Quantifying the Phenomenon of Momentum

Practice makes perfect. At least, that's what we all hear growing up. But is this statement true at the highest level of sports? If it is, how can one athlete be a champion and another be only a runner up? How can the greatest gymnast of all time get the "twisties"? Endless hours of practice can be overshadowed by one thing: a mental block. Doesn't it subsequently stand to reason that at any level of sports, being in the right head space and remaining confident in one's abilities is vital?

"Tennis is a mental game." A quote from 2023 Grand Slam Runner-Up himself, Novak Djokovic, showcasing that even one of the greatest tennis players believes that having a mental edge over their opponent can help them secure victory [12]. The implication that the mental strength of a player is just as vital to winning as the hours of training is indicative of the phenomenon of momentum. Being able to hold onto control of the match/maintain momentum and bounce back from the doubts that arise when an opponent is on a roll is essential to win a match.

We have been tasked to develop a viable model for this psychological marvel that will allow players and coaches to predict momentum swings in a match as well as identify which player is performing better at any given moment due to said momentum. In order to create an objective metric, we treat momentum as a latent variable dependent on the number of points scored in a given match. Additionally, we bias the scores to account for the intrinsic advantage possessed by the server of a point in tennis. We then create a continuous function with respect to time to model the flow of momentum in a tennis match.

After creating this metric, we employ A/B hypothesis testing to show that gaining momentum leads to a statistically significant advantage in a game of tennis that cannot be attributed to pure randomness. This is accomplished by calculating the total momentum associated with the winner and loser of each match in the 2023 Wimbledon tournament and determining whether or not they arise from the same distribution. With a strict significance level of 0.02, we show that possessing momentum leads to a statistically significant advantage with an empirical p-value of 0.0002. Because the p-value is so low, we conclude that the observed difference in the mean momentum between winners and losers is not due to random chance.

Upon instantiating momentum as a legitimate phenomenon that is worthy of our studies, we seek to predict momentum swings in a match and understand their causes. To accomplish this, we create and train an elastic net regression model and apply it to the Wimbledon data. This model serves as both a variable selection mechanism, by screening out irrelevant variables, and then as a method for predicting swings in matches based off of the selected variables. The variables selected by the elastic net model are analyzed in depth. We provide plausible explanations for the coefficients assigned to variables by the model, and use these explanations as coaching points that can be implemented to exploit momentum in live tennis matches. We find the elastic net model to be the best choice for this use case because it incorporates the λ_1 and λ_2 penalties found in Ridge Regression and LASSO regression models. This allows it to deal with multicollinearity in the independent variables and to screen our irrelevant variables.

To create the model, we split the data given into training, validation, and test data, with the training and validation data used to improve accuracy. We then test the effectiveness of our model in a match which has not been utilized for training. The results show that our model is able to successfully assess the game-flow of a match in real-time, offering valuable insights to coaches.

Contents

1 Introduction

Due to the continuous advancements in computing power and an ever-increasing amount of data, mathematicians are starting to dip their toes into the world of professional sports. Since the early 2000s, enthusiasts have been creating models and identifying patterns to try and make sense of sports [3]. Things like time of possession, home team advantage, and other statistics were being fitted into new equations in an attempt to predict the outcome of the game/match. But what about more abstract ideas in sports? For example: momentum. Many professional athletes will discuss this feeling in the context of giving it partial credit for their success. ”You go through phases, you want to ride the momentum” [9]. This quote from a professional soccer player is referencing his team, who have just won the last two games in a row, and their plans to use the feeling of repeated success to give them an advantage on their next game. Although this is a soccer specific example, the feeling of momentum applies to practically any sport.

Our data comes from the 2023 Wimbledon tournament and contains select matches with detailed records of the events that transpired. We will apply modern methods, such as regression via elastic net, to this data in an attempt to ascertain insight into the complex subject of momentum in tennis.

1.1 Paper Overview

We have been asked to develop a model to aid our understanding of momentum in sports. Once we are successful in doing so, we will be able to identify which player is performing better at a specific moment in time–and by how much. After this development, we will show that momentum has a real impact on the outcome of tennis matches, as opposed to random chance, and therefore is deserving of our studies. Finally, we will attempt to identify which variables momentum is dependent upon, and how coaches can capitalize on these findings. A written memo, detailing advice for controlling the flow of a game using momentum, will be provided at the end of the document for any coaches who wish to apply our work to their own profession. A test of our model will also be provided to demonstrate its practicality and accuracy in a live setting.

1.2 Basis for Momentum Metric

Momentum in sports is an undeniably controversial topic among researchers, with some procuring strong evidence for its existence, and others showing the contrary. In 2009, Lionel Page analyzed over 600,000 tennis matches and found that winning the first set has a significant and strong effect on the result of the second set. He concluded that there was strong evidence for momentum in tennis [11]. However, when Fry and Shukairy searched for similar signs of momentum in the NFL, they found that coming out on top during key plays did not necessarily translate to an advantage during later plays [5]. This agreed with a study by Kniffin and Mihalek that concluded leads in hockey from the first game do not imply wins in the second game of a two-game series in a statistically significant way [7]. Contrary to this, Chen and Fan [2] created a continuous metric for momentum in basketball and found that gaining momentum under this definition had significant effects on the outcome of games.

In light of these studies, we suggest that score frequency is vitally important to the study of momentum. It appears that sports such as tennis and basketball, where points are scored frequently, present evidence for the idea of momentum. Contrarily, sports such as soccer, hockey, and American football, where there are large gaps of time between scores, seem to show little evidence for momentum. This may be because scores in sports with high scoring frequency can be modeled as continuous with respect to time. Then, by viewing momentum as a latent variable (a variable that can only be indirectly inferred from other observable variables [10]) dependent on score, it can also be assumed to be continuous with respect to time. On the other hand, most attempts at modeling momentum in American football depend on a solution that analyzes probabilities of success given different state spaces [8]. A state-space-based model works well for sports that don't have rapid scores because it analyzes the game in discrete phases (or states), basing the next chunk of time solely off of the previous one [8][3]. A further complication in many sports where scores occur infrequently is that momentum may not be immediately reflected in the score, but rather in plays that are conducive to scoring.

This leads us to the model employed in this paper, which is inspired by Chen and Fan [3], and allows us to form a consistent function from a discrete data set. We chose this model after determining that in the 2023 Wimbledon men's games, a point was scored roughly every 45 seconds. Since every rally results in a score for one player or the other, and the rate of point-scoring is similar in timing to basketball, we concluded that this type of model would not only be applicable to our situation, but would also be a much better representation of momentum in tennis than a state space model.

1.3 Chosen Regression Model Basis

After instantiating a model for momentum, we then seek to gain insights about which variables affect momentum in a tennis match. Given the format of our data, where we are provided with detailed information about every point that is scored, we believe that a Linear Regression model is applicable. However, Linear Regression struggles when there is multicollinearity in the independent variables, and is not able to screen out irrelevant variables [14]. The three most popular improvements to Ordinary Linear Regression are Ridge Regression (RR), Least Absolute Shrinkage and Selection Operator (LASSO), and elastic net. Ultimately, it can be shown that RR and LASSO are just special cases of an elastic net [14]. This means that an elastic net model provides the advantages present in both RR and LASSO. By employing this method, we are able to form a regression model that can help us select relevant variables and deal with multicollinearity among those variables. For these reasons, it will be the regression model used in this paper.

2 Modeling Momentum

Before we can analyze how momentum can be exploited by players and coaches, we must first introduce a formal definition to work with. While momentum has been explored thoroughly in the psychological literature, we seek a definition that is dependent on empirical variables, rather than a player's head space. We will assume that when a player gains momentum, their level of play increases, which will be reflected in the score of the match. Because of this, we will focus on the score itself as a metric for the flow of play in a game.

This paper employs the continuous momentum definition proposed by Chen and Fan [2] to model momentum in the game of basketball, which has a similar scoring frequency to tennis. Chen and Fan define momentum for a player as

$$M(s,t,\gamma) = \begin{cases} y(t+s) - y(t) & \text{if } y(t+s) - y(t) \geq \gamma \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Where $y(t)$ represents the difference in points scored at time t, s represents an increment in time, and γ represents the threshold value of momentum. Upon examination of this metric, the intuition is readily available: a player gains momentum when they are able to outscore their opponent by a large margin over a set interval of time [2]. The value of γ should be chosen to capture significant swings in the match while filtering out normal game flow.

To adapt this model for tennis, we must also consider that at the 2023 Wimbledon tournament, players who served a point had a much higher likelihood of scoring the point (0.67 vs 0.33), which is a common phenomena in tennis. Because of this, players are expected to score when they serve, so it is likely that they gain less momentum from winning a point they served than from winning one they received. To accommodate this fact, we will define the score of a player, $s(t)$, as the number of points scored weighted by their probability of winning that point.

$$s(t) = \sum_{m=0}^{n(t)} \frac{1}{p_m} \tag{2}$$

Where $n(t)$ is the total number of points scored by that player at time t and p_m is the probability that the player wins the m^{th} point–which is 0.67 if they player is serving and 0.33 if the player is receiving the serve. Note that we use the overall p_m gathered from the entire field of players, rather than customizing p_m to be reflective of each player's skill at serving. This is simply because our dataset only provides a few matches at best for each player, which may not be reflective of their overall skill at serving. Given a larger dataset, it may be worthwhile to customize p_m for each individual player.

With this definition for the score, we can express $y(t)$ as

$$y(t) = s_1(t) - s_2(t) \tag{3}$$

Where $s_1(t)$ is the weighted score of player 1 at time t and $s_2(t)$ is the weighted score of player 2 at time t.

2.1 Explosiveness and Duration

Because scoring occurs at irregular time intervals, it may be useful to have a standardized metric for momentum, so that we can compare the relative strength of momentum acquired from different periods of time. This can be accomplished with the standardized version of momentum [2]

$$M(s,t,\mu) := \frac{y(t+s) - y(t)}{s} \geq \mu \tag{4}$$

Where the positive real number μ is a slope threshold. This standardized model for momentum is consistent with the previously defined version, and displays some interesting properties.

- **Explosiveness:** The largest possible values of μ represent the most explosive plays.
- **Duration:** The largest s such that the momentum does not fall below μ represents the duration of a momentum episode.

Using these properties alongside the standardized momentum model, we seek to create an algorithm that finds a set of momentum episodes in the data while maximizing for either explosiveness or duration [2]. We set $\mu = 0.02$ to search for momentum intervals in which a player is outscoring their opponent by 1.2 (biased) points per minute. Note that this value is arbitrary, but we chose it since it is able to identify significant swings in the match while filtering out the noise of normal match play. It may be worthwhile, however, to attempt to optimize the value of μ in some way in the future. We set $300 \leq s \leq 1320$ to search for momentum episodes that are roughly between 300 seconds long and 1320 seconds long (5 to 24 minutes). We chose these values because in the Wimbledon data, it is exceptionally rare that any meaningful momentum episode falls outside this duration range. Then, we can use the following algorithm to find momentum episodes:

Algorithm 1 Finding Momentum Episodes Based on Explosiveness or Duration

Input: $y_k = \{t_i | i = 1, 2, ..., I\}$, where t_i represents the starting time of the i^{th} score, and I is the total number of scores in the k^{th} match.
Output: Momentum dataset, $d = \{d_1, d_2, ..., d_N\}$, where N represents the total number of momentum episodes in the match.
for $i = 1 \rightarrow I$ **do** Find all points, s_i, in the interval $[t_i + 300, t_i + 1080]$ and determine if the explosiveness from $t_i \longrightarrow s_i$ is greater than μ
  if $|explosiveness|_{(t_i,s_i)} \geq \mu$ **then**
    $d_k = \{start = t_i, end = s_i, duration = s_i - t_i, explosiveness\}$
  else
    pass
  end if
end for
If there are any overlapping intervals, keep the ones with larger explosiveness/duration, depending on what is desired.

2.2 Visualizing Momentum in a Game

Let us now use this definition of momentum to model the flow of play in a tennis match. We will analyze the match between Carlos Alcaraz (Player 1) and Nicolas Jarry (Player 2), which is the first match in the dataset. We first find $s_1(t)$ and $s_2(t)$ in accordance with equation 2. Next, we find the intervals of momentum in accordance with algorithm (1) defined above.

For the purposes of visualization, we will be maximizing for episode duration, since it creates a more readable plot. Additionally, we will use equation (1), rather than the standardized momentum model for this example to also help with the readability. Note that both the regular and standardized model are consistent with each other, but if we wanted to search for momentum episodes with maximum explosiveness, we need to use the standardized model (equation 4). We set $\gamma = 20$ and $300 \leq s \leq 1320$

and then plot the momentum over top of the biased score difference of the two players over time. The resulting graph is displayed below.

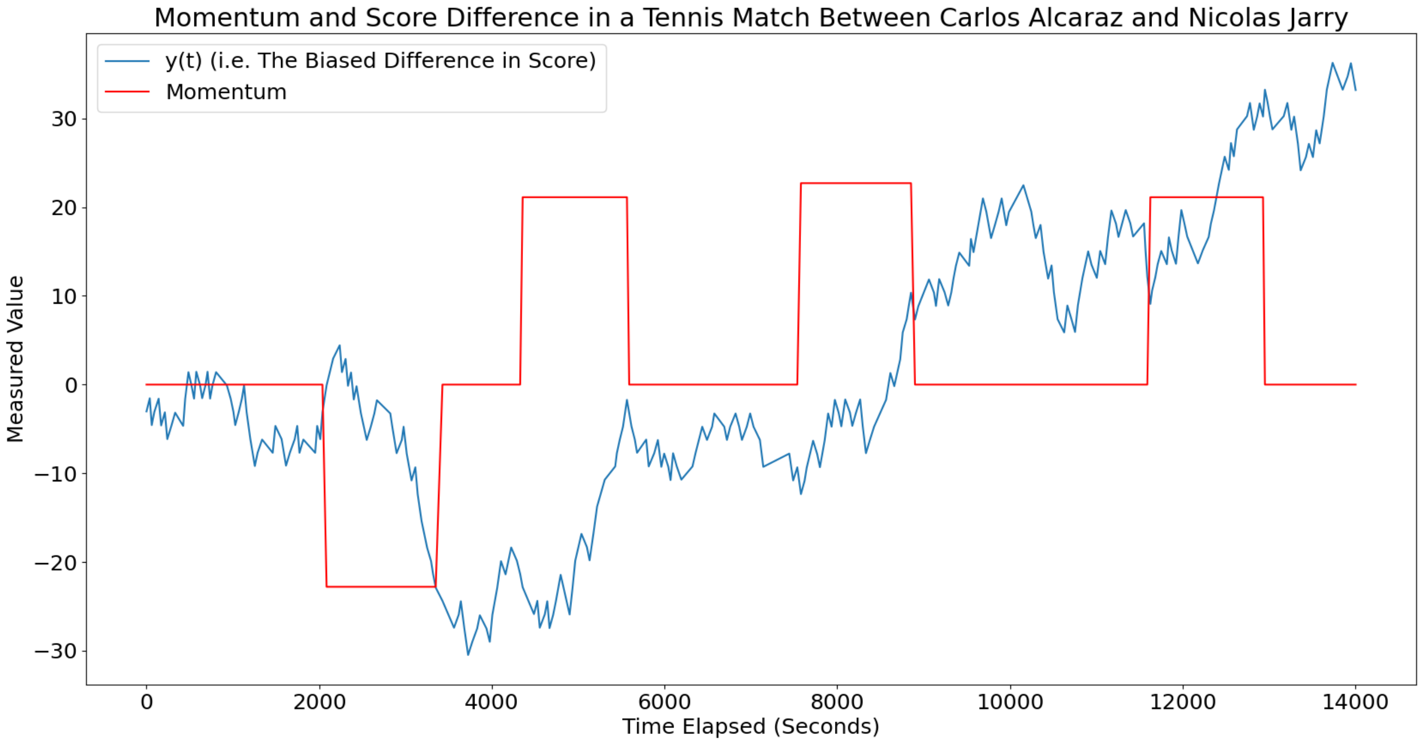

Figure 1: Momentum and Score Difference in a Tennis Match, using $\gamma = 20$ and Maximizing Momentum Episodes by their Duration

This chart is consistent with our expectations. It appears that momentum episodes primarily occur where the score difference between the two players changes rapidly. Additionally, we can see that the player with more momentum episodes was able to dominate the score of the match. First, Carlos Alcaraz took a considerable lead with a large momentum episode, until Nicolas Jarry was able to muster several momentum episodes of his own to catalyze a decisive victory.

3 Disproving the Possibility of Randomness

Some coaches and players may still be unconvinced that momentum plays a meaningful role in a tennis match, arguing that sharp changes in the flow of a game are random. Ultimately, if this is the case, then ”momentum” swings should not have a tangible effect on the outcome of tennis matches in the long run. We argue that a player who gains momentum plays significantly better than their opponent, meaning that swings in a match are not random, but rather due to a difference in skill during those episodes. To prove that this is the case, we must show that the player with more momentum during a match has a higher chance of winning.

Using our definition of momentum, we will employ A/B hypothesis testing to determine whether or not the swings in play and runs of success by one player are random or not. This method of testing is effective because if momentum does not have a significant effect on a match, then the momentum of winners vs losers comes from the same underlying distribution. To begin hypothesis testing, we must first determine our null and alternative hypotheses, significance level, and test statistic.

- **Null Hypothesis:** In the population, the distributions of the momentum of each player in the two group–winning and losing–are the same (They are different in the sample just due to chance).
- **Alternative Hypothesis:** In the population, the momentum of the winning player is higher, on average, than the momentum of the losing player.
- **Significance level:** 0.02
- **Test Statistic:** Winners Mean Momentum – Losers Mean Momentum.

We now calculate the observed difference between the mean momentum of the losers and winners. Later on, we will use this to calculate our empirical p-value that will be compared to our significance level. Using the episode duration optimized equation from above, we created new columns in the data set. One representing the total momentum of each player in the match, and one to store the match winner. The total momentum values of the winners of each match were averaged, and, after doing the same with the losers of every match, we found the observed difference between them to be 33.633. Now that we have determined our observed value, we will run simulations to see how likely it is to achieve the outcome of our observed test statistic (33.633).

Using our winners column (discussed in the paragraph above) we are able to determine which momentum value should be associated with the loser and which goes with the winner. The basis of A/B testing is to shuffle the rows that place each score into a certain category (winner or loser) and find the new simulated value of the test statistic. By repeating this process with many simulations, we can create a density histogram that allows us to see the distributions of the differences and subsequently determine the empirical p-value of the observed test statistic. If the momentum of the winners and losers originate from the same distribution, the shuffling of these rows will not produce a statistically significant different test stat from what we originally observed, and the subsequent empirical p-value would be relatively high. A relatively high empirical p-value (above our significance level) would indicate that we should fail to reject our null hypothesis due to a lack of proof that the momentum of the winners and losers come from different distributions.

We performed 10,000 simulations and created a density histogram that represents the distribution of the test statistics in comparison to our original observed statistic (Figure 2). The vertical red line on Figure 2 represents the observed test statistic. We can already see that the proportion of test statistics found in our 10,000 simulations that are equal to or greater than our observed value is very low. By calculating the empirical p-value, we can determine the outcome of our hypothesis test.

- **Simulated Test Statistics** = array/list of 10,000 test statistic values
- **Observed Test Statistic** = observed value from before (33.633)
- **Empirical p-value** = proportion of the Simulated Test Statistics greater than or equal to the Observed Test Statistic

We found the empirical p-value to be 0.0002. Since this is less than the significance level of 0.02, we can confidently reject the null hypothesis. In other words, we have found proof contrary to the belief that in the population, the distributions of the momentum of each player in the two groups–winning and losing–are the same. The small p-value indicates that the observed difference in means is highly unlikely to have occurred by random chance alone. Thus, we have disproven the postulation that swings in play and runs of success by one player are random.

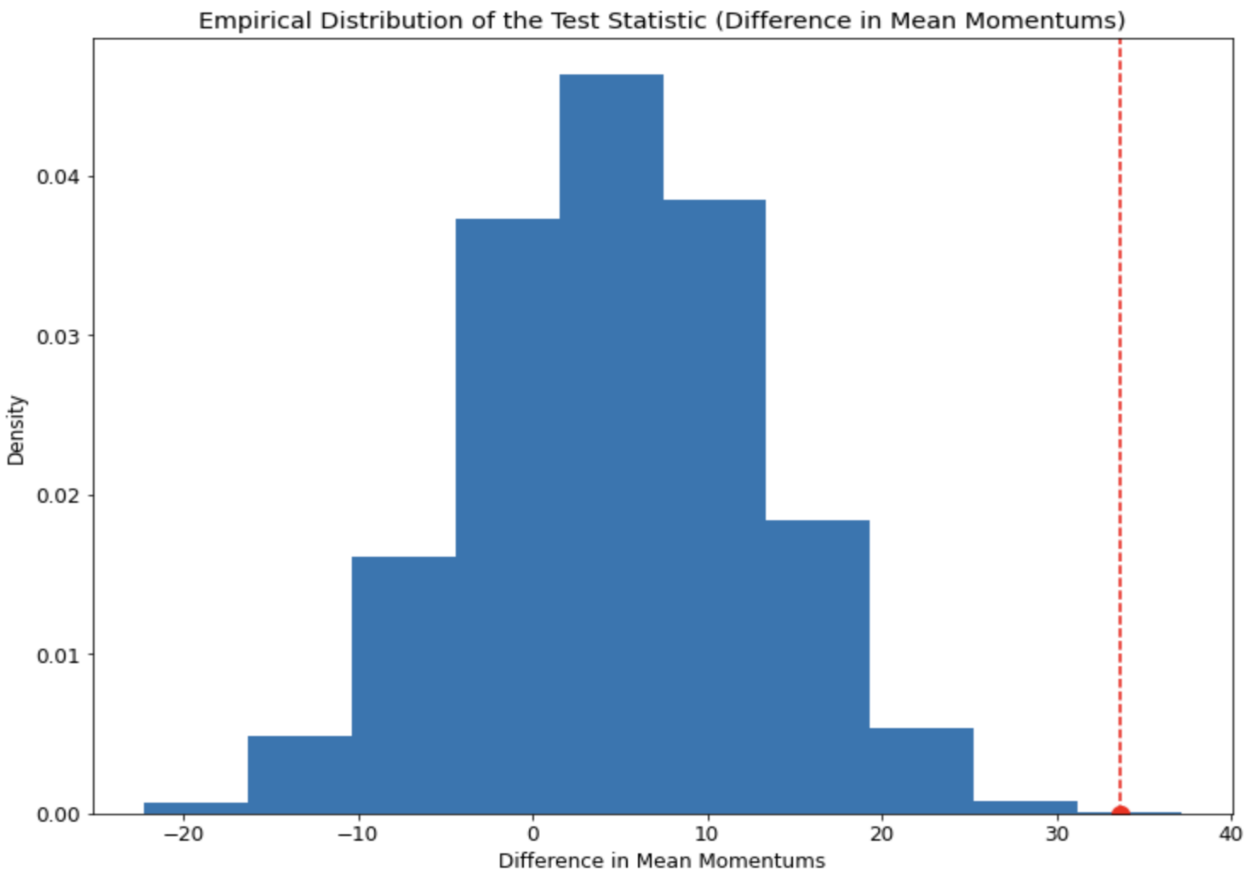

Figure 2: Density Histogram of the Difference Between the Mean Momentum of Winners and the Mean Momentum of Losers (10,000 simulations)

4 Predicting Momentum Episodes in a Tennis Match

Now that we have a established a model for momentum, and that momentum episodes are meaningful in affecting the outcome of a tennis match, we seek to understand what conditions lead to a player gaining momentum. One of the most popular statistical methods for predicting the result of a dependent variable (in our case: momentum) based on a set of independent variables is the Ordinary Least Squares (OLS) regression model. However, this model suffers when there is multicollinearity (the independent variables are correlated in some way) involved [14], which is almost certainly the case in our tennis data. There are many improved models that have been proposed, such as Ridge Regression and LASSO. We seek one which can handle multicollinearity, and is able to select relevant variables, so that we can better understand what conditions lead to momentum swings.

Ridge Regression and LASSO are both well-known for being empirically better than the OLS model. However, Ridge Regression keeps all variables, regardless of their relevance, which makes it difficult for us to decide which variables are most relevant to generating momentum. LASSO, on the other hand, is able to screen irrelevant variables, but struggles when the pairwise correlation between independent variables is high [14]. This is problematic for us, since we expect many independent variables to be highly correlated (e.g. rally_count and distance_run).

The elastic net model, first proposed by Zou and Hastie [14], is able to handle multicollinearity and select relevant variables, making it a well-suited model for our application. A full derivation of the elastic net model is outside the scope of this paper, but we will include the mathematics used in programming the model. First, consider the linear regression problem we wish to solve: given p predictor vectors, $\vec{x_1}, ..., \vec{x_p}$, the response $\vec{y}$ is predicted by

$$\hat{y} = \boldsymbol{X}\vec{\beta} \tag{5}$$

Where $\hat{y}$ is the $(n \times 1)$ prediction vector, $\boldsymbol{X}$ is the $(n \times p)$ matrix of observations on p independent variables, and $\vec{\beta}$ is the $(p \times 1)$ vector of parameters which is to be solved for. OLS works by minimizing the residual sum of squares between the predicted and actual data. Elastic-net, on the other hand, uses the following objective function:

$$L(\lambda_1, \lambda_2, \vec{\beta}) = ||\vec{y} - \boldsymbol{X}\vec{\beta}||_2 + \lambda_1||\vec{\beta}||_2 + \lambda_2||\vec{\beta}||_1 \tag{6}$$

Where the 2-norm of a vector (e.g. $\vec{a}$) is defined as $||\vec{a}||_2 = \sqrt{\sum_{j=1}^{p} a_j^2}$, the 1-norm is defined as $||\vec{a}||_1 = \sum_{j=1}^{p} |a_j|$, and λ_1 and λ_2 are the tuning parameters. The elastic net estimator $\hat{\beta}$ is the minimizer of the objective function. That is,

$$\hat{\beta} = \arg\min_{\beta} L(\lambda_1, \lambda_2, \vec{\beta}) \tag{7}$$

4.1 Training the Model

Because elastic net is a penalized linear regression model that includes both the λ_1 and λ_2 tuning parameters (i.e. penalties) during training, we are able to optimize the contributions of the λ_1 and λ_2 penalties. The λ_1 penalty on regression coefficients penalizes a model based on the sum of the absolute coefficient values (the 1-norm). A λ_2 penalty minimizes the size of all coefficients by penalizing a model based on the sum of the squared coefficient values (the 2-norm) [1]. Let the hyperparameter $\alpha = \frac{\lambda_1}{\lambda_1+\lambda_2}$. Then, from equations (6) and (7) we have

$$\hat{\beta} = \arg\min_{\beta} ||\vec{y} - \boldsymbol{X}\vec{\beta}||_2 + (\lambda_1 + \lambda_2)[(\alpha)||\vec{\beta}||_2 + (1 - \alpha)||\vec{\beta}||_1] \tag{8}$$

$$\text{elasticNetPenalty} = (\alpha)||\vec{\beta}||_2 + (1 - \alpha)||\vec{\beta}||_1 \tag{9}$$

Allowing us to tune our model via α. Another aspect of the elastic net model is that we can set the hyperparameter $\lambda := \lambda_1 + \lambda_2$, which now controls the weighting of the sum of both penalties to the loss function in our code. The default value of λ in `Sci-Kit Learn`, which is the elastic net implementation used in this paper, is 1.0 and indicates a fully weighted penalty while a value of 0 would exclude the penalties. Using this hyperparameter, we can further rewrite equation (8) as

$$\hat{\beta} = \arg\min_{\beta} ||\vec{y} - \boldsymbol{X}\vec{\beta}||_2 + \lambda[(\alpha)||\vec{\beta}||_2 + (1-\alpha)||\vec{\beta}||_1] \tag{10}$$

For our elastic net model, we found that the optimized weight of the λ_1 and λ_2 penalties were 0.21 and 0.79 respectively (meaning $\lambda = 1$). The optimized hyperparameter of α had a value of 0.31 to minimize the error of our model, which was Root Mean Squared Error (RMSE), defined below.

$$\text{RMSE} = \sqrt{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2} \tag{11}$$

In order to train our model, we begin with the dataset given to us containing all of the matches along with the columns we added to represent momentum. The first step is to split the data into **training**, **validation**, and **test** data. The training and validation data sets are used continuously to improve our model while the test data is only run once on our final version of the model. The model is created based off of only the training data and we check the accuracy of it based on the root mean squared error of both the training and validation data sets.

Once we create the basic elastic net model, we fit it with the values of the independent variables in the dataframe along with the dependent variable of momentum. Due to the fact that categorical data is uninterpretable by the model, we use **one hot encoding** to transform it into numeric values of 0s and 1s. This method was used to transform the columns of the **serve width**, **serve depth**, and **return depth**. The model will allow us to predict the momentum at a given point and therefore upcoming swings in the match, and the root mean squared error we generate comes from the differences between the predicted and actual values of the momentum. The goal, at this point, is to minimize the RMSE through the use of different independent variables that will increase our models' accuracy in predicting momentum and subsequent swings in the match.

4.2 Key Factors in Generating Momentum

After we optimize the model, we are able to identify which factors play the largest role in generating momentum. We train the elastic net model using all of the independent variables provided. During this process, elastic net screens out irrelevant variables by setting their coefficient to 0. So, we can define key variables as those which are assigned nonzero coefficients by elastic net. The resulting selections are displayed in the table below, along with their coefficients assigned by elastic net.

Variable	Coefficient	Description
p2 games won	-0.1690237	# of games won by player 2
p1 points won	-0.07873	# of points won by player 1
p2 points won	0.03382	# of points won by player 2
p1 distance run	-0.0110997	distance run by player 1 (meters)
rally count	0.228854	# of back-and-forth shots
server	0.127503	'1' if player 1 is serving and '2' if player 2 is serving
metric p1	0.06627	Biased score of player 1
metric p2	-0.01734	Biased score of player 2
p1 break pt	-0.233148	player 1 has an opportunity to win a game player 2 is serving
p2 break pt	0.10298	player 2 has an opportunity to win a game player 1 is serving
p1 net pt	0.46436	player 1 made it to the net
p2 net pt	-0.00947	player 2 made it to the net
p2 ace	-0.3606	player 1 was unable to return player 2's serve

A positive coefficient in the table above represents a positive relationship between the variable and player 1's momentum, while a negative coefficient represent a positive relationship between the variable and player 2's momentum. Additionally, the "metric p1" and "metric p2" fields are the biased score of each respective player, which was calculated using equation (2).

Upon further inspection, there are a number of interesting features to comment on. Some of the relationships shown above are readily apparent. For example, an "ace" for player 2, which occurs when player 1 is unable to return player 2's serve, appears to create momentum for player 2. Other relationships, however, are not so obvious. The "server" variable is equal to 1 when player 1 is serving and is equal to 2 when player 2 is serving. The positive coefficient associated with this variable means that momentum episodes are actually more likely when a player is receiving a serve. At first, this seems counter intuitive, since the server of a point usually has an advantage. However, this may be due to the fact that scoring off of an opponent's serve is a trigger for a momentum episode, since it is demoralizing for a server when they aren't able to cash in on their serve. There are potential coaching points for each of the variables above, which we will describe in detail in the memorandum at the end of this paper.

Based on the above findings, we will group variables into 3 designations based on their ability to trigger momentum episodes, so that coaches can gain greater insight into which variables are most important and which can be ignored. Variables in **Tier 1** are the most important for triggering momentum, variables in **Tier 2** are potentially important, and variables in **Tier 3** are likely not able to significantly affect momentum. These tiers are listed in the table below.

Tier 1	Tier 2	Tier 3
Winning games	Having a higher biased score than opponent	Unforced Errors
Winning when opponent has a break point	Winning sets	Winning own break points
Winning a point off the opponent's serve	Winning at the net	Serve speed
Playing from the net	Running less than opponent	Serve width/depth
Acing an opponent	Complacency when up on points	Return depth
Winning long rallies		Double faulting

5 Testing Model Performance in a Match

After using our training and validation sets to improve our model, we run our test set to get a final gauge of its accuracy. Since the test set data should only be run once, we must ensure that no aspect of it was included in the training or validation data. We can subsequently use the results of the models' predictions to give final statements on the effectiveness of our elastic net model.

As mentioned previously, our data was split into training, validation, and test data when we began creating a model to predict the momentum of the players. Our test data contains a random match (Christopher Eubanks vs Christopher O'Connell) from the data set given and we will use it to test our model.

First, we run the model on the data to produce a prediction for the momentum at every point. Next, we sort the predictions by their magnitude, using the highest predictions as potential turning points for Player 1 (Christopher Eubanks) and using the lowest predictions as potential turning points for Player 2 (Christopher O'Connell). When we did this, many of the high and low predictions were clustered together as consecutive strings of points. In these cases, we took a point from the middle of the cluster and used it as a potential turning point. After compiling these points, we graphed them alongside a graph of the actual momentum and the score difference over time. For readability, we used the same settings as Figure 1 for the actual momentum (displayed in red). This graph is displayed below.

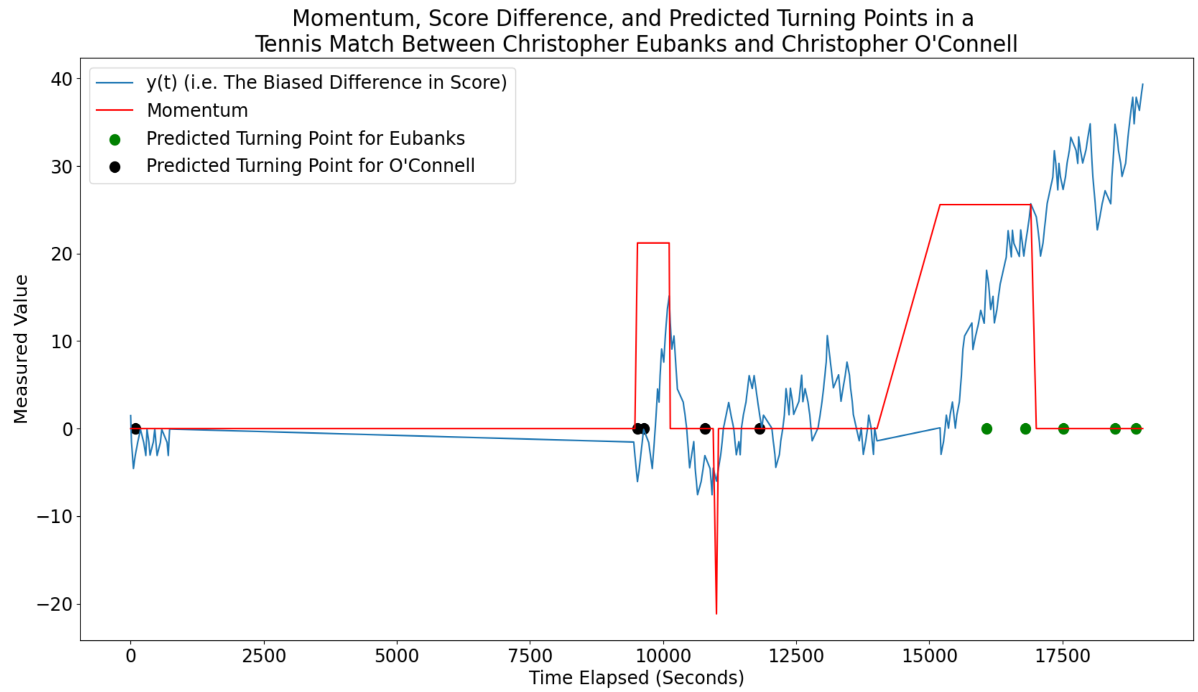

Figure 3: Actual Momentum, Score Difference, and Predicted Turning Points in a Tennis Match, obtained by Maximizing Momentum Episodes by their Duration

While the predicted turning points are not perfect, they are surprisingly accurate. Note that in this match there was a large intermission at the beginning, possible due to injury, technical issues, etc., which is why the score difference appears flat for a period of time. This graph shows that our model was able to predict Chistopher O'Connell's only momentum episode, albeit a very brief one. The rest of the points are false positives, but given the fact that only one momentum episode occurred for O'Connell, these misses are understandable. For Christopher Eubanks, our model missed his first momentum episode, but then correctly predicted that he would dominate the end of the match.

We believe that, based on these results, a coach with this model could significantly improve the outcome of the match for their player. On O'Connell's side of the ball, this would entail pushing his opponent harder at the predicted points, which may have stopped Eubanks's first momentum episode and possibly could have extended one of his own. For Eubanks, he appears to have played the match wonderfully according to the model, by stifling O'Connell's momentum early on and leaning into his own at the end of the match. Our model recommends similar play, which appears to have been the key to victory.

5.1 Generalization of Our Model

Our metric for momentum in equation 1, inspired by the work of Chen and Fan [2], is based on score, and could therefore be easily generalized to any sport with a high scoring frequency. This includes basketball, racquet games such as tennis or pickle ball, and volleyball. Additionally, by biasing scores to reflect advantages players may have that are intrinsic to their sport, such as being the server in tennis, we offer an additional contribution to the momentum metric. This may be useful in any sport when scores are not equally likely to create psychological momentum, even in those with low scoring frequencies. For example, a field goal in American football may not drive momentum in the same way that a large first down conversion does, despite the fact that it results in a score.

Our elastic net model can also be easily generalized to other sports as well. Similar sports such as table tennis and women's tennis follow the same conventions of allowing two serves and share similar rules to scoring, making our model incredibly applicable. Additionally, an elastic net model such as ours is likely generalizable to any sport with high scoring frequency, given the right data modifications. In fact, elastic net has already been used to model momentum in basketball [2] and could have additional applications other than momentum, such as setting player salaries or minimizing injuries. An interesting paper by Wang even uses elastic net to optimize health outcomes in athletes who play sports [13].

In lower scoring sports, such as soccer or hockey, a continuous model for momentum is less promising than one that is state-based. However, we believe that this model could be still be used with proper data collection and management. In particular, using elastic net as a method to screen out irrelevant variables could be useful as a preliminary step to a state-based model.

6 Weaknesses and Future Improvements

We acknowledge that our model is not perfect and, just like the majority of models that attempt quantify human feelings, there are areas that could stand to be improved. There are many external factors that can affect the performance of a player which can not be represented as points on a graph.

6.1 Environmental Factors

The data taken from the 2023 Wimbledon tournament to build this model primarily includes technical variables such as information about a player's serve, how far they ran, what type of shot they won with, etc. However, there are no environmental variables included, such as crowd size, crowd noise level at different times, or the amount of additional money a player would receive from the tournament purse for winning a match. Research in basketball shows that environmental factors such as crowd size and player salaries are actually more important triggers for generating momentum than any technical variable [2]. For any coach wishing to implement our model, they would do well to attempt to collect environmental data in addition to technical data.

6.2 Psychological Factors

Momentum in sports was first described and validated by sports psychologists [6], and while empirical variables are useful tools for determining momentum swings, we agree that momentum is likely a psychological phenomenon. While we believe our model is successful in aiding our understanding of momentum episodes, it is ultimately an abstraction of a player's personal head space.

For example, the ”Back to the Wall” Effect is the sensation when a player has nothing to lose, or is perhaps a severe underdog, they can drastically increase their level of performance [3]. Our model does not take into account the human drive for a miracle. However, research done on this curiosity has yielded inconclusive results [3] and since it remains unproven we decided not to include it as a variable in our model. While it is possible that every so often there could be a valid, psychological-based argument that the player not predicted to win by the model has won due to ”believing in themselves more” (or a similar argument), it does not occur often enough to alter the overall results of the model.

6.3 Low Correlation of Explanatory Variables

The tables below display the correlation between each independent variable and momentum.

	momentum
p2_games	-0.052154
p2_winner	-0.034597
p2_distance_run	-0.026001
p2_break_pt	-0.022126
point_victor	-0.021936
p2_net_pt_won	-0.020162
p2_break_pt_missed	-0.019404
p1_distance_run	-0.018395
p2_net_pt	-0.017790
serve_width_C	-0.016093
p1_unf_err	-0.013984
serve_depth_nan	-0.012986
serve_width_nan	-0.012986
p2_unf_err	-0.011864
p2_ace	-0.010841
p2_break_pt_won	-0.010613

	momentum
serve_width_W	-0.009947
speed_mph	-0.008614
p2_double_fault	-0.004703
game_victor	-0.002934
set_victor	-0.002380
return_depth_nan	-0.002355
serve_depth_CTL	-0.001473
p2_sets	0.000663
return_depth_D	0.000906
return_depth_ND	0.000987
serve_width_BW	0.002554
serve_no	0.002971
serve_depth_NCTL	0.004000
p1_ace	0.004676
server	0.006701
p1_double_fault	0.006957

	momentum
p1_break_pt_won	0.007484
p1_break_pt_missed	0.012636
p1_net_pt	0.012690
p1_break_pt	0.014721
p1_net_pt_won	0.016111
serve_width_B	0.016882
serve_width_BC	0.018137
p1_games	0.018389
rally_count	0.023578
p1_winner	0.027397
metric_p2	0.044648
p2_points_won	0.050934
p1_points_won	0.071314
metric_p1	0.077268
p1_sets	0.106453
momentum	1.000000

Figure 4: Correlation Between Explanatory Variables and Momentum

These correlations are depressingly low, with only a select few even rising above a significant value of 0.05. This presents a problem for predicting momentum episodes since the explanatory power of these variables seems sub-optimal. To accommodate this fact, we used elastic net as a variable selection mechanism, rather than the correlations. The hope is that certain combinations of these variables selected by the elastic net model may be superior to simply selecting the highest correlated variables; which appears to be valid given the results of our model. Nevertheless, future improvements to the model could involve inclusion of variables that are more correlated with momentum, such as crowd size, or could employ data engineering to improve the relationship between these variables and momentum.

As an additional result of these low correlations, the model may have left out variables that we think could impact momentum. Many of the variables chosen by the model are intuitive. For example, a high rally count will bring a confidence and momentum boost for the player that managed to win the point, which is confirmed by our model. However, other events, such as missing a serve or making an unforced error, would be expected to lower the momentum of the offending player and increase the momentum of their opponent, but those variables were not included. More work should be done to determine if they truly can be ignored when predicting momentum, or if there is a more subtle relationship present.

6.4 Momentum Across Successive Matches

Our model assesses momentum during a match. However, tennis players often play in several matches per tournament, and may even compete in several tournaments back-to-back. In these intense periods of competitions, it may be the case that players build or lose momentum across successive matches. Can momentum from one match carry into another? Is there a limit to how long a player can hold onto momentum across matches before competition fatigue sets in? These are interesting questions that could be an avenue for further research into momentum in tennis.

Memorandum to Tennis Coaches

To: Coaches of Competitors in the 2023 Wimbledon Tournament
From: Team #2400951
Re: Recommendations for Generating Momentum in Tennis Athletes
Date: Feb 5, 2024

Momentum has been studied as a psychological phenomenon since the 1980s, but providing objective advice for such an esoteric subject is difficult. However, with modern data science and computing, we are able to study momentum using empirical methods. In this memorandum, we present a justification for the study of momentum using data from the 2023 Men's Wimbledon tournament. We also identify key indicators as momentum triggers in athletes and provide coaches with advice for exploiting these triggers.

Section I reviews the existence of momentum and its effect on runs of success and changes in the flow of play during matches.

Section II identifies main triggers of momentum in the players of the 2023 Wimbledon tournament. Additionally, we provide detailed coaching points for each variable, and place them into one of three categories:

- Vital indicators
- Potentially important indicators
- Irrelevant indicators

Finally, Section III concludes this memo with the limitations of the suggested practices and advice for addressing these.

I. Justifying the Study of Momentum

Some coaches may be skeptical that momentum plays a meaningful role in a tennis match; crediting swings in the game to random chance. To test this, we created a well-defined metric for momentum that distinguishes normal game flow from momentum episodes. We then tested whether episodes actually gave players an advantage using A/B hypothesis testing. Our conclusion, with a confidence level of 99.98%, was that experiencing well-defined momentum episodes leads to a statistically significant increase in the chance of winning a match. This means that players and coaches competing in the sport of tennis have a vested interest in developing a comprehensive understanding of momentum.

II. Identifying Key Indicators

We created a metric for momentum that was able to successfully identify critical periods where one player gained the upper hand in tennis matches. After applying this metric to the 2023 Wimbledon data, we employed modern regression techniques to create a comprehensive ranking of empirical variables in the game of tennis that affect momentum. We placed these variables into tiers based on the weights given to them by our model. Variables in **Tier 1** are the most important for triggering momentum, variables in **Tier 2** are potentially important, and variables in **Tier 3** are likely not able to significantly affect momentum. These tiers are listed in the table below.

Tier 1	Tier 2	Tier 3
Winning games	Having a higher biased score than opponent	Unforced Errors
Winning when opponent has a break point	Winning sets	Winning own break points
Winning a point off the opponent's serve	Winning at the net	Serve speed
Playing from the net	Doing less running than opponent	Serve width/depth
Hitting an ace	Complacency when leading in points	Return depth
Winning long rallies		Double faulting

Many of these indicators are intuitive, such as acing an opponent. For the more subtle indicators, we have provided coaching points.

Winning Games: We emphasize that victory in games appears to be more important than winning individual points or even sets. This may be due to the fact that a set occurs over too long a period to generate momentum, and a point over too short a period.

Serves: Serving is typically viewed as a major advantage in tennis. However, coaches should view receiving a serve as a vital opportunity to generate momentum if the point is won.

Break Points: Contrary to serving in general, our research suggests that serving on the opponent's break point *does* provide an advantage to momentum generation. It appears to be an opportunity for a momentum triggering "comeback" in a game. In fact, winning an opponent's break point seem to be more important that winning one's own break points.

Playing From the Net: Getting to the net may intimidate opponents as it is a sign that a player is in control of a rally. Winning the point is less important than simply getting there.

Biased Score: The biased score gives additional weight to points won when receiving a serve. It seems that having a high biased score is more important than winning points outright.

Complacency: Players are less likely to trigger momentum episodes when they already have a large lead. This may be due to fatigue during the later stages of the match, or from feeling too secure in their point advantage over their opponent.

Double Faulting and Unforced Errors: Players should be coached to understand that these events rarely give the opponent any momentum and should not be feared.

III. Conclusion: Limitations and Implications

We conclude that momentum in tennis can give players a statistically significant advantage in a match. Furthermore, we conclude that it is possible to understand certain empirical triggers of momentum. Coaches should be mindful that our research does not include environmental variables, such as crowd size, which other studies suggest may be important. Additionally, we agree with sports psychologists that momentum is likely a reflection of a player's head space, meaning that coaches should understand how to subjectively motivate their players as well as understanding the objective metrics presented in this memo.

References

[1] Brownlee, Jason. “How to Develop Elastic Net Regression Models in Python.” MachineLearningMastery.com, June 11, 2020. https://machinelearningmastery.com/elastic-net-regression-in-python/.

[2] Chen, T., Fan, Q., Liu, K., & Le, L. (2020). Identifying key factors in momentum in basketball games. Journal of applied statistics, 48(16), 3116–3129. https://doi.org/10.1080/02664763.2020.1795819

[3] Chen, Tao, and Qingliang Fan. 2018. “A Functional Data Approach to Model Score Difference Process in Professional Basketball Games.” Journal of Applied Statistics 45 (1): 112–27. doi:10.1080/02664763.2016.1268106.

[4] COMAP, Inc. “2024 MCM Problem C: Momentum in Tennis.” www.comap.org, February 1, 2024. https://www.contest.comap.com/undergraduate/contests/mcm/contests/2024/problems /2024_MCM_Problem_C.pdf.

[5] Fry, Michael J. and Shukairy, F. Alan. ”Searching for Momentum in the NFL” Journal of Quantitative Analysis in Sports 8, no. 1 (2012). https://doi.org/10.1515/1559-0410.1362

[6] Iso-Ahola, S. E., & Mobily, K. (1980). “Psychological Momentum”: A Phenomenon and an Empirical (Unobtrusive) Validation of its Influence in a Competitive Sport Tournament. Psychological Reports, 46(2), 391-401. https://doi.org/10.2466/pr0.1980.46.2.391

[7] Kevin M. Kniffin, Vince Mihalek, ”Within-series momentum in hockey: No returns for running up the score.” Economics Letters, Volume 122, Issue 3, 2014, Pages 400-402, ISSN 0165-1765, https://doi.org/10.1016/j.econlet.2013.12.033.

[8] Leushuis, Carien. “Beating the Odds - A State Space Model for Predicting Match Results in the Australian Football League.” openjournals.maastricht.nl. Accessed February 2, 2024. https://openjournals.maastrichtuniversity.nl/Marble/article/view/613/428

[9] Liljenwall, Ari. “Sounders Look to Continue Momentum Heading into First Away Match of the Season” Seattle Sounders FC, March 11, 2023. https://www.soundersfc.com/news/sounders-look-to-continue-momentum-heading-into-first-away-match-of-the-season.

[10] Newsom. “Latent Variables.” web.pdx.edu, 2023. https://web.pdx.edu/ñewsomj/semclass/ho_lab %20data.pdf

[11] Page, Lionel. “The Momentum Effect in Competitions: Field Evidence from Tennis Matches.” CITESEERX. Accessed February 2, 2024. https://citeseerx.ist.psu.edu/viewdoc/summary.

[12] “Top Tennis Champions Motivational Mind Quotes, Past and Present.” Tennis Mind Quotes — ATP WTA Champions — MindTraining.net. Accessed February 1, 2024. https://mindtraining.net/motivational-quotes/sports-champions/tennis.php.

[13] Wang C. Optimization of sports effect evaluation technology from random forest algorithm and elastic network algorithm. 2023 Oct, 2018. PloS one, 18(10), e0292557. https://doi.org/10.1371/journal.pone.0292557

[14] Zou, Hui, and Trevor Hastie. "Regularization and Variable Selection via the Elastic Net." Journal of the Royal Statistical Society. Series B (Statistical Methodology) 67, no. 2 (2005): 301–20. http://www.jstor.org/stable/3647580.