
Optimal Actuation in a Peristaltic Crawler

Presenter: Andrew Gusty
Mentor: Dr. Emily Jensen

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- An example of such a system is a *peristaltic crawler*



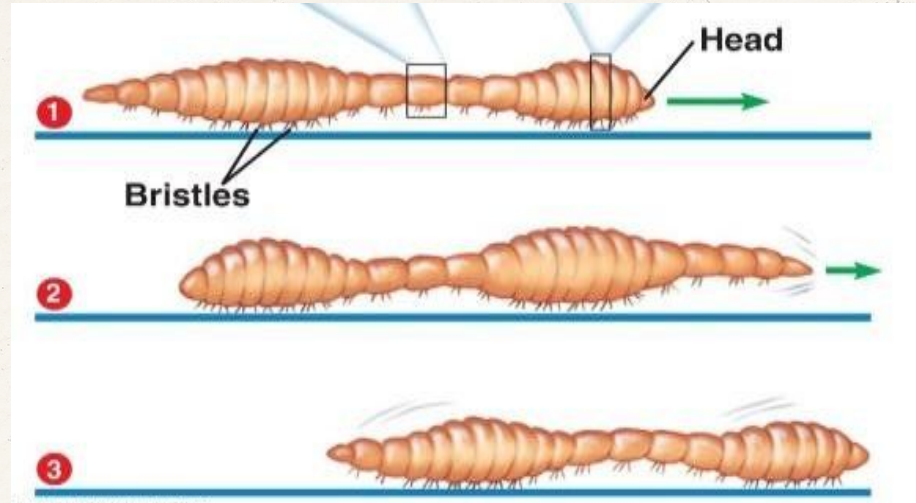
Purpose

- Our lab is interested in the efficient control of systems with wave-like dynamics
- An example of such a system is a *peristaltic crawler*
- My objective is to uncover the optimal actuation of such crawlers



Peristaltic Motion

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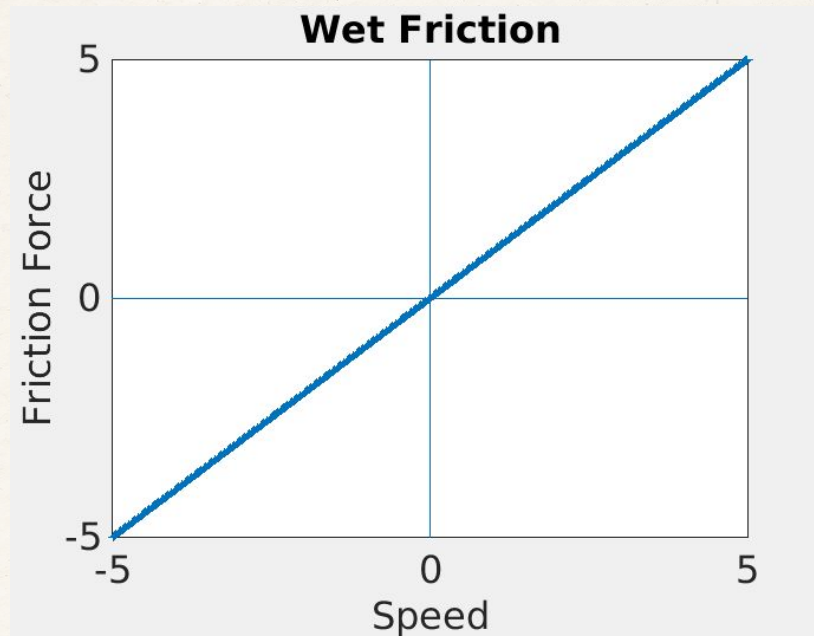
Longitudinal Waves + Friction Control = Movement

Equation of Motion

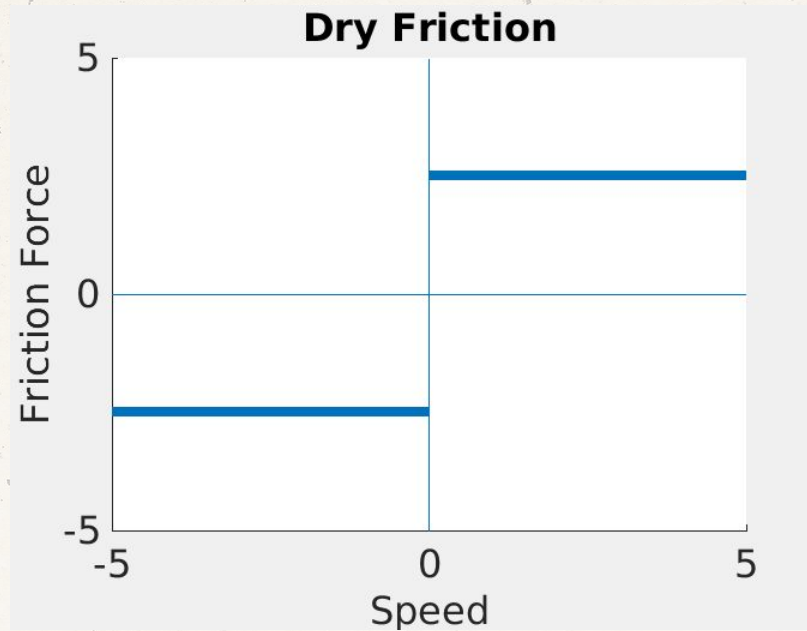
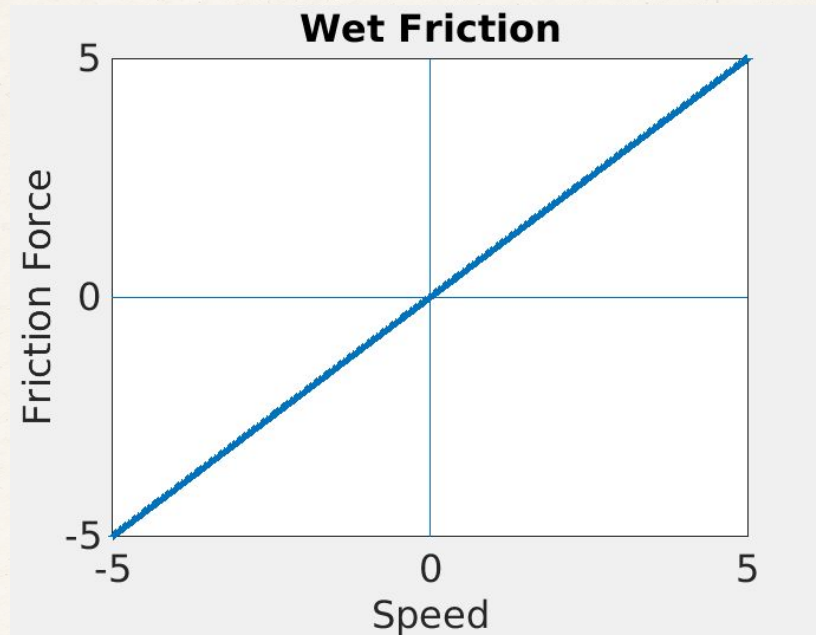
$$\underbrace{A_c E \frac{\partial^2 u}{\partial x^2}}_{\text{Elastic}} + \underbrace{A_m \frac{\partial f}{\partial x}}_{\text{Muscular}} - \underbrace{F \left(\frac{\partial u}{\partial t} \right)}_{\text{Friction}} - \underbrace{\rho \frac{\partial^2 u}{\partial t^2}}_{\text{Inertial}} = 0$$

Notation	Definition	Units
u	Displacement	m
x	Space	m
t	Time	seconds
f	Muscle Stress	$\frac{N}{m^2}$
E	Young's modulus	Pa
A_c	Body's cross-sectional area	m^2
A_m	Muscle's cross-sectional area	m^2
ρ	Density	kg/m

Modeling Friction

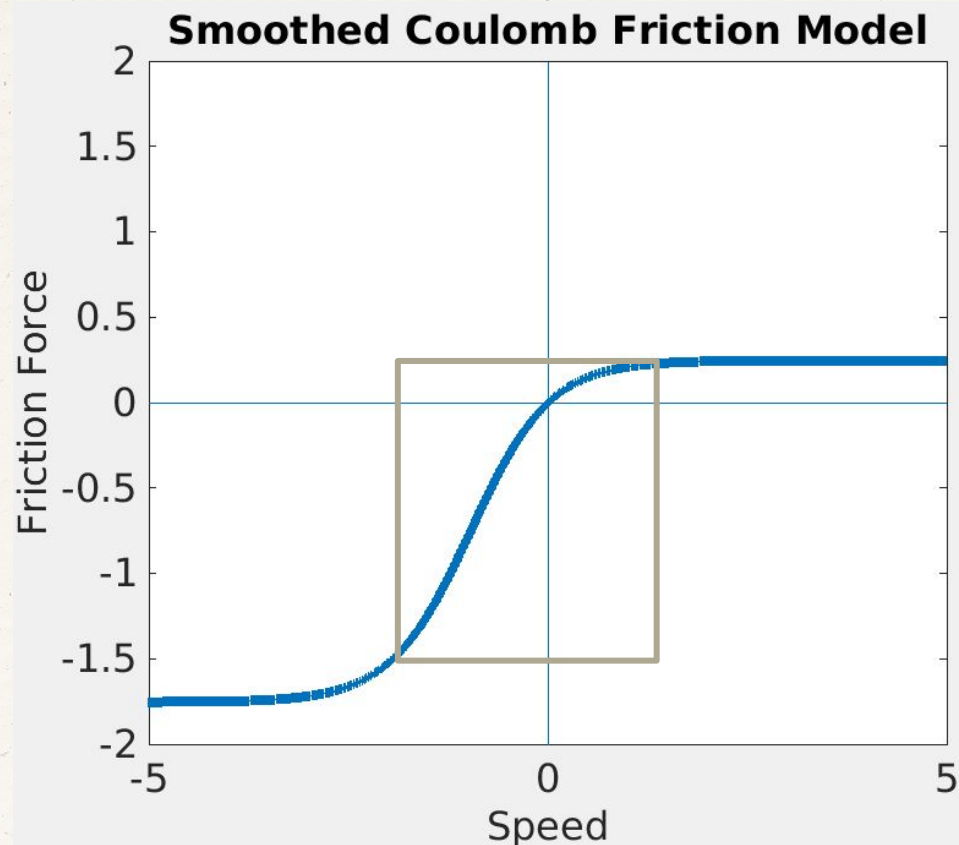


Modeling Friction



Friction Model

- Asymmetry creates preferential direction of motion
- Incorporates features of dry and wet friction
- Fairly realistic at small velocities
- Model permits Taylor Series approximation



Adding Friction to the Model

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Perturbation Theory

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$$(A_c E \partial_x^2 - c_F \partial_t - \rho \partial_t^2) u^{(0)} = \frac{\partial f(x, t)}{\partial x}$$

$$(A_c E \partial_x^2 - c_F \partial_t - \rho \partial_t^2) u^{(1)} = c_F n_F (u_t^{(0)})^2$$

Muscular Force

Periodic function that drives the motion of the crawler

$$f(x, t) = f_0 \sin(\omega t - kx)$$

Three Parameters:

- f_0 : strength of contractions
- ω : frequency of contractions
- k : wave number

Crawler Efficiency

$$\text{Velocity} = \frac{1}{\lambda} \int_0^\lambda u_t^{(1)} dx$$

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$$\|\text{Force Input}\|^2 = \int_0^{\lambda} (f(x, t))^2 dx$$

Crawler Efficiency

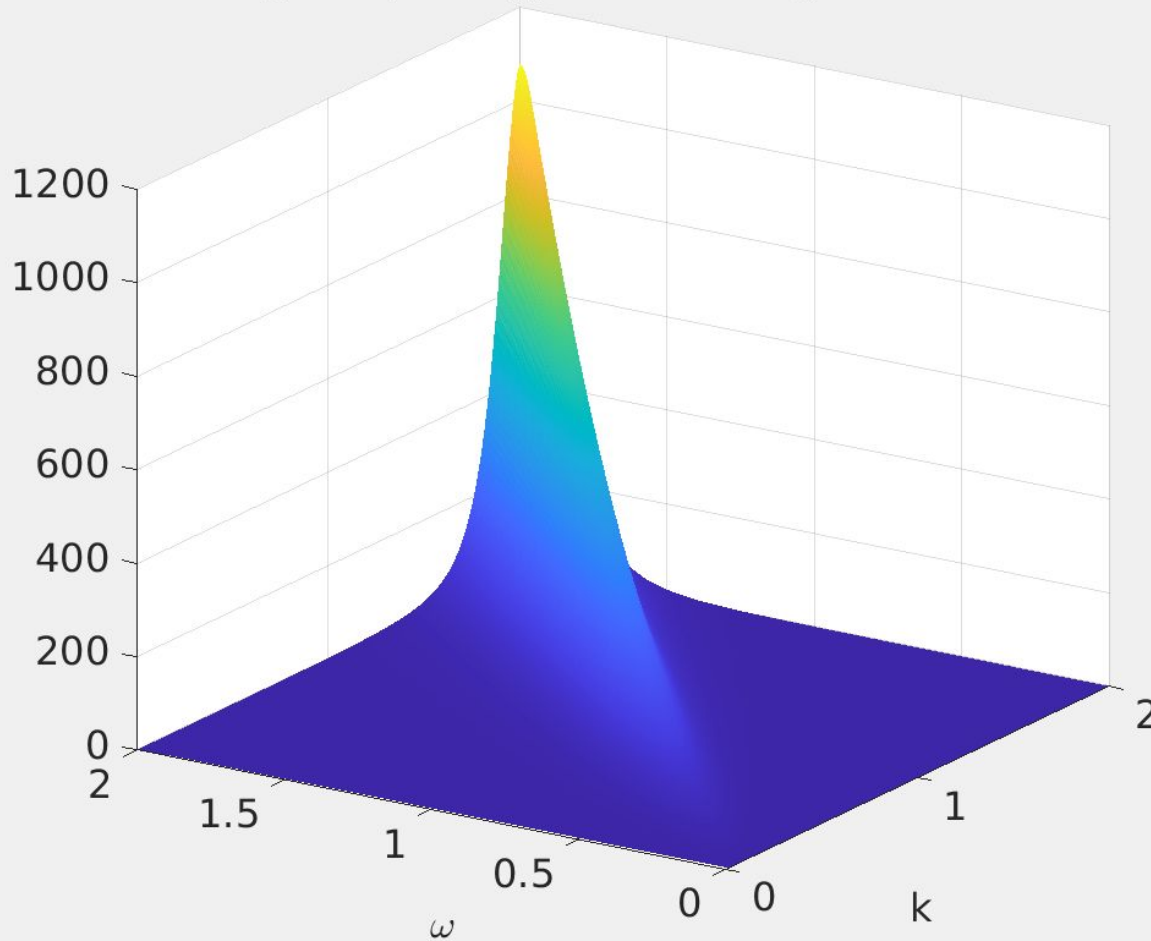
$$\text{Efficiency} = \frac{||\text{Velocity}||^2}{||\text{Force Input}||^2}$$

Relationship Between Crawler Efficiency, Forcing Frequency, and Forcing Wavenumber



Main Result

- Optimal relationship between ω and k is linear
- Efficiency does not depend on f_0
- Slope of optimal line depends on other parameters (which are set to unity in this graph)



Conclusions and Future Work

Accomplishments

- Developed Methods for Analyzing More Sophisticated Friction Models
- Uncovered relationship between forcing frequency and wavelength

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Future Work

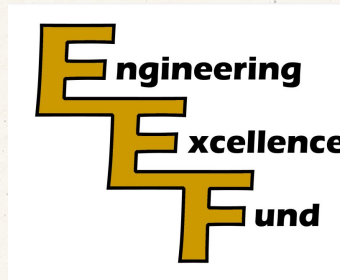
- Numerical methods for better approximation of dry friction
- Dimensional analysis of parameters
- Use these results for distributed control of peristaltic crawlers

Acknowledgements

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Questions