Dynamic Control-Aware Noise Modeling for Qiskit (Physically Motivated, Executable, and Calibratable)

1 Scope

These notes specify a physically motivated, control-aware framework intended for implementation and verification in Qiskit. The framework models observable effects of control and noise processes and couples them dynamically. It comprises: (i) stateful system and environment parameters; (ii) control primitives (modulated entangling drive, Ramsey probe with explicit delay, controller confidence update); (iii) algorithmic layers (Bell, GHZ, Grover, QAOA with optional optimization); (iv) measurement parsing with optional SPAM error; and (v) performance metrics tied to a noise model. Control choices influence a driven, damped environment proxy and thereby the noise; the evolving environment feeds back onto subsequent operations. Classical controller metrics guide control selection but do not directly modulate physical noise channels.

Clarification: the environment proxy aggregates effects such as resonator photon population, TLS defect excitation, and low-frequency flux/charge noise; it is not a literal single mode. The model is designed to capture memory effects and control-hardware interaction phenomenologically rather than to simulate bath dynamics from first principles.

2 System and Environment

The register has n qubits. All normalized frequencies $\omega \in [0,1]$ reference a hardware frequency ω_q (physical frequency $\omega\,\omega_q$). Time is in seconds unless noted. Dimensionless rates (e.g., γ_0, c_A) can be converted to physical rates (Hz or rad/s) via a characteristic frequency ω_q or timescale τ_{ref} by multiplying with ω_q (for angular rates) or dividing by τ_{ref} as calibrated. A global factor $\iota \in [0,\infty)$ denotes a quasi-static thermal load or calibration-drift knob; $\iota = 0$ corresponds to a baseline, well-calibrated state.

2.1 Per-Qubit Control and Crosstalk

Per-qubit system drive parameters $(\boldsymbol{\omega}_s, \boldsymbol{\phi}_s, \boldsymbol{A}_s) \in [0,1]^n \times \mathbb{R}^n \times [0,1]^n$ define a local drive Hamiltonian

$$H_{\mathrm{drive}}(t) = \sum_{q=0}^{n-1} A_{s,q}\big[\cos(\omega_{s,q}t + \phi_{s,q})\,\sigma_x^{(q)} + \sin(\omega_{s,q}t + \phi_{s,q})\,\sigma_y^{(q)}\big].$$

This two-quadrature form aligns with microwave control; in an appropriate rotating frame it reduces to an effective single-quadrature picture.

A static linear crosstalk matrix $G \in \mathbb{R}^{n\times n}$ maps intended controls to effective parameters: $\boldsymbol{A}_{\mathrm{eff}} = G_A\,\boldsymbol{A}_s$, $\boldsymbol{\phi}_{\mathrm{eff}} = G_\phi\,\boldsymbol{\phi}_s$, $\boldsymbol{\omega}_{\mathrm{eff}} = G_\omega\,\boldsymbol{\omega}_s$. If amplitude crosstalk dominates, set $G_\phi = I$, $G_\omega = I$ and use $G \equiv G_A$. We assume a static, linear crosstalk model as a first-order approximation. A potential extension could model G as dependent on the drive frequencies or on the environment state A_e. Crosstalk also induces coherent rotations on non-driven qubits; see the noise section for the induced coherent error model.

Entanglement is introduced via digital interactions (e.g., CX or ZX/ZZ rotations) layered with drive-induced rotations.

2.2 Environment as a Driven, Damped Complex Field (Proxy)

The environment proxy $(\boldsymbol{\omega}_e, \boldsymbol{E})$ represents, per qubit/control line, a dominant driven, damped field. For each q, use the complex phasor $E_q = X_{e,q} + iP_{e,q} = A_{e,q}e^{i\phi_{e,q}}$ and the effective complex drive $D_q := A_{\mathrm{eff},q}e^{i\phi_{\mathrm{eff},q}}$ at frequency $\omega_{\mathrm{eff},q}$.

Define instantaneous detuning and relaxation rate (elementwise):

$$\Delta_q := |\omega_{\mathrm{eff},q} - \omega_{e,q}|,$$

$$\Gamma_q(\Delta_q, \iota, A_{\mathrm{eff},q}) := (\Gamma_0 + \Gamma_1\,\iota) \underbrace{(1 + \kappa A_{\mathrm{eff},q}^2)}_{\text{drive-induced heating/power broadening}} \underbrace{e^{-k_\Gamma\,\Delta_q}}_{\text{off-resonant drive less effective}},$$

with example defaults $\Gamma_0 = 0.02$, $\Gamma_1 = 0.12$, $k_\Gamma = 2.0$, $\kappa = 0.5$.

Environment updates over duration Δt:

$$E_q \leftarrow D_q + \big(E_q - D_q\big)\, e^{-\Gamma_q\,\Delta t},$$
$$\omega_{e,q} \leftarrow \omega_{\mathrm{eff},q} + \big(\omega_{e,q} - \omega_{\mathrm{eff},q}\big)\, e^{-\Gamma_q\,\Delta t}.$$

Extract amplitude and phase afterward: $A_{e,q} \leftarrow |E_q|$ clipped to $[0, 1]$, $\phi_{e,q} \leftarrow \arg(E_q)$.

3 Control Primitives and Noise Coupling

Each primitive acts for a duration Δt and triggers environment updates and noise insertion. Maintain an internal controller diagnostic CQ $\in [0, 1]$ (initialized 0); it influences control selection but does not directly appear in physical noise probabilities.

Instantaneous dephasing rate $\gamma_{\phi,q}$ and baseline $T_{1,q}$ define time-dependent noise per qubit q.

Noise channels per segment of duration Δt: - Depolarizing with global and local components:

$$\bar{A}_e := \frac{1}{n}\sum_q A_{e,q}, \quad p_{\mathrm{depol},q} = \mathrm{clip}\Big(p_{\mathrm{dep,glob},0} + c_{\mathrm{dep,glob}}\,\bar{A}_e + p_{\mathrm{dep,loc},0} + c_{\mathrm{dep,loc}}\,A_{e,q},\ 0, 1\Big).$$

- Coherent detuning error per qubit as a Z rotation:

$$R_z(\epsilon_{z,q}), \quad \epsilon_{z,q} := 2\pi\,\Delta_q\,\Delta t.$$

- Coherent crosstalk-induced rotations and parasitic ZZ: For a drive on qubit k with amplitude $A_{\mathrm{eff},k}$,

$$\delta\theta_{j\leftarrow k} = \chi\,G_{jk}\,A_{\mathrm{eff},k}\,\Delta t, \quad R^{(j)}_{x/y}(\delta\theta_{j\leftarrow k}),$$

and for spectator j,

$$R^{(j,k)}_{ZZ}\big(\zeta\,G_{jk}\,A_{\mathrm{eff},k}\,\Delta t\big), \quad \chi, \zeta \geq 0.$$

Aggregate over active k during the segment. - Phase damping per qubit:

$$p_{\phi,q} = 1 - e^{-\gamma_{\phi,q}\,\Delta t}, \quad \gamma_{\phi,q} := \gamma_0 + \gamma_A A_{e,q} + \gamma_\Delta \Delta_q,$$

with defaults $\gamma_0 = 0.01$, $\gamma_A = 0.10$, $\gamma_\Delta = 0.05$. The $\gamma_\Delta \Delta_q$ term models shot-to-shot fluctuations in the AC Stark shift induced by a detuned drive. - Amplitude damping (T1-like) per qubit:

$$p_{\mathrm{amp},q} = 1 - \exp\left(-\Delta t \left(\frac{1}{T_{1,q}} + c_A\, A_{e,q}\right)\right),$$

where $T_{1,q} > 0$ and $c_A \geq 0$.

For segmented primitives with beat duration δt, compute $p_{\phi,q}(\delta t)$, $p_{\mathrm{amp},q}(\delta t)$, $p_{\mathrm{depol},q}$, apply $R_z(\epsilon_{z,q}(\delta t))$, and insert coherent crosstalk rotations and parasitic ZZ per beat from current state.

3.1 Modulated Entangling Drive (MED): Direct CR-Like Model

This primitive models a digital approximation of a CR-like interaction. For higher fidelity, the underlying analog Hamiltonian $H(t)$ could be simulated directly, with this Trotterized version serving as a computationally efficient alternative. The beat duration δt is the Trotter step.

Inputs: $s \in [0, 1]$, beats $T \geq 1$, beat duration $\delta t > 0$, total $\Delta t = T\,\delta t$.

Per beat $t = 0, \ldots, T-1$ and per qubit q define

$$\mathrm{drive\_angle}_{t,q} = s\, A_{\mathrm{eff},q}\, \pi\, \sin\!\big(2\pi\, \omega_{\mathrm{eff},q}\, (t + \tfrac{1}{2})\, \delta t + \phi_{\mathrm{eff},q}\big).$$

For each nearest-neighbor pair $(q, q+1)$, model a simplified CR step driven by control q on target $q+1$:

$$\text{Apply } R_x^{(q)}(\mathrm{drive\_angle}_{t,q}) \text{ and simultaneously on target } q{+}1: \ R_x^{(q+1)}(\kappa_{IX}\, \mathrm{drive\_angle}_{t,q}) \text{ and } R_z^{(q+1)}(\kappa_{ZX}\, \mathrm{drive\_a}$$

with $\kappa_{ZX} \geq 0$ and $\kappa_{IX} \geq 0$.

After each beat, apply: - Depolarizing with $p_{\mathrm{depol},q}$ on 1- and 2-qubit gates, - Per-qubit coherent $R_z(\epsilon_{z,q})$ with $\epsilon_{z,q} = 2\pi\, \Delta_q\, \delta t$, - Coherent crosstalk rotations $\{R^{(j)}_{x/y}(\sum_k \delta\theta_{j\leftarrow k})\}$ and parasitic R_{ZZ} terms, - Per-qubit phase damping with $p_{\phi,q} = 1 - e^{-\gamma_{\phi,q}\delta t}$, - Per-qubit amplitude damping with $p_{\mathrm{amp},q} = 1 - \exp\!\big(-\delta t(1/T_{1,q} + c_A A_{e,q})\big)$.

Controller metric update (per beat; diagnostic only):

$$Q_{\mathrm{beat}} := \min\left\{1,\ (0.75 + 0.25\, s)\, \exp\left(-\frac{1}{n}\sum_q \eta\, \Delta_q\right)\right\},\ \eta = 2.0,$$

$$\mathrm{CQ} \leftarrow \mathrm{clip}\Big(\mathrm{CQ}\, e^{-\bar{\gamma}_\phi \delta t} + (1 - e^{-\bar{\gamma}_\phi \delta t})\, Q_{\mathrm{beat}},\ 0, 1\Big), \quad \bar{\gamma}_\phi = \frac{1}{n}\sum_q \gamma_{\phi,q}.$$

Environment update per beat using the complex-field rule with $\Gamma_q = \Gamma_q(\Delta_q, \iota, A_{\mathrm{eff},q})$ and duration δt.

3.2 Ramsey Probe with Explicit Delay

Inputs: interrogation frequencies $\tilde{\boldsymbol{\omega}} \in [0, 1]^n$ (default $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}_{\mathrm{eff}}$), delay $\tau \geq 0$, pulse duration $\delta t_{\pi/2}$.

Procedure (true Ramsey): 1) Set $\boldsymbol{\omega}_s \leftarrow \tilde{\boldsymbol{\omega}}$. Apply first $\pi/2$ pulse: $\prod_q R_x(\pi/2)^{(q)}$ (duration $\delta t_{\pi/2}$). 2) Free evolution of duration τ; relative phase accumulates at detuning $\delta_q := \tilde{\omega}_q - \omega_{e,q}$ and is revealed by the second pulse. No terminal R_z is applied. 3) Apply second $\pi/2$ pulse with the same phase reference as step 1: $\prod_q R_x(\pi/2)^{(q)}$ (duration $\delta t_{\pi/2}$). The measured expectation values oscillate as $\langle X\rangle_q \sim e^{-(\Gamma_{2,q}\tau)} \cos(2\pi\delta_q\tau)$ and $\langle Y\rangle_q \sim e^{-(\Gamma_{2,q}\tau)} \sin(2\pi\delta_q\tau)$, where $\Gamma_{2,q}$ emerges from the inserted decoherence channels.

Noise: - After each pulse and during the delay, insert depolarizing with $p_{\mathrm{depol},q}$, phase damping $p_{\phi,q} = 1 - e^{-\gamma_{\phi,q}\Delta t_{\mathrm{seg}}}$, amplitude damping $p_{\mathrm{amp},q} = 1 - \exp(-\Delta t_{\mathrm{seg}}(1/T_{1,q} + c_A A_{e,q}))$, coherent detuning $R_z(\epsilon_{z,q})$ with $\epsilon_{z,q} = 2\pi\,\Delta_q\,\Delta t_{\mathrm{seg}}$, and coherent crosstalk rotations induced by the pulses. - Associate time-dependent noise directly to the `delay` instruction of duration τ; apply only dephasing, amplitude damping, and detuning R_z during the delay (no crosstalk if no active drive).

Controller metric during delay (diagnostic):

$$Q_{\mathrm{Ramsey}} := \exp\left(-\frac{1}{n}\sum_q \eta_R|\tilde{\omega}_q - \omega_{e,q}|\right),\ \eta_R = 2.0, \quad \mathrm{CQ} \leftarrow \mathrm{clip}\Big(\mathrm{CQ}\,e^{-\bar{\gamma}_\phi\tau} + (1-e^{-\bar{\gamma}_\phi\tau})\,Q_{\mathrm{Ramsey}},\ 0, 1\Big).$$

Environment updates after each pulse and during free evolution with the corresponding durations using the complex-field rule.

3.3 Controller Confidence Update (Controller Only)

The controller confidence update models an automated calibration agent's confidence about attractor bitstrings. Maintain a confidence $S_b \in [0,1]$ for bitstring $b \in \{0,1\}^n$ in target set $\mathcal{A}$. Let $\epsilon_0 = 0.01$ and $\epsilon := \epsilon_0\,\iota$. Define

$$\lambda_b := \frac{1}{1+\epsilon/\epsilon_0}\,e^{-k_A \bar{A}_e}, \quad k_A = 0.5, \quad \bar{A}_e = \frac{1}{n}\sum_q A_{e,q},$$

$$S_b \leftarrow \mathrm{clip}\big(\lambda_b\,S_b + (1-\lambda_b), 0, 1\big), \quad \mathcal{A} \leftarrow \mathcal{A} \cup \{b\}.$$

Beliefs $\{S_b\}$ guide controller decisions (e.g., choice of $\boldsymbol{\omega}_s$ or algorithm parameters) but do not directly enter physical noise formulas. In a closed-loop simulation, the controller could use ResonanceFidelity (CQ) to trigger recalibration routines; for example, if CQ drops below a threshold, launch a Ramsey probe to update the controller's estimate of $\boldsymbol{\omega}_e$.

4 Algorithmic Layers

Noise inserted per gate or per block duration.

Bell pairs (even n, $m = n/2$):

$$\left(\prod_{i=0}^{m-1} H^{(i)}\right)\left(\prod_{i=0}^{m-1} \mathrm{CX}_{i,\,i+m}\right).$$

GHZ:

$$H^{(0)}\prod_{i=0}^{n-2} \mathrm{CX}_{i,\,i+1}.$$

Grover (marked $11\ldots1$): initialize with Hadamards; per iteration apply the phase oracle on $11\ldots1$ and diffusion; iterations

$$R = \max\Big(1,\ \Big\lfloor \frac{\pi}{4}\sqrt{2^n}\Big\rfloor\Big).$$

QAOA (MaxCut ring), depth $p = 2$ default:

$$\text{init } H^{\otimes n}; \quad \text{for } \ell = 1, 2: \left(\prod_{q=0}^{n-2} R_{ZZ}(\gamma_\ell)_{q,q+1}\right)\left(\prod_{q=0}^{n-1} R_x(2\beta_\ell)^{(q)}\right),$$

with heuristics $\gamma_\ell = 2\pi\bar{\omega}_{\text{eff}}$ and $\beta_\ell = \frac{\pi}{4\ell}$, $\bar{\omega}_{\text{eff}} = \frac{1}{n}\sum_q \omega_{\text{eff},q}$. Optionally optimize $(\gamma_\ell, \beta_\ell)$ to maximize $\mathcal{F}_{\text{QAOA}}$ or a scalarized control objective. A primary goal is to track the optimal (γ, β) as a function of the evolving environment state $(\boldsymbol{\omega}_e, \boldsymbol{A}_e)$.

5 Measurement and Parsing

Measurement returns counts; parser strips whitespace and maps to $\{0,1\}^L$. For a data register of length k use

$$\text{tail}_k(x) := \text{last } k \text{ bits of } x.$$

Endianness: rightmost bit is least significant in Qiskit. Optional SPAM: independently flip each bit with probability $p_{\text{spam}} \in [0,1]$ before computing metrics.

6 Metrics

Let parsed counts $C : \{0,1\}^L \to \mathbb{N}$ with $N = \sum_x C(x)$; if $N = 0$, metrics are 0.

Alignment (entropy reduction):

$$p_x = C(x)/N, \quad H(C) = -\sum_{x:C(x)>0} p_x \log_2 p_x, \quad H_{\max} = n, \quad \text{Align} := 1 - \frac{H(C)}{H_{\max}} \ (0/0 := 0).$$

Stability for target subspace $\mathcal{A} \subseteq \{0,1\}^k$:

$$\text{Stab} := \begin{cases} \frac{1}{N}\sum_x C(x)\, \mathbf{1}\{\text{tail}_k(x) \in \mathcal{A}\}, & \mathcal{A} \neq \varnothing, \\ 0, & \mathcal{A} = \varnothing. \end{cases}$$

ResonanceFidelity (diagnostic; replaces CQ): internal CQ $\in [0,1]$.

EnvironmentQuiescence: EnvQ $:= \text{clip}(1 - \bar{A}_e, 0, 1)$ with $\bar{A}_e = \frac{1}{n}\sum_q A_{e,q}$.

Vector performance for analysis:

$$\boldsymbol{M} := \big(\text{Align},\ \text{Stab},\ \text{CQ},\ \text{EnvQ}\big).$$

Scalarized objective (default weighted sum) for optimization:

$$\text{EffCtrl} := \text{clip}\Big(w_1\,\text{Align} + w_2\,\text{Stab} + w_3\,\text{CQ} + w_4\,\text{EnvQ},\ 0, 1\Big),$$

with defaults $(w_1, w_2, w_3, w_4) = (0.4, 0.4, 0.15, 0.15)$. Multiplicative alternative:

$$\text{EffCtrl}_\Pi := \Big(\max\{10^{-6}, \text{Align}\} \cdot \max\{10^{-6}, \text{Stab}\} \cdot \max\{10^{-6}, \text{CQ}\} \cdot \max\{10^{-6}, \text{EnvQ}\}\Big)^{1/4}.$$

7 Task-Oriented Fidelities

GHZ, target $\mathcal{T} = \{00\ldots0,\ 11\ldots1\}$:

$$\mathcal{F}_{\text{GHZ}} := \frac{1}{N}\sum_x C(x)\, \mathbf{1}\{\text{tail}_n(x) \in \mathcal{T}\}.$$

Bell pairs (even n, $m = n/2$, pairs $(i, i+m)$). For $x \in \{0,1\}^n$:

$$p_{ab}^{(i)} := \frac{1}{N}\sum_x C(x)\,\mathbf{1}\{x_i = a,\ x_{i+m} = b\},\ a,b \in \{0,1\}.$$

Per-pair fidelity

$$\mathcal{F}_{\text{pair}}^{(i)} := p_{00}^{(i)} + p_{11}^{(i)}, \quad \mathcal{F}_{\text{BellPairs}} := \frac{1}{m}\sum_{i=0}^{m-1} \mathcal{F}_{\text{pair}}^{(i)}.$$

Grover (marked $w = 11\ldots1$):

$$\mathcal{F}_{\text{Grover}} := \frac{1}{N}\sum_x C(x)\,\mathbf{1}\{\text{tail}_n(x) = w\}.$$

QAOA (MaxCut ring; edges $(q, q+1)$, $q = 0, \ldots, n-2$):

$$\langle \widehat{Z_q Z_{q+1}} \rangle = \frac{1}{N}\sum_x C(x)\,(-1)^{x_q \oplus x_{q+1}}, \quad \mathcal{F}_{\text{QAOA}} := \frac{1}{n-1}\sum_{q=0}^{n-2}\left(\frac{1 - \langle \widehat{Z_q Z_{q+1}} \rangle}{2}\right).$$

8 Execution Template

A typical run: (i) MED with chosen $s, T, \delta t$; (ii) Ramsey at $\boldsymbol{\omega}_{\text{eff}}$ with delay τ; (iii) controller confidence updates (e.g., 0^n and 1^n); (iv) an algorithm layer (optionally optimize QAOA and track (γ, β) vs. $(\boldsymbol{\omega}_e, \boldsymbol{A}_e)$); (v) optional interleaved MED or probes; (vi) after each segment, insert noise parameterized by current $(\boldsymbol{\gamma}_\phi, \boldsymbol{T}_1, \boldsymbol{A}_e, G_A, \boldsymbol{A}_{\text{eff}})$; apply coherent R_z errors, coherent crosstalk rotations, and parasitic ZZ; measure per shot; apply optional SPAM; and (vii) compute $\boldsymbol{M} = (\text{Align}, \text{Stab}, \text{CQ}, \text{EnvQ})$, EffCtrl, and task fidelities. In closed loop, use low CQ to trigger recalibration (e.g., a Ramsey probe to update $\boldsymbol{\omega}_e$).

9 Assumptions and Conventions

- Frequency normalization: $\omega \mapsto \omega\,\omega_q$. - Endianness: use tail_k for lower-order bits. - Clipping: $\text{clip}(x, 0, 1) = \min\{1, \max\{0, x\}\}$. - Internal state: $\mathcal{A}$ accumulates attractors; S_b are controller stabilization confidences. - Stochasticity: from sampling, optional SPAM, and simulator randomness. - Coefficients: all numeric constants are calibration knobs with roles noted parenthetically above.

10 Proposed Qiskit Implementation Architecture

Implement a Qiskit `BackendV2`-compatible fake backend class `ControlAwareBackend` encapsulating per-qubit $(\boldsymbol{\omega}_s, \boldsymbol{\phi}_s, \boldsymbol{A}_s)$, effective parameters $(\boldsymbol{\omega}_{\text{eff}}, \boldsymbol{\phi}_{\text{eff}}, \boldsymbol{A}_{\text{eff}})$ via (G_ω, G_ϕ, G_A), environment $(\boldsymbol{\omega}_e, \boldsymbol{E})$, ι, G_A, $T_{1,q}$, and $(\text{CQ}, \mathcal{A}, \{S_b\})$. - Front-end orchestrator: `QuantumOrchestrator` to build circuits with segments: `med(s,T,delta_t)`, `ramsey(tilde_omega_vec,tau,t_pi2)`, `update_confidence(b)`, `run_ghz()`, `run_bell()`, `run_grover()`, `run_qaoa(p, params=None, optimize=False)`. Use `transpile(..., backend=ControlAwareBackend())`. - Backend `run` responsibilities: 1) Parse circuit, detect segment barriers, 2) For each segment/beat, compute $(\{p_{\phi,q}\}, \{p_{\text{amp},q}\}, \{p_{\text{depol},q}\}, \{\epsilon_{z,q}\}, \{\delta\theta_{j\leftarrow k}\}, \{\zeta G_{jk} A_{\text{eff},k}\})$, 3) Update environment via complex-field rule for the segment duration, 4) Inject coherent rotations (R_z, crosstalk $R_{x/y}$, parasitic R_{ZZ}) and Kraus errors; attach noise to `delay` instructions, 5) Update CQ via primitive-specific rule (diagnostic and available to the controller), 6) Execute on a density-matrix or shot-based simulator. - Performance: the most performant approach is a custom transpiler pass that iterates through the circuit, computes state-dependent error parameters per instruction/delay, and inserts concrete `QuantumError`/Kraus operators and coherent rotations directly into the circuit, avoiding repeated `NoiseModel` construction at runtime.

11 Verification Plan

- Environment: relaxation increases with ι and A_{eff}; decays with detuning Δ_q; amplitude and phase co-relax via complex update; frequency tracks drive with same rate. - Noise: $p_{\phi,q}$ increases with $A_{e,q}$ and detuning; amplitude damping rises with $A_{e,q}$ and duration while preserving baseline $T_{1,q}$; $p_{\text{depol},q}$ grows with both local $A_{e,q}$ and global $\bar{A}_e$; coherent R_z captures detuning phase; crosstalk includes spectator rotations and parasitic ZZ. - Ramsey: oscillations in $\langle X \rangle, \langle Y \rangle$ vs. τ at detuning; environment drifts during τ; noise active on `delay`; no terminal R_z is inserted. - MED: CR-like ZX and IX terms proportional to control-qubit drive amplitude; crosstalk yields coherent spectator rotations and parasitic ZZ. - Algorithms: GHZ mass on $\{0^n, 1^n\}$; Bell pairs correlated; Grover peaks at $11\ldots1$; QAOA parameters shift as $(\boldsymbol{\omega}_e, \boldsymbol{A}_e)$ evolve. - Metrics: vector $\boldsymbol{M}$ exposes trade-offs; CQ (ResonanceFidelity) improves near-resonant $\boldsymbol{\omega}_{\text{eff}} \approx \boldsymbol{\omega}_e$; multiplicative EffCtrl_{Π} penalizes weak components.