Optimal Actuation in a Peristaltic Crawler

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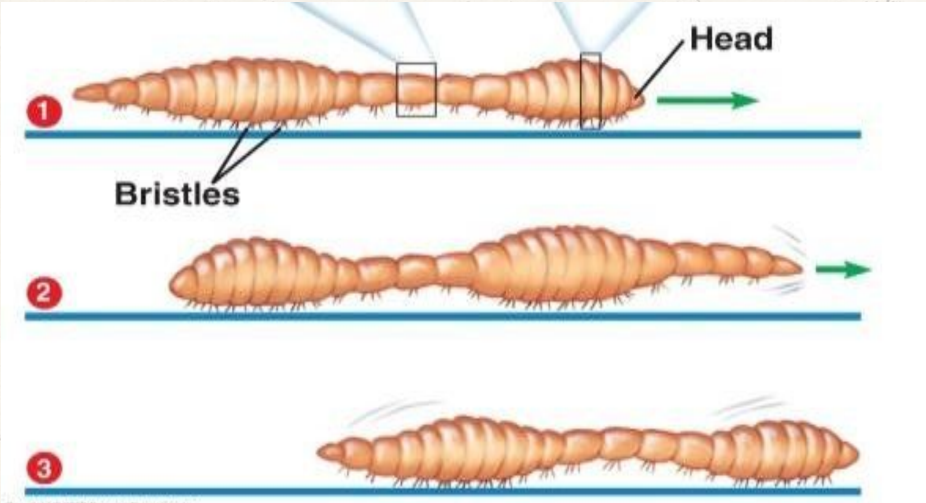
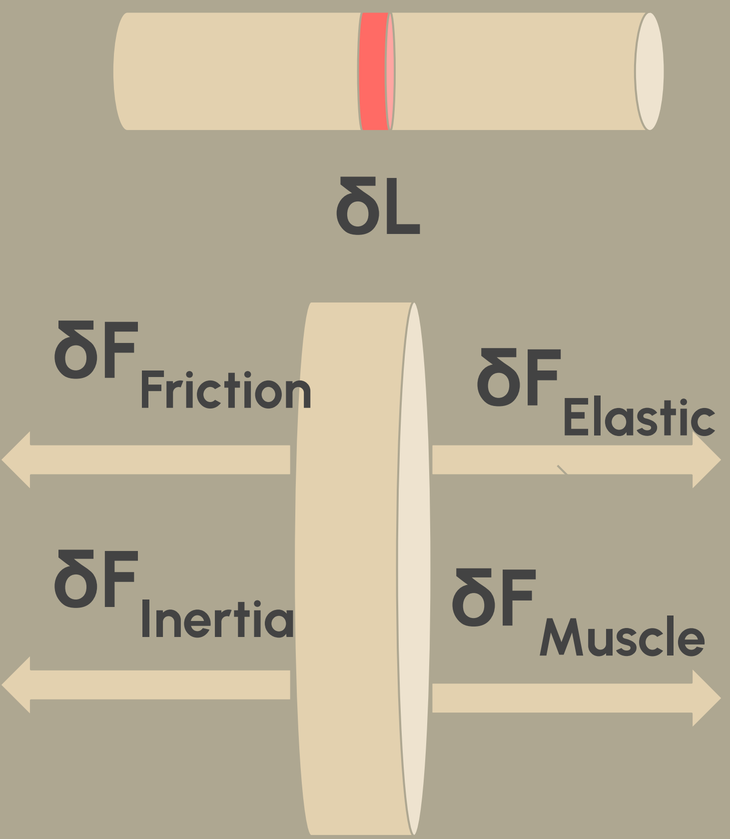
- An example of such a system is a *peristaltic crawler*
- My objective is to uncover the optimal actuation of such crawlers

Peristaltic Motion



Longitudinal Waves + Friction Control = Movement

(1) Niu el al. (2015). Enabling earthworm-like soft robot development using bioinspired IPMC-scissor lift actuation structures: Design, locomotion simulation and experimental validation.



Derivation

Elastic Force:

$$\lim_{l \to 0} \frac{\delta F_{\text{elastic}}}{\delta l} = A_c E \frac{\partial^2 u}{\partial x^2}$$

Muscle Force:

$$\lim_{l \to 0} \frac{\delta F_{\text{muscle}}}{\delta l} = A_m \frac{\partial f}{\partial x}$$

Inertial Force:

$$\rho \frac{\partial^2 u}{\partial x^2}$$

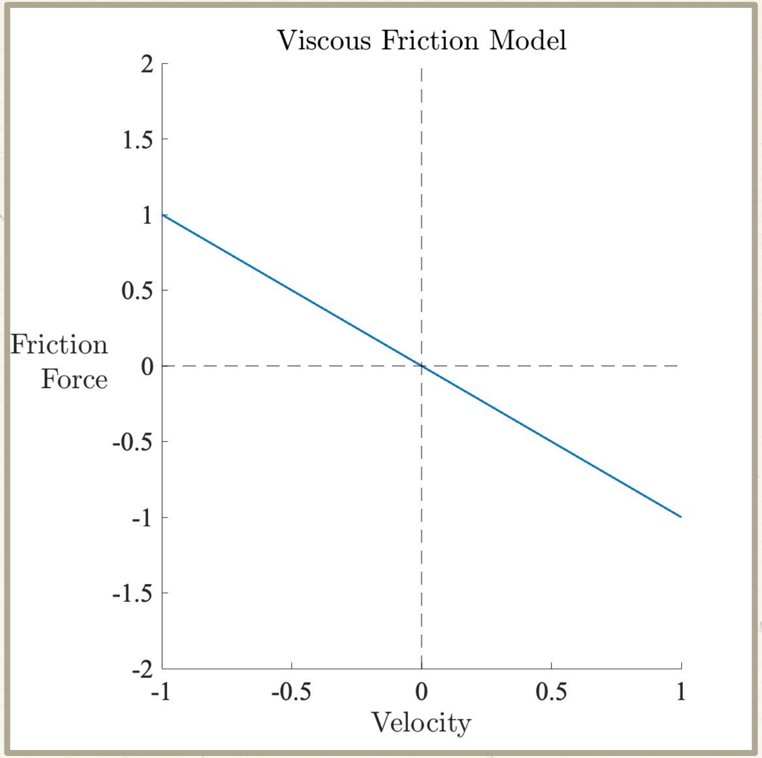
(2) Juncal, Arbelaiz (2019). A Bioinspired Controller for Crawling Locomotion

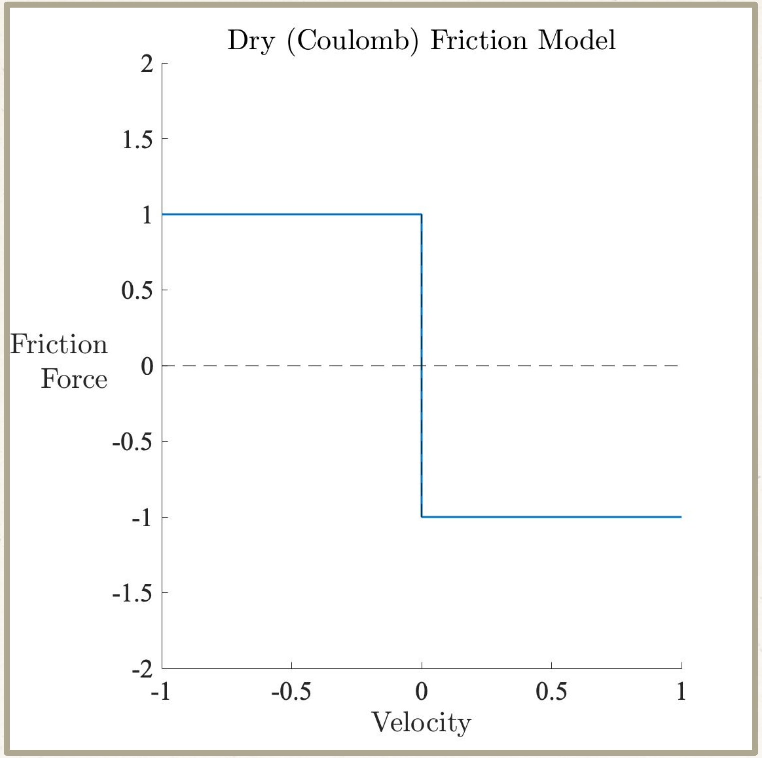
Equation of Motion

$$\underbrace{A_c E \frac{\partial^2 u}{\partial x^2}}_{\text{Elastic}} + \underbrace{A_m \frac{\partial f}{\partial x}}_{\text{Muscular}} - \underbrace{F\left(\frac{\partial u}{\partial t}\right)}_{\text{Friction}} - \underbrace{\rho \frac{\partial^2 u}{\partial t^2}}_{\text{Inertial}} = 0$$

Notation	**Definition**	**Units**
u	Displacement	m
x	Space	m
t	Time	seconds
f	Muscle Stress	$\frac{N}{m^2}$
E	Young's modulus	Pa
A_c	Body's cross-sectional area	m^2
A_m	Muscle's cross-sectional area	m^2
ρ	Density	kg/m

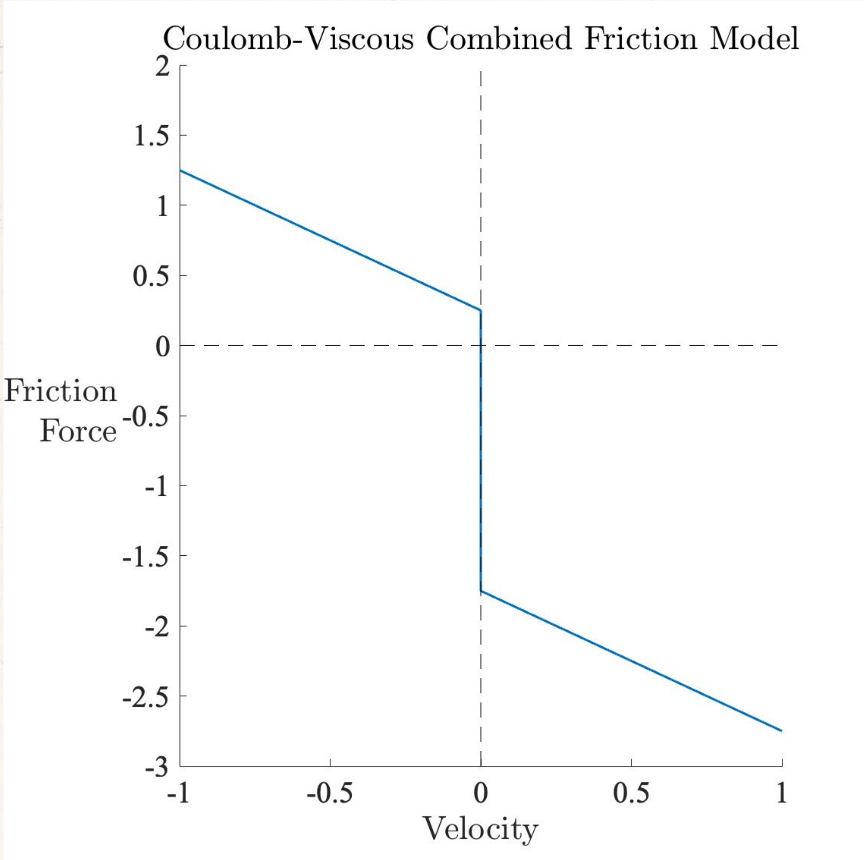
Modeling Friction





Friction Model

- Asymmetry creates preferential direction of motion
- Incorporates features of dry and wet friction
- Consistent with physical environments of crawlers



(3) Review and Comparison of Dry Friction Force Models *E. Penestri et al. (2016)*

(4) Mechanics of peristaltic locomotion and the role of anchoring. *Tanaka et al.* (2012)

Adding Friction to the Model

Small parameter, ϵ signifies weak dry friction effects

$$F_{friction}(u_t) = \underbrace{c_v u_t}_{\text{Viscous Friction}} + \underbrace{\epsilon c_d(\operatorname{sgn}(u_t) - n_F)}_{\text{Dry Friction}}$$

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$$F_{friction}(u_t) = \underbrace{c_v u_t}_{\text{Viscous Friction}} + \underbrace{\epsilon c_d(\mathrm{sgn}(u_t) - n_F)}_{\text{Dry Friction}}$$

$$\underbrace{A_c E \frac{\partial^2 u}{\partial x^2}}_{\text{Elastic}} + \underbrace{A_m \frac{\partial f}{\partial x}}_{\text{Muscular}} - \underbrace{c_F u_t}_{\text{Wet Friction}} + \underbrace{\epsilon c_d(\mathrm{sgn}(u_t) - n_F)}_{\text{Dry Friction}} - \underbrace{\rho \frac{\partial^2 u}{\partial t^2}}_{\text{Inertial}} = 0$$

Muscular Force

Periodic function that drives the motion of the crawler

$$f(x,t) = f_0 \sin\left(\omega t - kx\right)$$

Three Controllable Parameters:

- f_0: strength of contractions
- ω: frequency of contractions
- k: wave number

Nondimensionalizing

- τ is time normalized by natural undamped frequency of crawler
- □ is space normalized by wavenumber of actuator
- μ is nondimensional displacement of crawler

$$\omega_n = k\sqrt{\frac{A_c E}{\rho}}$$

$$\tau = \omega_n t, \quad \chi = kx, \quad \mu = \frac{u f_0}{\rho \omega_n^2}$$

Nondimensional Dynamics

$$\mu_{\chi\chi} - \mu_{\tau\tau} - \zeta\mu_{\tau} + \epsilon\pi_r(\mathrm{sgn}(\mu_{\tau}) - n_F) = cos(\chi - \alpha\tau)$$

- ζ is known as the damping coefficient
- π_r and α are nondimensional coefficients

$$\zeta = \frac{c_1}{k\sqrt{\rho A_c E}}, \quad \pi_r = \frac{c_2 f_0}{k^2 \rho A_c E},$$

$$\alpha = \frac{\omega}{\omega_n}$$

Well-Orderedness

An asymptotic series $\sum_{n=0}^{\infty} a_n \phi_n(\epsilon)$ is well-ordered if $\phi_{n+1}(\epsilon) = o(\phi_n(\epsilon))$ as $\epsilon \to 0$,

$$\text{i.e., for all } c > 0,\ |\phi_{n+1}(\epsilon)| < c|\phi_n(\epsilon)| \text{ for } \epsilon \text{ close to } 0$$

...This notion of orderedness allows us to break the nonlinear PDE into a system of linear PDEs

Perturbation Theory

If solution is analytic with respect to epsilon, we can break it into a well-ordered asymptotic series

$$\mu(\chi, \tau, \epsilon) = \mu(\chi, \tau, 0) + \epsilon \frac{\partial \mu}{\partial \epsilon}(\chi, \tau, 0) + \frac{\epsilon^2}{2!} \frac{\partial^2 \mu}{\partial \epsilon^2}(\chi, \tau, 0) + \cdots$$

$$\mu(\chi, \tau, \epsilon) = \mu^{(0)}(\chi, \tau) + \epsilon \mu^{(1)}(\chi, \tau) + \epsilon^2 \mu^{(2)}(\chi, \tau) + \cdots$$

Perturbation Theory

It can be shown that the asymmetric friction brings about a singular perturbation problem, which can be treated using multiple "slow" and "fast" timescales.

This modifies our derivative operator:

$$\tau_1 = \tau,$$

$$\tau_2 = \epsilon\tau$$

$$\frac{\partial\mu}{\partial\tau} = \frac{\partial\mu}{\partial\tau_1} + \epsilon\frac{\partial\mu}{\partial\tau_2}$$

$$\frac{\partial^2\mu}{\partial\tau^2} = \frac{\partial^2\mu}{\partial\tau_1^2} + 2\epsilon\frac{\partial^2\mu}{\partial\tau_1\tau_2} + \epsilon^2\frac{\partial^2\mu}{\partial\tau_2^2}$$

Perturbation Theory

$$\mu_{\chi\chi} - \mu_{\tau\tau} - \zeta\mu_\tau + \epsilon\pi_r(\mathrm{sgn}(\mu_\tau) - n_F) = cos(\chi - \alpha\tau)$$

$$\mu^{(0)}\chi\chi - \mu^{(0)}\tau_1\tau_1 - \zeta\mu^{(0)}\tau_1 = cos(\chi - \alpha\tau)$$

$$\mu^{(1)}\chi\chi - \mu^{(1)}\tau_1\tau_1 - \zeta\mu^{(1)}\tau_1 = \zeta\mu^{(0)}_{\tau_2} + 2\mu^{(0)}_{\tau_1\tau_2} + \pi_r(\mathrm{sgn}(\mu^{(0)}_{\tau_1}) - n_F)$$

Using $\mu_\tau \approx \mu^{(0)}_{\tau_1}$

Solving O(1) Equation

The nonhomogeneous term suggests a trial solution of the form:

$$\mu^{(0)}(\chi, \tau_1, \tau_2) = A\cos(\chi - \alpha\tau_1) + B\sin(\chi - \alpha\tau_1) + C\tau_2$$

$$A = \frac{\alpha^2 - 1}{(\alpha^2 - 1)^2 + (\zeta\alpha)^2}, \quad B = \frac{\zeta\alpha}{(\alpha^2 - 1)^2 + (\zeta\alpha)^2}$$

$$C = \frac{\pi_r n_f}{\zeta}$$

Fourier Expansion of $O(\epsilon)$ Equation

- The dry friction force is now a traveling square wave
- ϕ, as well as Fourier coefficients, will vary with degree of asymmetry in the friction

$$\pi_r \operatorname{sgn}(\mu_\tau^{(0)}) = \sum_{n=1,\ \text{odd}}^{\infty} \frac{4\pi_r}{\pi n} sin(n(\chi - \alpha\tau) + n\phi)$$

Fourier Expansion of O(ε) Equation

This suggests a Fourier series trial solution for the 1st-order equation:

$$\mu^{(1)} = c_0 + \sum_{n=1,\ \mathrm{odd}}^{\infty} A_n \cos(n(\chi - \alpha\tau)) + B_n \sin(n(\chi - \alpha\tau))$$

Crawler Velocity

$$\text{Velocity} = \frac{1}{\lambda}\int_0^\lambda u_t \, dx$$

Crawler Efficiency

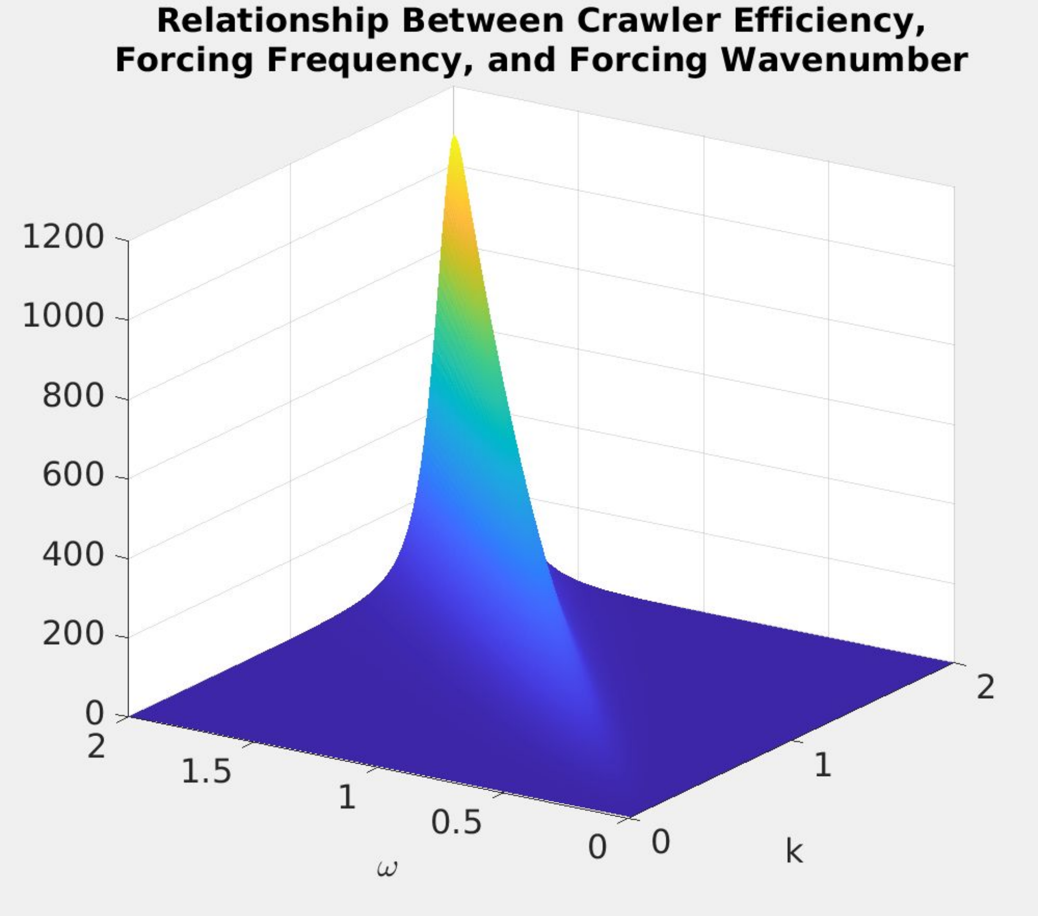
$$\text{Velocity} = \frac{1}{\lambda} \int_0^\lambda u_t \, dx$$

$$||\text{Force Input}||^2 = \int_0^\lambda (f(x,t))^2 \; dx$$

Crawler Efficiency

$$\text{Efficiency} = \frac{||\text{Velocity}||^2}{||\text{Force Input}||^2}$$

Main Result



- Optimal relationship between ω and k is linear
- Optimal frequency is at resonance, $\omega = \omega_n$
- Slope of optimal line depends on other parameters (which are set to unity in this graph)

Conclusions and Future Work

Accomplishments

- Developed Methods for Analyzing More Sophisticated Friction Models
- Uncovered relationship between forcing frequency and wavelength

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Future Work

- Numerical methods for better approximation of strongly nonlinear friction
- Use these results for optimal control of peristaltic crawlers

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