A Simple Expenditure System for Agricultural Commodities

A simple demand system for aggregate agricultural commodities where commodities have one of two forms, staples (S) or non-staples (N), and for which all other income is allocated to a single aggregate materials (M) sector, is given by

Where Qi is the quantity demand of good i, Pi is the price of good i, Y is income, wi is Pi/Pm, x is Y/Pm and Qi(ws,wn,x), i=s,n, are functions.

The system is homogeneous of degree zero in prices (Pi) and income (Y). Scaling all prices and income leave the quantities demanded unaffected. The system exhausts income by virtue of the fact that Qm is determined indirectly as a residual variable.

Price elasticities for s and n can be calculated as

The income elasticity of demand for goods s and n is given by

The price elasticity of demand for goods s and n with respect to Pm is given by

For good M, price elasticities are a bit more complicated. Price and income elasticities are given in Table 1, with .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table . Price and Income Elasticities | | | | |
|  | Ps | Pn | Pm | Y |
| Qs | ss | sn | -ss-sn-s | ssy+sny+s |
| Qn | ns | nn | -ns-nn-n | nsy+nny+n |
| Qm | –(s/m)(1+ss)  – (n/m)ns | –(s/m)sn  – (n/m)(1+nn) | (s/m)ss+sn+hs)+  (n/m)(ns+nn+hn) – 1 | (1–shs –nhn)/m |

This system can be shown to exhibit the neoclassical relationships:

1. , and

Price and income elasticities can be linked back to underlying Hicksian utility-constant, price elasticities through the Slutsky equation,

Where . (Hicksian compensated demand elasticity) and

This adds one additional constraint on the system, namely that ii <0, where, ii = ii - ii.

In addition there are a set of relational constraints, implied by equations through which include:

Price and income elasticities can be defined in terms of ij and i, with N(N-1) degrees of freedom, N being the number of goods in this case. That is, (N-1)2 values for ij can be chosen, and N-1 values for i can be chosen arbitrarily.

With that flexibility it would be possible to choose values for ij and i such that they were consistent with observed behavior.

We could add additional observational constraints such as,

* Absolute food demand rises with income
  + S S + N N < 0.5 (expenditure on food increases with income, prices constant and expenditure on food increases less rapidly with income than expenditures on other things, prices constant)
  + S S + N N < N N (expenditure on food increases less rapidly with income than expenditures on other things, prices constant)
* At high income levels, food demand saturates
* Share of staples in diet declines with income
* Staples are a Giffen good at low income levels.
* Demand for Staples increases when the price of Non-Staples increases

These constraints are observational characteristics and not theoretical requirements.

ALGEBRA NOTES

# Calculate ms Elasticity

* + 1. Qm = (Ym–QsPs-QnPn)/Pm
    2. dQm/dPs = [-Qs-Ps dQs/dPs-PndQn/dPs]/Pm
    3. (dQm/Qm)/(dPs/Ps) = Ps[-Qs-Ps dQs/dPs-Pn dQn/dPs]/QmPm
    4. ms = Ps[-Qs- Qs(Ps/Qs)(dQs/dPs)- Qn(Pn/Qn)(dQn/dPs)]/QmPm
    5. ms = -PsQs- PsQs(Ps/Qs)(dQs/dPs)- PnQn(Ps/Qn)(dQn/dPs)]/QmPm
    6. ms = [-PsQs- PsQs ****ss – PnQn ****ns]/QmPm
    7. ms = –(s/m)(1+****ss) – (n/m)****ns

# Calculation of mm

1. Qm = (Y–QsPs-QnPn)/Pm
2. dQm/dPm = –(Ps/Pm) dQs/dPm–(Pn/Pm) dQn/dPm-((Y–QsPs-QnPn)/Pm2)
3. (dQm/dPm)(Pm/Qm)= –(Pm/Qm)(Ps/Pm) dQs/dPm –(Pm/Qm)(Pn/Pm) dQn/dPm-(Pm/Qm)(Qm/Pm)
4. **mm** = –(Pm/Qm)(Qs/Qs )(Ps/Pm) dQs/dPm –(Pm/Qm)(Qn/Qn) (Pn/Pm) dQn/dPm-1
5. **mm** = –(1/Qm)(QsPs)/Pm) (Pm/Qs) dQs/dPm – (Qn Pn/QmPm) (Pm/Qn) dQn/dPm-1
6. **mm** = –(QsPs/PmQm)sm –(PnQn/PmQm)nm-1
7. **mm** = –(s/m)sm –(s/m)nm-1

# Calculation of my

1. Qm = (Y–QsPs-QnPn)/Pm
2. dQm/dY= (1/Pm)(1– Ps(dQs/dY) -Pn(dQs/dY))
3. dQm/dY = (1/Pm)(1– Ps(dQs/dY) -Pn(dQs/dY))
4. (dQm/dY)(Y/Qm) = (Y/QmPm)(1– Ps(dQs/dY) -Pn(dQs/dY))
5. ****m = (1/m)(1– Ps(dQs/dY) -Pn(dQs/dY))
6. ****m = (1/m)[1– (Qs/Qs)(Y/Y)Ps(dQs/dY) -(Qn/Qn)(Y/Y)Pn(dQs/dY)]
7. ****m = (1/m)[1– Ps (Qs/Y)(Y/Qs) (dQs/dY) -(Qn/Qn)(Y/Y)Pn(dQs/dY)]
8. ****m = (1/m)[1 – s ****s -– n ****n]
9. ****m = (1 – s ****s -– n ****n)/m

# Test all j mj = -m.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Price and Income Elasticities | | | | |
|  | Ps | Pn | Pm | Y |
| Qs | ess | esn | -ess-esn-hs | hs |
| Qn | ens | enn | -ens-enn-hn | hn |
| Qm | –(s/m)(1+ess)– (n/m)ens | –(s/m)esn – (n/m) (1+enn) | (s/m) ess+esn+hs)+ (n/m) (ens+enn+hn ) – 1 | (1 – s hs -– n hn)/m |

1. all j mj = –(s/m)(1+ess)– (n/m)ens –(s/m)esn+ (n/m) (1+enn)]/m +(s/m) ess+esn+hs)+ (n/m) (ens+enn+hn ) – 1
2. all j mj = –(s/m) –(s/m)ess – (n/m)ens –(s/m)esn+ (n/m) +(n/m)enn +(s/m)ess +(s/m)esn+ (s/m) hs + (n/m)ens+ (n/m)enn+(n/m)hn – 1
3. all j mj = –(s/m) - (n/m) + (s/m)hs +(n/m)hn – 1
4. all j mj = –(/m) + (s/m)hs +(n/m)hn

# Test all i i ij = -i.

## First test i=s,n

1. all i i is = sess + nens –s(1+ess)– nens
2. all i i is = -s

## First test i=m

1. all i i ij = s(-ess-esn-hs) + n(-ens-enn-hn ) + m((s/m) ess+esn+hs)+ (n/m) (ens+enn+hn ) – 1)
2. all i i ij = s(-ess-esn-hs) + n(-ens-enn-hn ) + s ess+esn+hs)+ n (ens+enn+hn ) – m
3. all i i im = – m

# Calculate Eij Derivatives

* + 1. E = Eij(wi,x), where wi=Pi/Pm and x=Y/Pm
    2. dEij(wi,x) = (Eij/wj)[(wj/Pj)dPj + (wj/Pm)dPm] + (Eij/x) [(x/Y) dY + (x/Pm) dPm]
    3. dEij(wi,x)/Eij(wi,x) = [(Eij/wj)/Eij(wi,x)] [dPj/Pm–(Pj/Pm2)dPm] + [(Eij/x)/Eij(wi,x)] [dY/Pm-(Y/Pm2)dPm]
    4. [dEij(wi,x)/Eij(wi,x)]/[dPj/Pj] = (Pj/Pm)(Eij/wj)/Eij(wi,x) = eij
    5. [dEij(wi,x)/Eij(wi,x)]/[dY/Y] = (Y/Pm)(Eij/x)/Eij(wi,x) = eijy
    6. [dEij(wi,x)/Eij(wi,x)]/[dPm/Pm] = -(Pj/Pm)(Eij/wj)/Eij(wi,x) - (Y/Pm)(Eij/x)/Eij(wi,x) = -eij - eijy

# Calculation of ms

1. Qm = (Ym–QsPs-QnPn)/Pm
2. dQm/dPs =– Qs/Pm –(Ps/Pm) dQs/dPs–(Pn/Pm) dQn/dPs
3. (dQm/dPs)(Ps/Qm) = – (Ps/Qm)(Qs/Pm) – (Ps/Qm)(Ps/Pm) dQs/dPs – (Ps/Qm)(Pn/Pm) (dQn/dPs)
4. **ms** = –(s/m) – (QsPs/Qm)(Ps/Pm)(dQs/dPs)/Qs – (Ps/Qm)(Pn/Pm) (dQn/dPs)
5. **ms** = –(s/m) – (QsPs/PmQm)(Ps/Qs)(dQs/dPs)– (Ps/Qm)(Pn/Pm) (dQn/dPs)
6. **ms** = –(s/m) – (s/m)(ess)– (Ps/Qm)(Pn/Pm) (dQn/dPs)(Ps/Qn) (Qn/Ps)
7. **ms** = –(s/m) – (s/m)(ess)– (Ps/PmQm)(ens)(PnQn/Ps)
8. **ms** = –(s/m) – (s/m)(ess)– (PnQn/PmQm)(ens)
9. **ms** = –(s/m) – (s/m)(ess)– (n/m)(ens)
10. **ms** = –(s/m) – (s/m)(ess)– (n/m)(ens)
11. **ms** = –[s(1+ess)+ nens)]/m

# Calculation of mm

1. Qm = (Y–QsPs-QnPn)/Pm
2. dQm/dPm = –(Ps/Pm) dQs/dPm–(Pn/Pm) dQn/dPm-((Y–QsPs-QnPn)/Pm2)
3. (dQm/dPm)(Pm/Qm)= –(Pm/Qm)(Ps/Pm) dQs/dPm –(Pm/Qm)(Pn/Pm) dQn/dPm-(Pm/Qm)(Qm/Pm)
4. **mm** = –(Pm/Qm)(Qs/Qs )(Ps/Pm) dQs/dPm –(Pm/Qm)(Qn/Qn) (Pn/Pm) dQn/dPm-1
5. **mm** = –(1/Qm)(QsPs)/Pm) (Pm/Qs) dQs/dPm – (Qn Pn/QmPm) (Pm/Qn) dQn/dPm-1
6. **mm** = –(QsPs/PmQm)sm –(PnQn/PmQm)nm-1
7. **mm** = –(s/m) -ess-esn-hs)– (n/m) (-ens-enn-hn)– 1
8. **mm = (s/m) ess+esn+hs)+ (n/m) (ens+enn+hn) – 1**