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Aim:

To write a program in PYTHON to implement DCT transform

Theory:

If T is the transformation matrix and T' is its transpose, forward transform of any 2D matrix f , is given by $F = T f T'$

Similarly, inverse transform is given by $f_{new} = T' F T$

Discrete Cosine Transform

The N X N Cosine Transform Matrix is given by the following expression $C(u,v) = \begin{cases} \sqrt{1/N}, & u=0, 0 \leq v \leq N-1 \\ \sqrt{2/N} \cos(\pi(2v+1)u/2N), & 1 \leq u \leq N-1, 0 \leq v \leq N-1 \end{cases}$

Conclusion:

1. Understand the significance of transforms in image processing
2. Implement forward and inverse DCT
3. Appreciate the energy conservation and energy compaction property of DCT
4. Realize and comprehend the effect of DCT in compression

```
import numpy as np
from scipy.fftpack import dct, idct
import random
import cv2
import matplotlib.pyplot as plt
```

#Function to implement 2D DCT and IDCT

```
def dct2(a):
    return dct(dct(a, axis=0, norm='ortho'), axis=1, norm='ortho')
```

```
def idct2(a):
    return idct(idct(a, axis=0, norm='ortho'), axis=1, norm='ortho')
```

DCT and IDCT on a random matrix

Generate a random integer matrix of size x X y

```
x=8
y=8
f= np.random.randint(9,11,size=(x,y))
F= dct2(f)
print(f)
print(F)
```

```
[[10  9  9 10 10 10 10  9]
 [ 9  9 10 10 10  9  9  9]
 [10  9  9  9  9  9  9 10]
 [ 9 10 10  9 10 10 10 10]
 [10  9 10 10 10 10 10  9]
 [ 9  9 10 10 10 10 10  9]
 [10 10  9  9  9 10 10  9]
 [ 9 10  9 10  9  9 10 10]]
[[ 7.63750000e+01 -3.65785240e-01 -4.89961112e-01  4.23826698e-01
 -1.25000000e-01  5.83864389e-01 -2.02948538e-01  1.82138582e-01]
 [-2.68240079e-01  1.70985926e-01 -5.19268845e-01  1.61087823e-01
  8.39994096e-01 -5.19606398e-02  5.62569800e-01  4.98168374e-01]
 [-4.89961112e-01  1.35179967e-01  2.13388348e-01  3.07748308e-02
  3.54662087e-01  2.98348201e-01 -7.65165043e-01 -6.79595585e-01]
 [ 7.01611815e-01  5.27014470e-02 -1.36006227e+00  1.07054312e+00
 -2.80194019e-01  8.76044103e-01  1.96784560e-01  2.87395619e-01]
 [ 8.75000000e-01 -4.18576301e-01 -2.80213456e-02  6.08829528e-02
 -1.25000000e-01  4.06806885e-02 -3.94290254e-01 -8.32600029e-02]
 [ 1.68129583e-01 -2.14172314e-01  1.10313323e+00  8.76044103e-01
 -6.12434706e-01 -4.66989729e-01 -5.09770951e-02 -2.27628639e-01]
 [-2.02948538e-01 -4.34399265e-01  2.34834957e-01 -1.59455643e-01
  5.29589279e-01  2.38642234e-01  3.66116524e-02 -1.83952547e-01]
 [-3.08254059e-01  4.98168374e-01  9.36235568e-01  8.32290726e-01
  5.27565125e-01  5.87864518e-01  2.33115677e-01 -2.74539316e-01]]
```

#Find Energy of the original image

```

fenergy= np.sum(f**2)
print(fenergy)

5849

#Find Energy of the DCT of the image
Fenergy= np.sum(F**2)
print(Fenergy)

5849.000000000002

#Energy in the DC coefficient
F[0,0]**2

5833.140625000002

#Retain coefficients with energy more that 0.01% of the Total energy
P= 0.0001*Fenergy
print(P)

0.5849000000000002

Fnew= np.where(F**2 > P, F,0)
print(Fnew)

[[76.375      0.          0.          0.          0.          0.
   0.          0.          ]
 [ 0.          0.          0.          0.          0.8399941  0.
   0.          0.          ]
 [ 0.          0.          0.          0.          0.          0.
  -0.76516504  0.          ]
 [ 0.          0.         -1.36006227  1.07054312  0.          0.8760441
   0.          0.          ]
 [ 0.875      0.          0.          0.          0.          0.
   0.          0.          ]
 [ 0.          0.          1.10313323  0.8760441  0.          0.
   0.          0.          ]
 [ 0.          0.          0.          0.          0.          0.
   0.          0.          ]
 [ 0.          0.          0.93623557  0.83229073  0.          0.
   0.          0.          ]]

# Finding the nonzero values in the compressed matrix
ind,val= np.unique(Fnew,return_counts=True)
nonzeros = np.sum(val[np.where(ind!=0)])
print(nonzeros)

11

#Compression Ratio
Original = x*y
Compressed = nonzeros
CR= Original/Compressed
print(CR)

5.818181818181818

#retrieving the estimate of the original matrix
fnew= idct2(Fnew)
fnew= fnew.astype(int)
print(fnew)

[[10  9  9  9 10  9  9  9]
 [ 8  9  9 10  9  9  9  9]
 [ 9  9  9  8  9  9  9 10]
 [ 9  9  9  9  9  9  9 10]
 [ 9  9  9  9  9 10  9  9]
 [ 9  9  9 10  9  9  9  8]
 [ 9  9  8  8  9  9  9  9]
 [ 9 10  9  9  9  9  9  9]]

#Obtain Mean Square Error between f and fnew
mse= np.sum(((f-fnew)**2))/(x*y)
print(mse)

0.5

# Retain first row of the transformed matrix
Fnew1= np.zeros((x,y), dtype =float)
Fnew1[0,:]= F[0,:]
print(Fnew1)

[[76.375      -0.36578524 -0.48996111  0.4238267  -0.125      0.58386439
  -0.20294854  0.18213858]
 [ 0.          0.          0.          0.          0.          0.
   0.          0.          ]

```

```

    0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
[ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
]]

```

```

fnew1= idct2(Fnew1)
fnew1= fnew1.astype(int)
print(fnew1)

```

```

[[9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]]

```

```

# Retain first row and first column of the transformed matrix
Fnew2= np.zeros((x,y), dtype =float)
Fnew2[0,:]= F[0,:]
Fnew2[:,0]= F[:,0]
print(Fnew2)

```

```

[[76.375      -0.36578524 -0.48996111  0.4238267  -0.125      0.58386439
  -0.20294854  0.18213858]
 [-0.26824008  0.      0.      0.      0.      0.
  0.      0.      ]
 [-0.48996111  0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.70161181  0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.875      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.16812958  0.      0.      0.      0.      0.
  0.      0.      ]
 [-0.20294854  0.      0.      0.      0.      0.
  0.      0.      ]
 [-0.30825406  0.      0.      0.      0.      0.
  0.      0.      ]
]]

```

```

fnew2= idct2(Fnew2)
fnew2= fnew2.astype(int)
print(fnew2)

```

```

[[9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]
 [9 9 9 9 9 9 9]]

```

```

#Retain the first four coefficients
Fnew3= np.zeros((x,y), dtype =float)
Fnew3[0:2,0:2]= F[0:2,0:2]
print(Fnew3)

```

```

[[76.375      -0.36578524  0.      0.      0.      0.
  0.      0.      ]
 [-0.26824008  0.17098593  0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
 [ 0.      0.      0.      0.      0.      0.
  0.      0.      ]
]]

```

```

fnew3= idct2(Fnew3)
fnew3= fnew3.astype(int)
print(fnew3)

```

```

[[9 9 9 9 9 9 9]

```

```
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
[9 9 9 9 9 9 9]
```

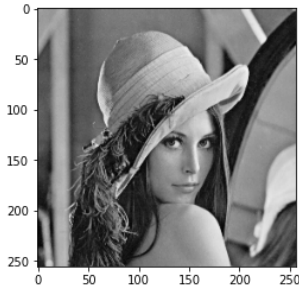
DCT on image with and without compression

```
img = cv2.imread('/content/lena_gray_256.tif',0) # Read the image
m,n= img.shape #Find and store size of image in m,n
print(m,n)
```

```
256 256
```

```
plt.imshow(img, cmap="gray")
```

```
<matplotlib.image.AxesImage at 0x7fea43752bd0>
```



```
#Find DCT of the given image using DCT function
```

```
N=8 #Size of the block
```

```
imgdct = np.zeros((m,n), dtype=int)
```

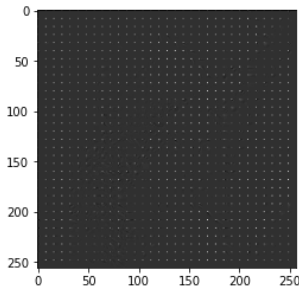
```
for row in range(m//N):
```

```
    for col in range(n//N):
```

```
        imgdct[row*N:(row+1)*N,col*N:(col+1)*N]= dct2(img[row*N:(row+1)*N,col*N:(col+1)*N])
```

```
plt.imshow(imgdct, cmap="gray")
```

```
<matplotlib.image.AxesImage at 0x7fea455e0e10>
```



```
#Find DCT of the given image using DCT function
```

```
N=2 #Size of the block
```

```
imgdct = np.zeros((m,n), dtype=int)
```

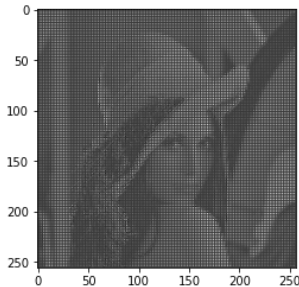
```
for row in range(m//N):
```

```
    for col in range(n//N):
```

```
        imgdct[row*N:(row+1)*N,col*N:(col+1)*N]= dct2(img[row*N:(row+1)*N,col*N:(col+1)*N])
```

```
plt.imshow(imgdct, cmap="gray")
```

```
<matplotlib.image.AxesImage at 0x7fea43347550>
```



```
#Find DCT of the given image using DCT function
```

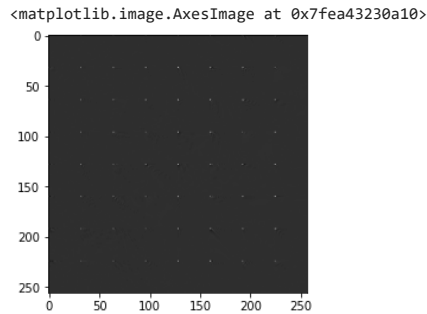
```
N=32 #Size of the block
```

```
imgdct = np.zeros((m,n), dtype=int)
```

```
for row in range(m//N):
```

```
    for col in range(n//N):
```

```
imgdct[row*N:(row+1)*N,col*N:(col+1)*N]= dct2(img[row*N:(row+1)*N,col*N:(col+1)*N])
plt.imshow(imgdct, cmap="gray")
```



```
#Total energy of the image
energy= np.sum(img**2)
print(energy)
```

```
6941402
```

```
#For compression, retaining only P percent coefficients with max magnitude.
P= 0.0001*energy
Inew= np.where(imgdct**2 > P, imgdct,0)
```

```
# Finding the nonzero values in the compressed matrix
ind,val= np.unique(Inew,return_counts=True)
nonzeros = np.sum(val[np.where(ind!=0)])
print(nonzeros)
```

```
5585
```

```
#finding nonzero values in the compressed matrix
nonzeros= np.sum([Inew!=0])
print(nonzeros)
```

```
5585
```

```
#Compression Ratio
CR= (m*n)/nonzeros
print("total Coefficient in the input image is ", m*n)
print("total coefficients retained in the output image is ", nonzeros)
print("compression ratio = input size/output size = ", CR)
```

```
total Coefficient in the input image is 65536
total coefficients retained in the output image is 5585
compression ratio = input size/output size = 11.734288272157565
```

```
#Find IDCT of the given image using IDCT function
inew = np.zeros((m,n), dtype=int)
for row in range(m//N):
    for col in range(n//N):
```

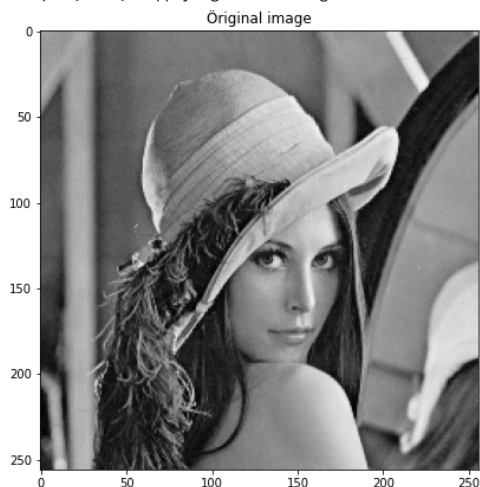
```
inew[row*N:(row+1)*N,col*N:(col+1)*N]= idct2(Inew[row*N:(row+1)*N,col*N:(col+1)*N])
```

```
plt.imshow(inew,cmap="gray")
```



```
fig = plt.figure(figsize=(15,15),facecolor='w')
plt.subplot(2,2,1)
plt.imshow(img, cmap="gray")
plt.title("Original image")
plt.subplot(2,2,2)
plt.imshow(inew, "gray")
plt.title("Applying Dct on original with coefficients with energy more that 0.01%")
```

```
Text(0.5, 1.0, 'Applying Dct on original with coefficients with energy more that 0.01%')
```



Here you can see a faint difference between the two images.

```
#For compression, retaining only P percent coefficients with max magnitude.
P= 0.0005*energy
Inew1= np.where(imgdct**2 > P, imgdct,0)
```

```
# Finding the nonzero values in the compressed matrix
ind,val= np.unique(Inew,return_counts=True)
nonzeros = np.sum(val[np.where(ind!=0)])
print(nonzeros)
```

```
2595
```

```
#finding nonzero values in the compressed matrix
nonzeros= np.sum([Inew!=0])
print(nonzeros)
```

```
2595
```

```
#Compression Ratio
CR= (m*n)/nonzeros
print("total Coefficient in the inpur image is ", m*n)
print("total coefficients retained in the output image is ", nonzeros)
print("compression ratio = input size/output size = ", CR)
```

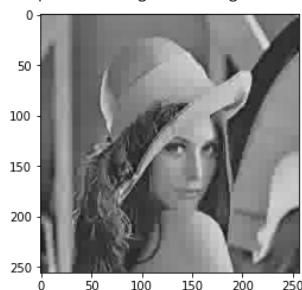
```
total Coefficient in the inpur image is 65536
total coefficients retained in the output image is 2595
compression ratio = input size/output size = 25.254720616570328
```

```
#Find IDCT of the given image using IDCT function
inew1 = np.zeros((m,n), dtype=int)
for row in range(m//N):
    for col in range(n//N):
```

```
        inew1[row*N:(row+1)*N,col*N:(col+1)*N]= idct2(Inew1[row*N:(row+1)*N,col*N:(col+1)*N])
```

```
plt.imshow(inew1,cmap="gray")
```

```
<matplotlib.image.AxesImage at 0x7fea43587c10>
```



```
#For compression, retaining only P percent coefficients with max magnitude.
P= 0.1*energy
Inew2= np.where(imgdct**2 > P, imgdct,0)
```

```

# Finding the nonzero values in the compressed matrix
ind,val= np.unique(Inew,return_counts=True)
nonzeros = np.sum(val[np.where(ind!=0)])
print(nonzeros)

2595

#finding nonzero values in the compressed matrix
nonzeros= np.sum([Inew!=0])
print(nonzeros)

2595

#Compression Ratio
CR= (m*n)/nonzeros
print("total Coefficient in the input image is ", m*n)
print("total coefficients retained in the output image is ", nonzeros)
print("compression ratio = input size/output size = ", CR)

total Coefficient in the input image is 65536
total coefficients retained in the output image is 2595
compression ratio = input size/output size = 25.254720616570328

#Find IDCT of the given image using IDCT function
inew2 = np.zeros((m,n), dtype=int)
for row in range(m//N):
    for col in range(n//N):

        inew2[row*N:(row+1)*N,col*N:(col+1)*N]= idct2(Inew2[row*N:(row+1)*N,col*N:(col+1)*N])

plt.imshow(inew2,cmap="gray")

<matplotlib.image.AxesImage at 0x7fea434fd7d0>

#For compression, retaining only P percent coefficients with max magnitude.
P= 0.34675*energy
Inew3= np.where(imgdct**2 > P, imgdct,0)

# Finding the nonzero values in the compressed matrix
ind,val= np.unique(Inew,return_counts=True)
nonzeros = np.sum(val[np.where(ind!=0)])
print(nonzeros)

2595

#finding nonzero values in the compressed matrix
nonzeros= np.sum([Inew!=0])
print(nonzeros)

2595

#Compression Ratio
CR= (m*n)/nonzeros
print("total Coefficient in the input image is ", m*n)
print("total coefficients retained in the output image is ", nonzeros)
print("compression ratio = input size/output size = ", CR)

total Coefficient in the input image is 65536
total coefficients retained in the output image is 2595
compression ratio = input size/output size = 25.254720616570328

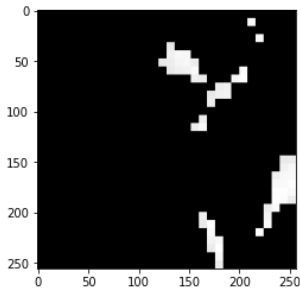
#Find IDCT of the given image using IDCT function
inew3 = np.zeros((m,n), dtype=int)
for row in range(m//N):
    for col in range(n//N):

        inew3[row*N:(row+1)*N,col*N:(col+1)*N]= idct2(Inew3[row*N:(row+1)*N,col*N:(col+1)*N])

plt.imshow(inew3,cmap="gray")

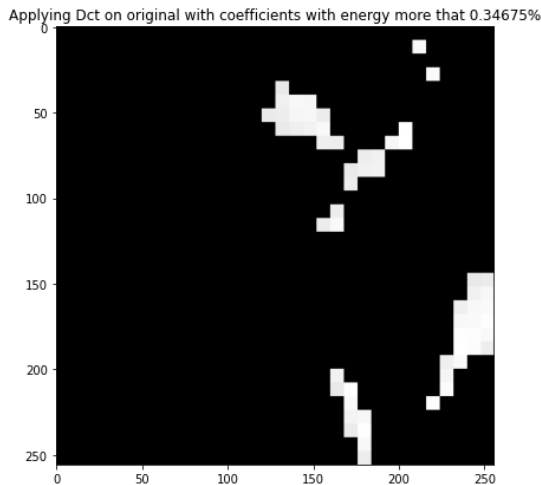
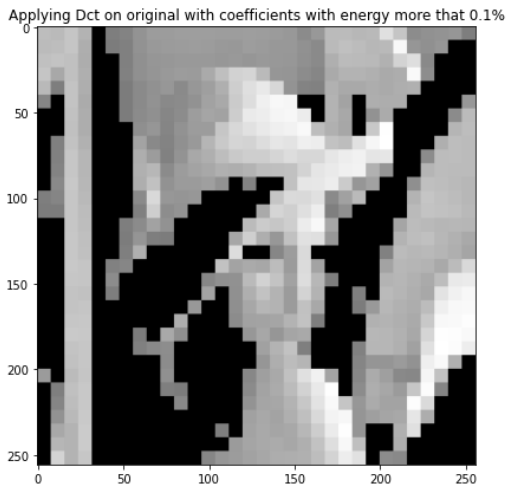
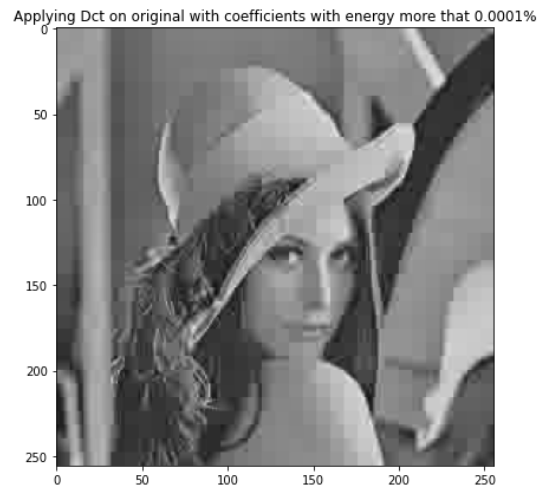
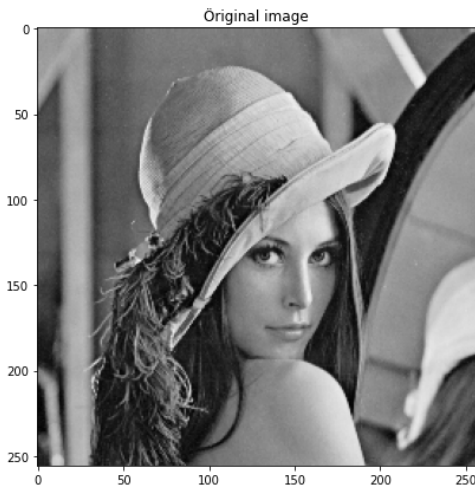
```

```
<matplotlib.image.AxesImage at 0x7fea43462e50>
```



```
fig = plt.figure(figsize=(15,15),facecolor='w')
plt.subplot(2,2,1)
plt.imshow(img, cmap="gray")
plt.title("Original image")
plt.subplot(2,2,2)
plt.imshow(inew, "gray")
plt.title("Applying Dct on original with coefficients with energy more that 0.0001%")
plt.subplot(2,2,3)
plt.imshow(inew1, "gray")
plt.title("Applying Dct on original with coefficients with energy more that 0.0005%")
plt.subplot(2,2,3)
plt.imshow(inew2, "gray")
plt.title("Applying Dct on original with coefficients with energy more that 0.1%")
plt.subplot(2,2,4)
plt.imshow(inew3, "gray")
plt.title("Applying Dct on original with coefficients with energy more that 0.34675%")
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:11: MatplotlibDeprecationWarning: Adding an axes using the same arguments as a previous axes
# This is added back by InteractiveShellApp.init_path()
Text(0.5, 1.0, 'Applying Dct on original with coefficients with energy more that 0.34675%')
```



So from the above 4 images we can infer that with energy more than 0.0001% the image is almost similar to the original image only the greyish tinge is a bit less with 0.1% the image is very blurred and we can see patches of grey and black and white and with 0.34675% we can see only black and white and the image is completely distorted

Mean Square Error(Mse)

```
def mse(img, inew):
    #the squared image between the 2 images is the sum of the squared difference between the two images
    err = np.sum((img.astype("float") - inew.astype("float")) ** 2)
    err /= float(img.shape[0] * inew.shape[1])
    return err
```

#the lower the error, the more "similar" it is.

m = mse(imgdct, img)

print("The Mean Square error is: ",m)

The Mean Square error is: 34331.6139831543

```
def mse(img, inew1):
    #the squared image between the 2 images is the sum of the squared difference between the two images
    err = np.sum((img.astype("float") - inew1.astype("float")) ** 2)
    err /= float(img.shape[0] * inew1.shape[1])
    return err
```

#the lower the error, the more "similar" it is.

m = mse(imgdct, inew1)

print("The Mean Square error is: ",m)

The Mean Square error is: 34068.562744140625

```
def mse(img, inew2):
    #the squared image between the 2 images is the sum of the squared difference between the two images
    err = np.sum((img.astype("float") - inew2.astype("float")) ** 2)
    err /= float(img.shape[0] * inew2.shape[1])
    return err
```

#the lower the error, the more "similar" it is.

m1 = mse(imgdct, inew2)

print("The Mean Square error is: ",m1)

The Mean Square error is: 31761.30108642578

```
def mse(img, inew3):
    #the squared image between the 2 images is the sum of the squared difference between the two images
    err = np.sum((img.astype("float") - inew3.astype("float")) ** 2)
    err /= float(img.shape[0] * inew3.shape[1])
    return err
```

#the lower the error, the more "similar" it is.

m = mse(imgdct, inew3)

print("The Mean Square error is: ",m)

☞ The Mean Square error is: 19943.941802978516