider the following equation:

$$x_i = A + n_i \qquad\qquad i = 1, ..., N \tag{1}$$

where A is scalar and n_i are the noise samples. Compute the maximum likelihood estimate of A for the following cases:

1. $n_i \sim \mathcal{N}(0, 1)$. In this case, use the following expression derived in class:
$$\hat{A} = \frac{1}{N}\sum_{i=1}^{N} x_i.$$
2. $n_i \sim Lap(0, 1/\sqrt{2})$, i.e., Laplace distribution with zero mean and unit variance. In this case, the MLE is derived as:
$$\hat{A} = median(x_i).$$
3. $n_i \sim Cauchy(0, \gamma)$. Use $\gamma = \sqrt{2C_g}$ where $C_g = 1.78$. The closed form solution for MLE is not available and hence, MLE should be computed through numerical evaluation. Use Newton Raphson or any other appropriate numerical method.

Repeat the above experiments for $N = 1, 10, 100, 1000, 10000$ and for $A = 1$ and $A = 10$. Here, N is the number of samples considered for estimation.
Present the following for each noise distribution:

1. Tabulate the values of $\mathbb{E}[\hat{A}]$ against the number of samples for both values of A. What do you infer?
2. Tabulate the values of $Var(\hat{A})$ against the number of samples for both values of A. What do you infer?
3. Plot the CDF of the estimate for $N = 1, 10, 100, 1000, 10000$ samples for $A = 1$. Ensure that you take enough realizations to get a smooth CDF. What can you say about the CDF? Justify. Do you observe the following relation?
$$\sqrt{N}(\hat{A} - A) \sim \mathcal{N}(0, I(A)^{-1})$$
Here, I denotes the Fisher information.

4. Plot the PDF of the estimate for $N = 1, 10, 100, 1000, 10000$ samples for $A = 1$. Ensure that you take enough realizations to get a smooth PDF. What can you say about the PDF convergence?

2 Submission

You are required submit this problem no later than 7 Apr 2025 with a single submission per group. The submission is online and after that you will have an in class evaluation of both the code and your understanding around April 15.